Gluon Distributions and Fits Using Dipole Cross-Sections

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Currently much work on calculating/modelling dipole cross-sections and fitting to structure function data. Variety of approaches match data very well.

However, picture of steeply growing quantities at small x tamed by saturation in conflict with DGLAP picture of small/negative gluon ($F_L(x, Q^2)$) at small x, Q^2. Need to understand this.

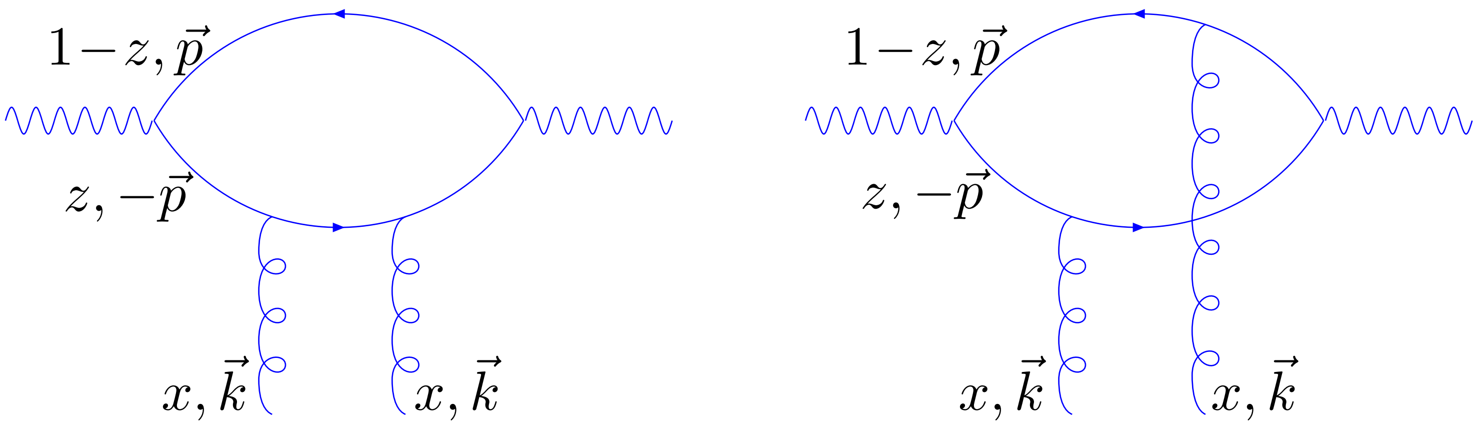
Approach – QCD factorization theory is correct and quantitative at high enough Q^2 (perhaps not too small x). Work back to dipole cross-sections.

Are dipole motivated fits truly quantitative? Are results to be taken too seriously in detail?

Does a large/steep dipole cross-section mean a large/steep gluon distribution?

Evidence for saturation really only a side issue.

Relationship between dipole cross-section and gluon distribution.



Within LO k_T-factorization theory can write $\gamma^\star p$ cross-section (Bialas, Navelet, Peschanski) as

$$\sigma_L \propto \int_0^1 dz[z(1-z)]^2 \int \frac{d^2k}{k^4} \int d^2p \left(\frac{1}{\hat{Q}^2 + p^2} - \frac{1}{\hat{Q}^2 + (p+k)^2} \right)^2 f(x, k^2)$$

where $f(x, k^2)$ is the unintegrated gluon distribution, $\hat{Q}^2 = z(1-z)Q^2$. A similar result holds for σ_T.

In the limit $x \to 0$, i.e LO in the k_T-factorization theory, this formula can be simplified.

Integrating over z and p we have the standard k_T-factorization expression

$$F(x, Q^2) = \int \frac{d^2k}{k^2} \frac{\alpha_S 2N_f}{6\pi} h(k^2/Q^2) f(x, k^2).$$

Taking the double Mellin transformation $\int dQ^2 Q^{2-2\gamma}$ and $\int dx\, x^N$ we have the familiar expression,

$$\tilde{F}(N, \gamma) = \frac{\alpha_S 2N_f}{6\pi} \tilde{h}(\gamma) \tilde{f}(N, \gamma)/\gamma \equiv \alpha_S \tilde{h}(\gamma) \tilde{g}(N, \gamma),$$

where $g(x, Q^2) = \int_0^{Q^2} \frac{d^2k}{k^2} f(x, k^2)$ is the integrated gluon distribution.

If $g(N, Q^2) \sim (Q^2)^{\gamma(\alpha_S, N)}$

$$F(N, Q^2) = \frac{\alpha_S 2N_f}{6\pi} h(\gamma(\alpha_S, N)) g(N, Q^2)$$

$$\to F(x, Q^2) = \frac{\alpha_S 2N_f}{6\pi} h(\gamma(\alpha_S, \ln(1/x))) \otimes g(x, Q^2).$$

Alternatively using the identity

$$\frac{1}{\hat{Q}^2+p^2} = \frac{1}{2\pi}\int d^2r\,\exp(ip\cdot r)K_0(\hat{Q}r)$$

integrating over p^2, and letting $x \to 0$ one can equivalently write

$$\sigma = \frac{2\pi}{3}\int_0^1 dz \int d^2r|\Psi(r,z,Q)|^2 \int \frac{d^2k}{k^4}\alpha_S f(x,k^2)(1-J_0(kr)).$$

This can be interpreted as

$$\sigma = \int_0^1 dz \int d^2r|\Psi(r,z,Q)|^2\hat{\sigma}(x,r^2),$$

where $\hat{\sigma}(x,r^2) = \frac{2\pi}{3}\int \frac{d^2k}{k^4}\alpha_S f(x,k^2)(1-J_0(kr))$ is the dipole-proton cross-section. In the small r^2 limit it is often written as

$$\hat{\sigma}(x,r^2) = \frac{2\pi}{3}r^2\int \frac{d^2k}{k^2}\alpha_S f(x,\mu^2) \sim \frac{2\pi\alpha_S}{3}r^2 g(x,\mu^2), \qquad \mu^2 \approx 10/r^2.$$

This is only approximate and only reasonable for very small r.

We previously had

$$F(N,Q^2) = \frac{\alpha_S 2N_f}{6\pi} h(\gamma(\alpha_S,N))g(N,Q^2)$$

Taking Mellin transformations of intermediate expressions $\rightarrow$

$$F_i(N,Q^2) = \frac{\alpha_S 2N_f}{6\pi} h_{id}(\gamma(\alpha_S,N))h_{dg}(\gamma(\alpha_S,N)g(N,Q^2)$$

$$\equiv \frac{\alpha_S 2N_f}{6\pi} h_i(\gamma(\alpha_S,N)) \otimes g(N,Q^2).$$

So the effective coefficient function for the hard cross-section $h_i(\gamma(\alpha_S,N))$ is the product of a photon-dipole coefficient function $h_{id}(\gamma(\alpha_S,N))$ and a dipole-gluon coefficient function $h_{dg}(\gamma(\alpha_S,N))$, both of which are calculable. For $dF_2/d\ln Q^2$

$$h_{2d}(\gamma) = \frac{(1+3/2\gamma-3/2\gamma^2)}{1-\gamma} \frac{4^{-\gamma}\Gamma^4(2-\gamma)\Gamma^2(1+\gamma)}{\Gamma(4-2\gamma)\Gamma(2+2\gamma)} \qquad h_{dg}(\gamma) = \frac{4^{\gamma}\Gamma(1+\gamma)}{(1-\gamma)\Gamma(2-\gamma)}$$

$$h_{2g}(\gamma) = \frac{3/2(2-3\gamma+3\gamma^2)}{3-2\gamma} \frac{\Gamma^3(1+\gamma)\Gamma^3(1-\gamma)}{\Gamma(2+2\gamma)\Gamma(2-2\gamma)}.$$

For example, in LO BFKL

$$\gamma(\alpha_S/N) = \frac{\bar{\alpha}_S}{N} + 2.4(\frac{\bar{\alpha}_S}{N})^4 + 2(\frac{\bar{\alpha}_S}{N})^6 + 17(\frac{\bar{\alpha}_S}{N})^7 + \cdots .$$

For $dF_2/d\ln Q^2$,

$$h_2(\gamma(\bar{\alpha}_S/N) = 1 + 2.17\gamma + 2.30\gamma^2 + 5.07\gamma^3 + 3.58\gamma^4 + 8.00\gamma^5 + \cdots$$

This leads to a steep growth of $dF_2/d\ln Q^2$ even relative to the gluon. How is this broken down. Find

$$h_{dg}(\gamma(\bar{\alpha}_S/N) = 1 + 2.23\gamma + 3.49\gamma^2 + 3.95\gamma^3 + 4.22\gamma^4 + 4.06\gamma^5 + \cdots$$

$$h_{2d}(\gamma(\bar{\alpha}_S/N) = 1 - 0.07\gamma - 1.05\gamma^2 + 3.77\gamma^3 - 4.94\gamma^4 + 6.53\gamma^5 + \cdots$$

Approx all growth of $dF_2/d\ln Q^2$ relative to $g(x,Q^2)$ generated by dipole-gluon cross-section, which is itself steep relative to gluon.

Calculating $f_g(x,k^2)$ and $xg(x,Q^2)$ taking e.g. simple GBW model within this picture (too simplistic for $xg(x,Q^2)$ since higher twists involved with saturation),

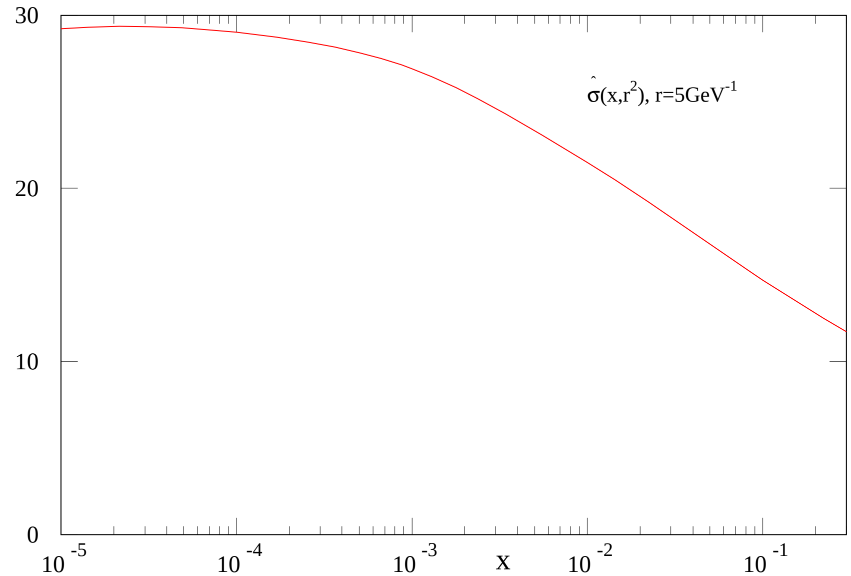
$$\hat{\sigma}(x,r^2) = \sigma_0(1 - \exp(r^2/4(x_0/x)^\lambda)) \rightarrow$$

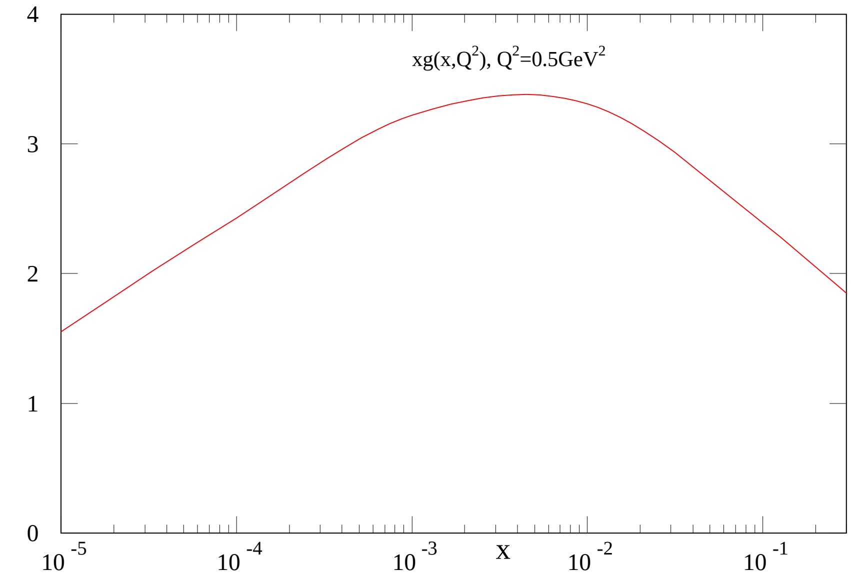
$$f_g(x,k^2) = \frac{3\sigma_0}{4\pi^2\alpha_S} k^4 (x/x_0)^\lambda e^{-k^2(x/x_0)^\lambda}.$$

$$xg(x,Q^2) = \frac{3\sigma_0}{4\pi^2\alpha_S}\Big[-Q^2 e^{-Q^2(x/x_0)^\lambda} + (x_0/x)^\lambda(1 - e^{-Q^2(x/x_0)^\lambda})\Big].$$

$\sigma_0 = 29.2mb, x_0 = 4 \times 10^{-5}, \lambda = 0.28.$

So, flat $\hat{\sigma}(x,r^2)$ must come from valence-like $xg(x,Q^2)$ and $f_g(x,k^2)$.





Phenomenology

How does this work out in practice? Perform fit to data from starting point of a well-defined gluon distribution. Use ($\eta = \log(1/x)$)

$$xg(x,Q^2) = A\left(\frac{5}{5+\eta}\right)^2 \exp\left(-1.5\log\left(\frac{\log((Q^2+Q_0^2)/\Lambda^2_{QCD})}{\log((Q_0^2)/\Lambda^2_{QCD})}\right)\right)$$

$$\times\left(I_0\left(2.4\left(\frac{\eta^2(1-\exp(-\eta/4))}{\eta+2.3}(1-\exp(-\eta))^4\log\left(\frac{\log((Q^2+Q_0^2)/\Lambda^2_{QCD})}{\log((Q_0^2)/\Lambda^2_{QCD})}\right)\right)^{0.5}\right)-1\right).$$

This is basically the *double asymptotic scaling* solution (Ball, Forte) to gluon evolution with $\gamma_{gg}(\alpha_s(Q^2),N) = \bar{\alpha}_s(Q^2)(1/N-1)$.

$$g(x,Q^2) \propto I_0\left(\left(2.4\eta\log\left(\frac{\log((Q^2)/\Lambda^2_{QCD})}{\log((Q_0^2)/\Lambda^2_{QCD})}\right)\right)^{0.5}\right)\exp\left(-1.5\log\left(\frac{\log((Q^2)/\Lambda^2_{QCD})}{\log((Q_0^2)/\Lambda^2_{QCD})}\right)\right).$$

with a modification for the $(1-x)^n$ high-x behaviour, and Q_0^2 introduced to allow a smooth approach to $Q^2=0$. $A\left(\frac{5}{5+\eta}\right)^2$ is the *input*, with A the normalization.

$\Lambda_{QCD} = 0.12\text{GeV}, Q_0^2 = 0.5\text{GeV}^2$.

Q_0^2 and the input $(5/(5+\eta))^2$ are chosen to give roughly the right shape. The whole is then the same shape in x as a standard LO or NLO gluon distribution and for $Q^2 \gg Q_0^2$ evolves in a quantitatively correct way. Clearly for $Q^2 \sim Q_0^2$ the evolution is slowed, but in an x-independent way – just $Q^2 \to Q^2 + Q_0^2$. $\alpha_S(\mu^2)$ is also slowed with same scale.

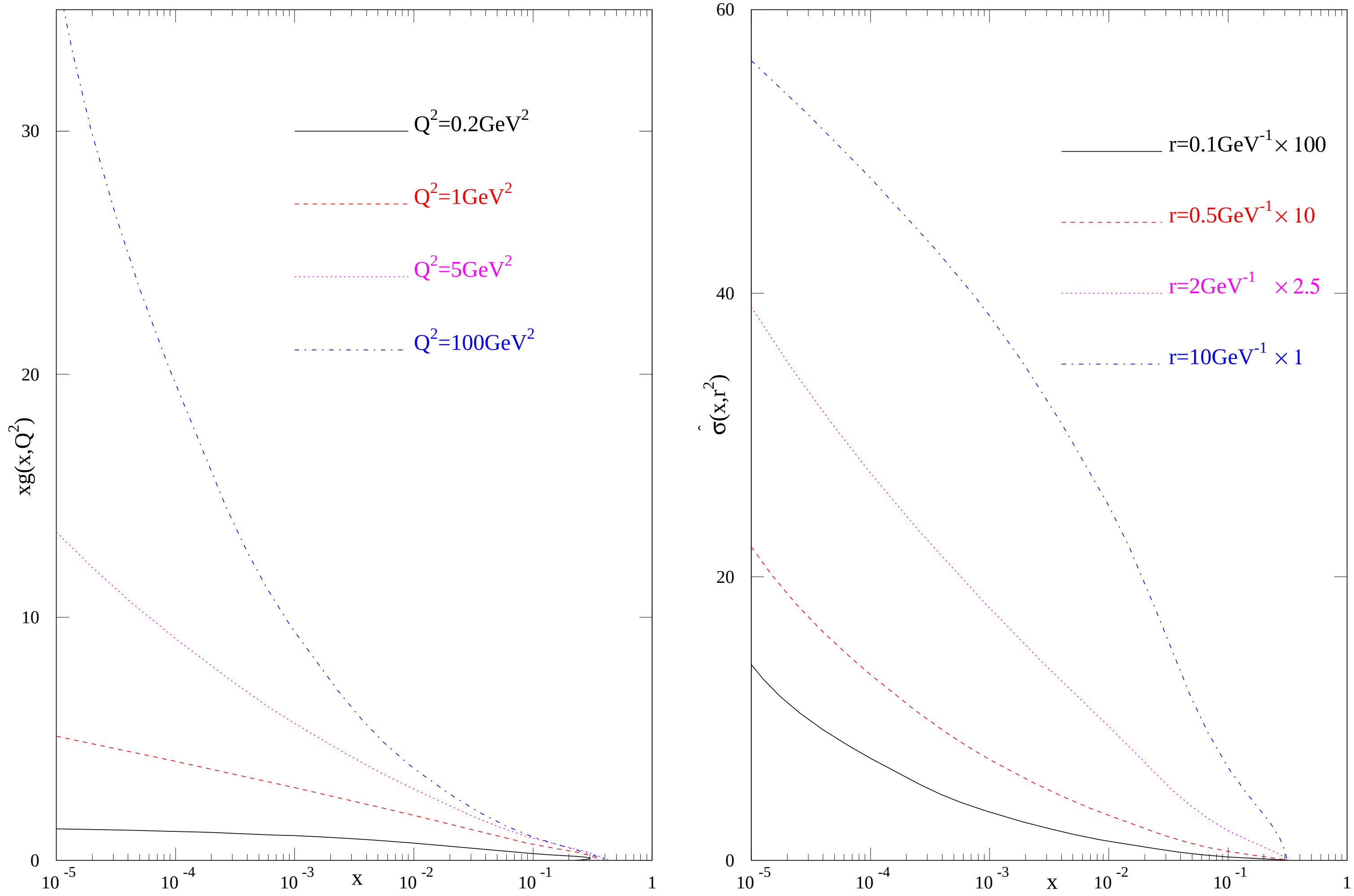
$$\alpha_S(\mu^2) = \frac{4\pi}{\beta_0 \log((\mu^2 + Q_0^2)/\Lambda_{QCD}^2)}.$$

This expression for the gluon is converted into a dipole cross-section using

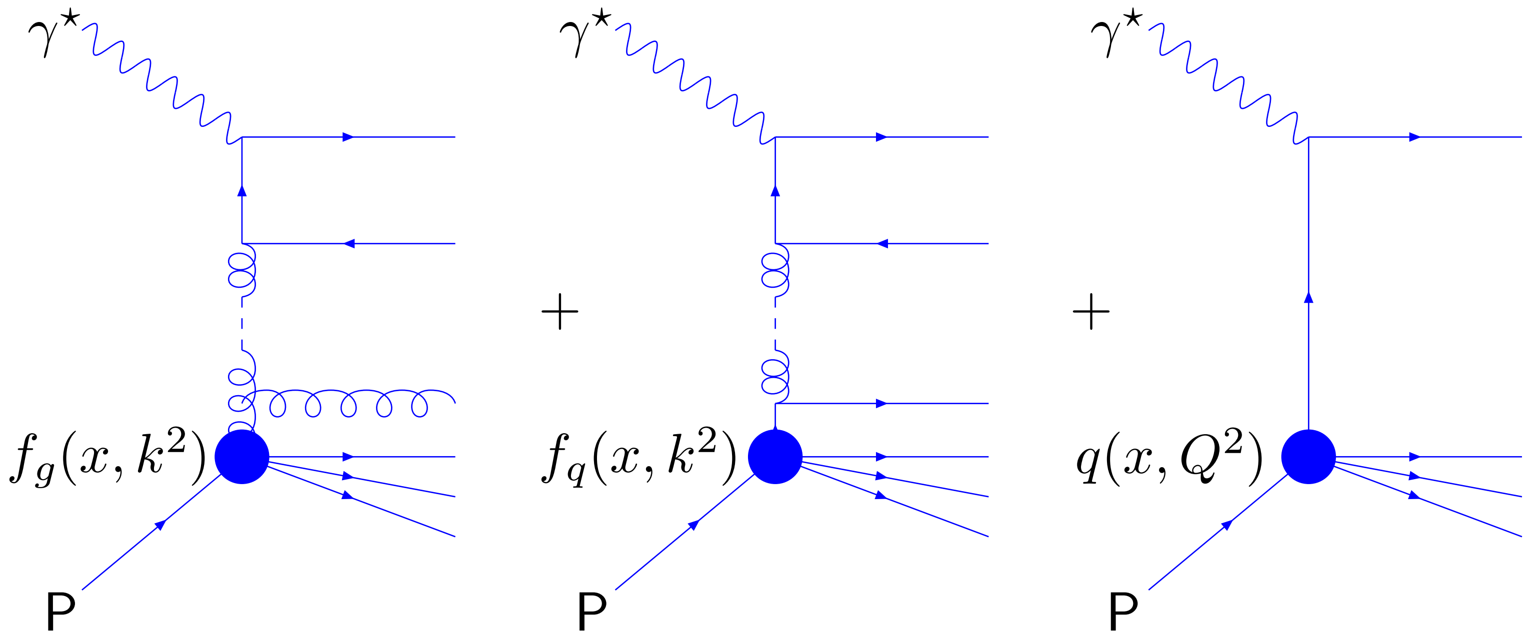
$$\hat{\sigma}(x, r^2) = \frac{2\pi}{3} \int \frac{d^2k}{k^4} \alpha_S(k^2) f(x, k^2)(1 - J_0(kr)),$$

and put into a fit to data. A is the only real free parameter associated with the gluon.

Comparison of $xg(x,Q^2)$ at various Q^2 with $\hat{\sigma}(x,r^2)$ at various r. Clearly the dipole cross-section is generally steeper, particularly comparing $Q^2 = 0.2\mathrm{GeV}^2$ with $r = 10\mathrm{GeV}^{-1}$.



Details of fit. Really 3 types of diagram enter.



In the LO k_T-factorization theorem the first two diagrams contribute as $f_g(x,k^2) + 4/9 f_S(x,k^2)$, i.e. really not just gluon contributing to dipole cross-section.

Also photon scattering from nonperturbative quark. Include as $f \times Q^2/(Q^2 + Q_0^2)$, where f free. In practice very small.

Also mass of light quarks $m_q \sim 90\text{MeV}$ in wavefunctions. Not very sensitive.

Very Important - Heavy quarks.

Often ignored in dipole fits. But charm about 40% of $dF_2/d\ln Q^2$ for $Q^2 > m_c^2$.

Omission $\rightarrow \hat{\sigma}(x, r^2), g(x, Q^2)$ up to 1.67 times too big. Saturation corrections $\propto g^2(x, Q^2)$ enormously exaggerated.

Try DGLAP fit without heavy quark contribution to $F(x, Q^2)$. Gluon bigger? α_S bigger?

Well....yes. But global fit terrible – $\chi^2 = 2$ per point for 2000 points. Cannot get $dF_2/d\ln Q^2$ consistently correct at all.

NLO and NNLO DGLAP good enough and constraining enough to say that charm has to be there.

Very nervous of good qualitative results (e.g. geometric scaling) present without heavy quarks. Should be wrong until corrected?

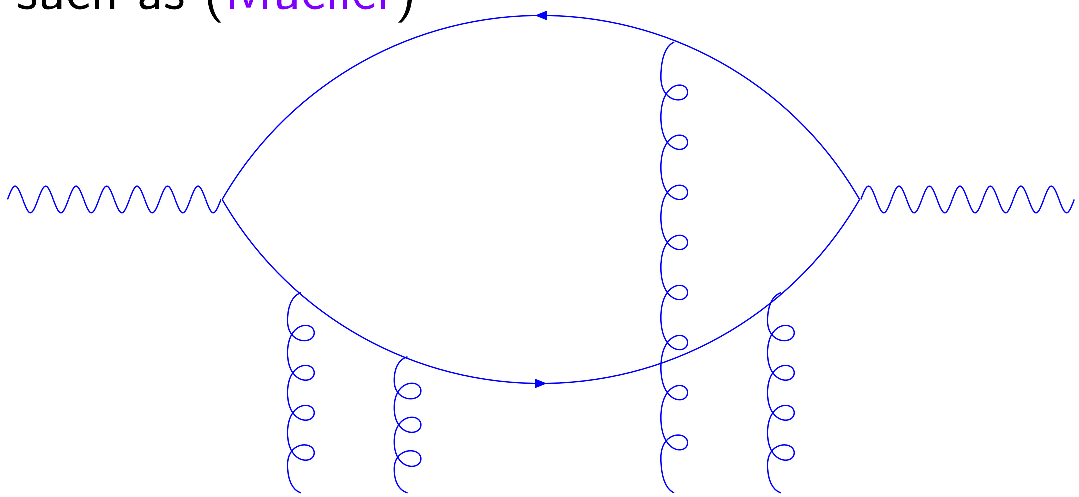
In fit $m_c = 1.3\mathrm{GeV}$.

Fit H1, ZEUS and E665 data $0.5\mathrm{GeV}^2 \leq Q^2 \leq 50\mathrm{GeV}^2$.

Does not work for $Q^2 < 0.5\mathrm{GeV}^2$. Real nonperturbative region?

Does not work well for $Q^2 > 50\mathrm{GeV}^2$. Discuss later.

Try also $\hat{\sigma}(x, r^2) = \sigma_0(1 - \exp(-\hat{\sigma}_{simp}(x, r^2)/\sigma_0)$. Similar to GBW. Comes about from processes such as (Mueller)



i.e. multiple dipole scattering. Nothing to do with gluon saturation?

Fit best if $\sigma_0 > 60mb$ ($R_p = 1fm \rightarrow 60mb$), with A varying by $< 3\%$.

Quality of fit $\chi^2 = 1.1$ per point. Very good for 3 different data sets.
Simple GBW dipole model $\chi^2 = 2.5$ per point.

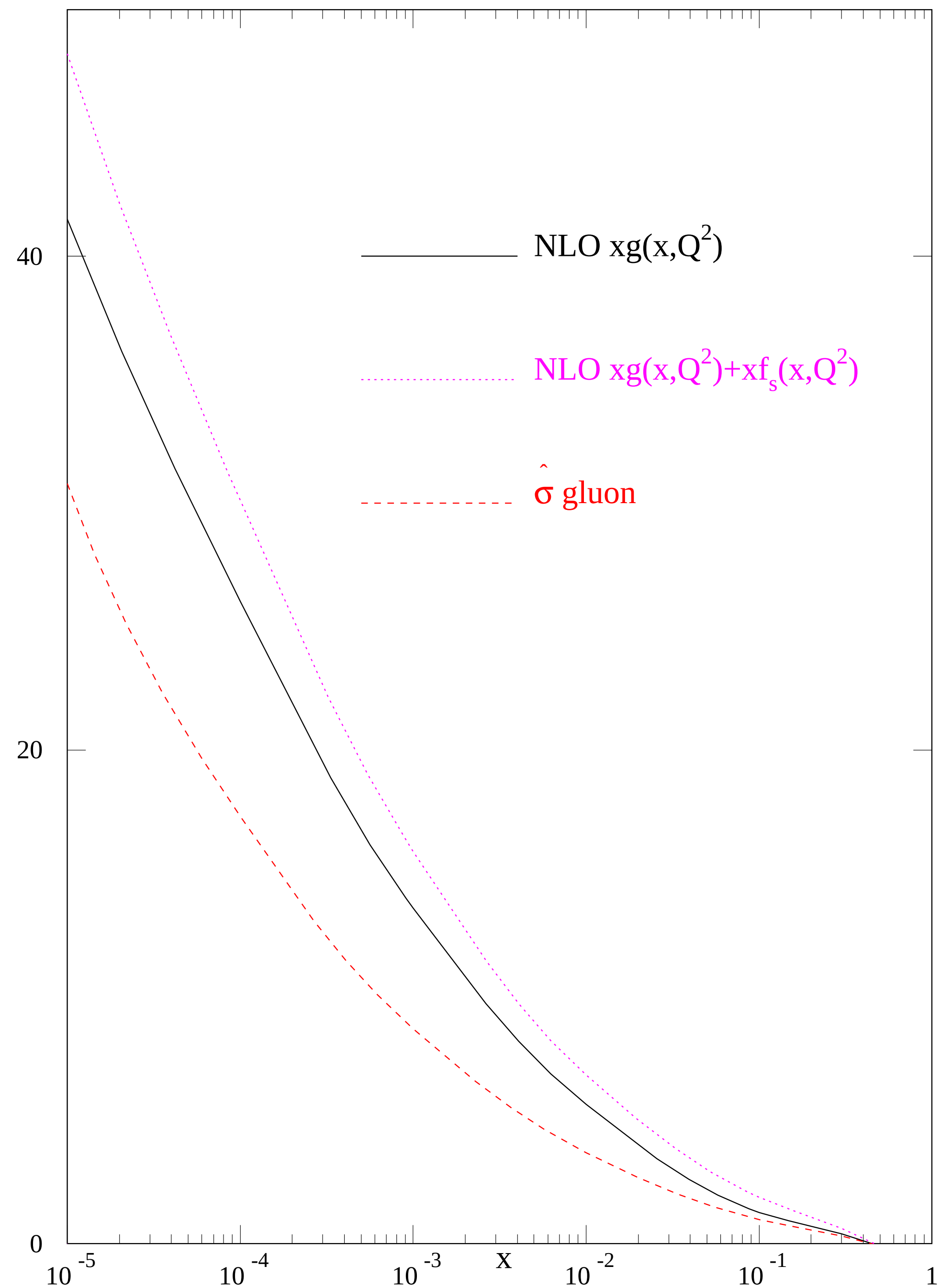
Value of $A = 10.3 \rightarrow$ gluon shown.

Compared to MRSTNLO gluon. Approx $0.65 - 0.75$ of this.

Should really be compared to $g(x, Q^2) + 4/9 f_S(x, Q^2)$. (MRSTNLO gluon relatively small). Biggest change at high x. Factor now $0.5 - 0.65$.

Does not match onto standard $DGLAP$ gluon at high Q^2.

At low Q^2 much smaller than DGLAP at moderate x but eventually bigger at very small x.



Can we understand this relative behaviour? Shouldn't they match at high Q^2?

Comes from effective coefficient functions/splitting functions.

Consider $dF_2/d\ln Q^2$ controlled by $\gamma^{DIS}(\alpha_S(Q^2), N)$.
For my model $\gamma_{gg}(\alpha_s(Q^2), N) = \bar{\alpha}_s(Q^2)(1/N - 1)$ (correct for DGLAP).

$$\gamma^{DIS}_{dip}(\alpha_S(Q^2), N) \approx \frac{\alpha_S(Q^2) 2N_f}{6\pi}\left(1+2.17\bar{\alpha}_S(Q^2)\left(\frac{1}{N}-1\right)+2.30\bar{\alpha}^2_S(Q^2)\left(\frac{1}{N}-1\right)^2+\cdots\right)$$

At highish Q^2 and moderate x first two terms most important. But exact result

$$\gamma^{DIS}_{exact}(\alpha_S(Q^2), N) = \frac{\alpha_S(Q^2) 2N_f}{6\pi}\left(1-1.08N+\cdots+2.17\bar{\alpha}_S(Q^2)\left(\frac{1}{N}-3.05+\cdots\right)+\cdots\right).$$

First terms a lot bigger in dipole approach $\rightarrow$ gluon smaller at moderate x. Terms more similar at smaller x (smaller N).

Affects even smaller x. Higher orders in α_S contain terms in $1/N^m \rightarrow \ln^{m-1}(1/x)$. Appear like

$$\int_x^1 \frac{dz}{z} \alpha_S^n \ln^m(1/z)\, g(x/z, Q^2),$$

so large contribution from $\ln^m(1/z)$ from largest x in $g(x, Q^2)$, where it is much smaller than it should be.

Difference in dipole and DGLAP gluons largely due to effective splitting functions. Verified by modifying DGLAP splitting functions directly in normal global fit $\rightarrow$ dipole-like gluon.

At $Q^2 > 50\text{GeV}^2$ contribution at moderate x just too large for normal gluon evolution. At $x \sim 0.01$ gluon must fall with Q^2 (incorrect) to fit data (achieved accidentally in some dipole models).

But very good fit at $Q^2 \sim 20 - 50\text{GeV}^2$ achieved with wrong gluon. Terms in splitting function by no means negligible at $Q^2 \sim 1\text{GeV}^2$. Perhaps DGLAP gluons not accurate here, but accuracy of dipole fits spoiled. Error in $\mathcal{O}(\alpha_S)$ and $\mathcal{O}(\alpha_S^2)$ more important as gluons get flatter.

Conclusions

Must use heavy quarks in all fits! (Odd if they make things worse.)

Dipole cross-section is considerably steeper than gluon distribution.

Gluon for good fit is small and not too steep at low Q^2. For $x > 10^{-5}$ never have $\frac{\alpha_s(Q^2)\pi x g(x,Q^2)}{Q^2 R_p^2} \sim 1$, i.e. naive saturation requirement, for sensible values of R_p.

Predicted cross-sections too big at small x for $Q^2 < 0.5\mathrm{GeV}^2$. Some reduction necessary here (possibly saturation).

However, extracted gluon too small to match to DGLAP. Fail at high Q^2. Due to inaccuracies in effective splitting functions. Affects size and shape of gluon at all x. Same problems exist at smaller Q^2.

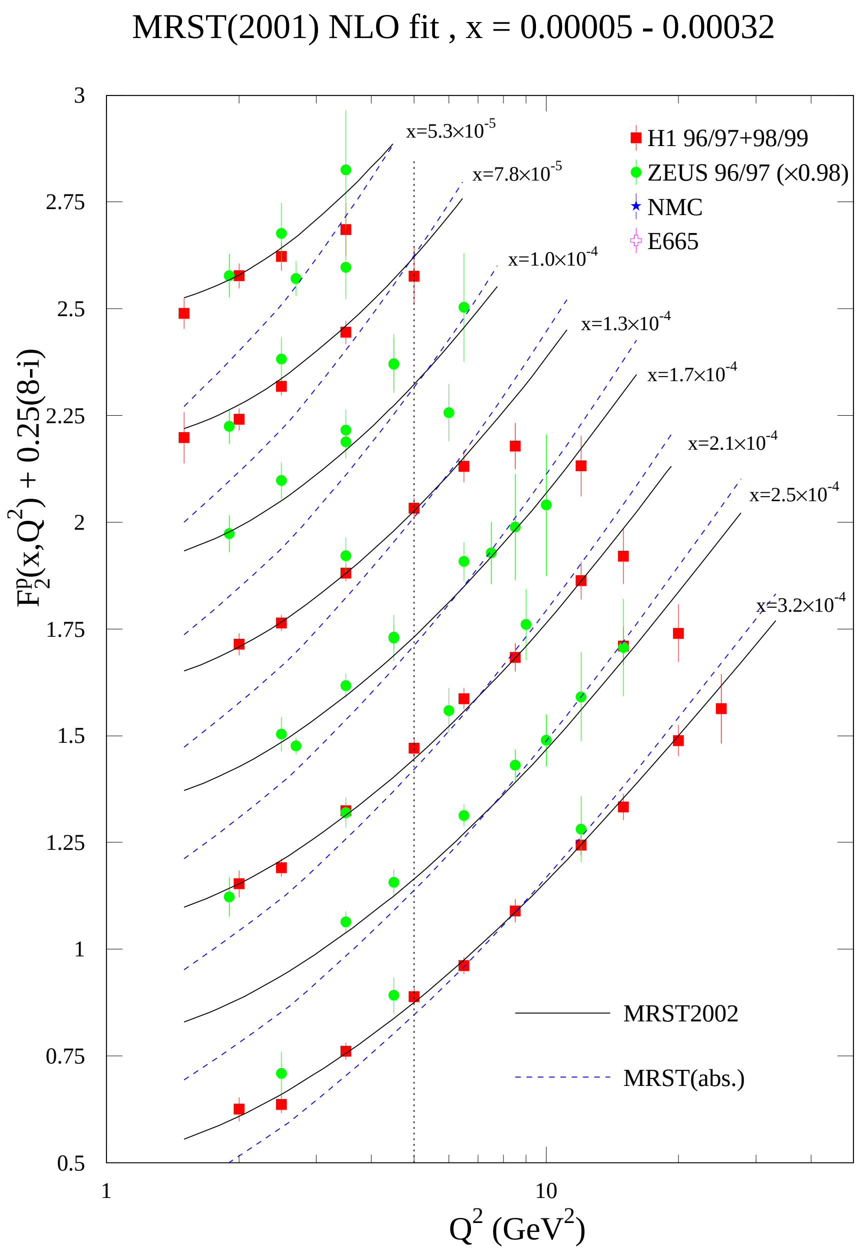
Don't really believe quantitative size and shape of extracted dipole cross-sections (and resulting gluons).

More sophisticated (beyond LO k_t-factorization theory) calculations needed for semi-quantitative results.

But simple dipole picture needs extension beyond LO k_t-factorization theory?

Saturation corrections do not help at NLO or NNLO.

MRST fit with slightly steep input gluon and fairly large shadowing corrections extrapolated to $Q^2 \leq 5\mathrm{GeV}^2$



Comparison of the gluon distribution obtained from the dipole model fit to $xg(x,Q^2) + 4/9xf_s(x,Q^2)$ obtained from a conventional NLO global fit and $xg(x,Q^2) + 4/9xf_s(x,Q^2)$ from a NLO global fit where the quark-gluon splitting functions have been used in the same small-x limit as in the dipole approach (NLODIP).

Also shown is the general probable form of a "correct" gluon which extrapolates from NLO at high x to something a little smaller at small x. All partons are shown for $Q^2 = 50\mathrm{GeV}^2$.