EFFECTIVE ACTION AND COLOR GLASS CONDENSATE

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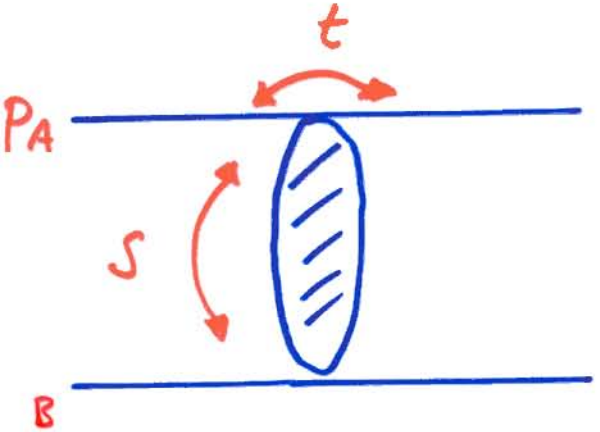
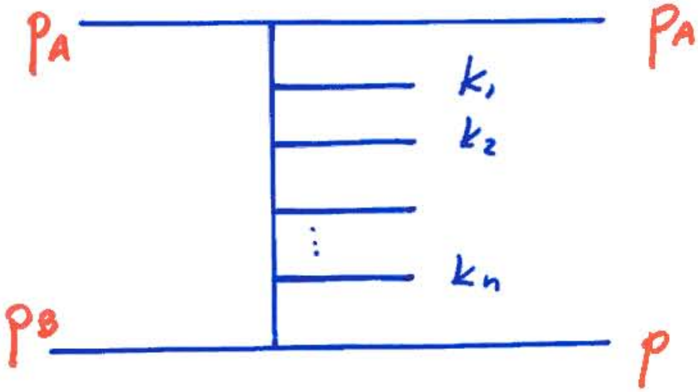
Outline

- High energy limit BFKL
- Corrections : high density
- Color Glass Condensate. JIMWLK equation
- Effective action at high energy
- Relation to other approaches : Verlinde & Verlinde model
- Outlook : remaining problems

High energy limit



More precisely



$s \gg t \gg \Lambda_{QCD}$

$s \gg s_i \quad 2k_{i-1} k_i \gg t \quad q$

multi Regge kinematics

) The resulting effective theory at high energy should be simpler than full QCD

) Different degrees of freedom

Indeed, the effective theory reduces

$\Downarrow$

2 dimensional field theory

$$(x^+, x^-) \longleftrightarrow x_\perp$$

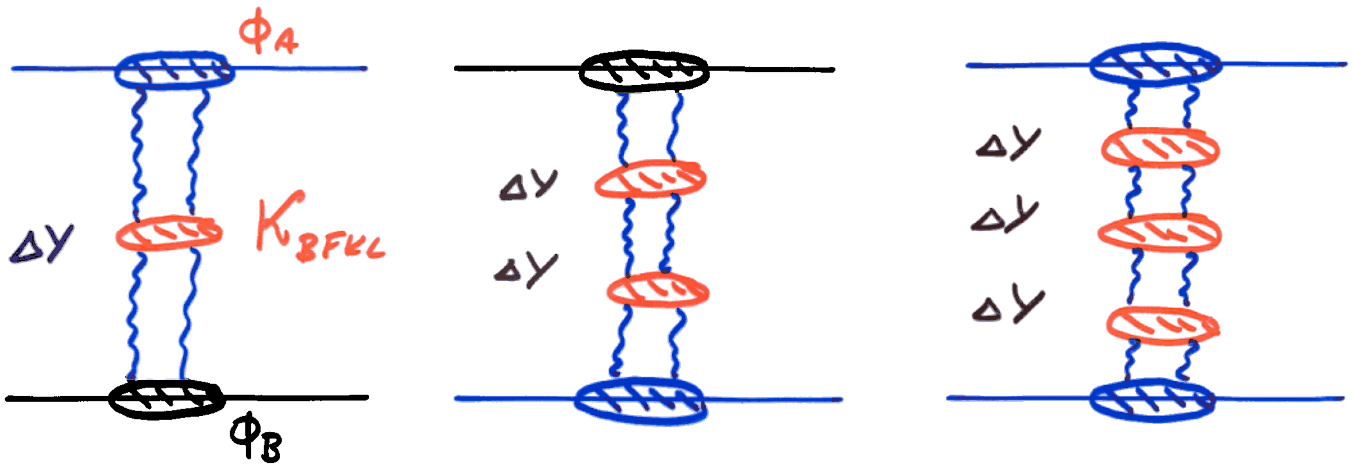
decoupling

Ad.2. Degrees of freedom at high energy

reggeized gluons $\longrightarrow$ ex. Lipatov

Wilson lines $\longrightarrow$ ex. Balitsky

Basic equation at high energy BFKL $\quad Y \sim \ln \frac{s}{s_0}$



$$\frac{dG}{dY} = K \otimes G$$

- Kernel K_{BFKL} depends on transverse momenta
- $2 \longleftrightarrow 2$ process in t channel
- $G \sim s^{\omega_{I\!P}}$ fast growth of the gluon density with energy

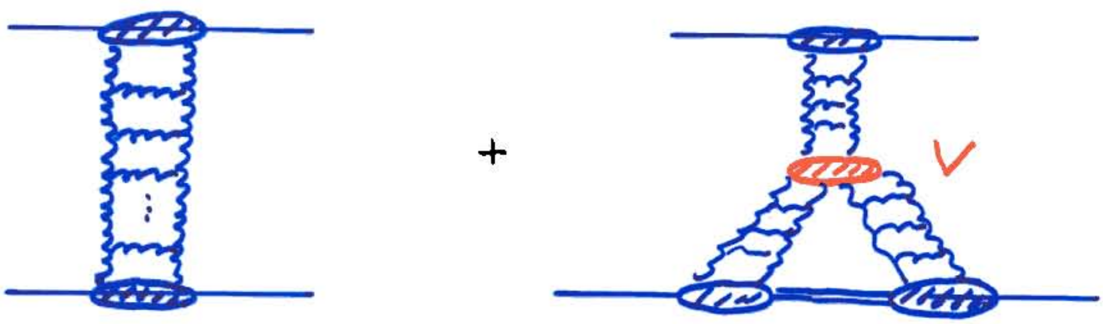
Corrections

NLL very large

Lipatov, Fadin
Camici, Ciafaloni

High density corrections

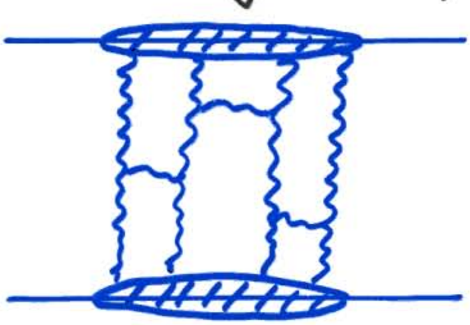
Gribov, Levin,
Ryskin Kovchegov
Mueller, Balitsky



$2 \to 4$
process

$$\frac{dG}{dY} \qquad K \otimes G \qquad V G^2$$

More generally

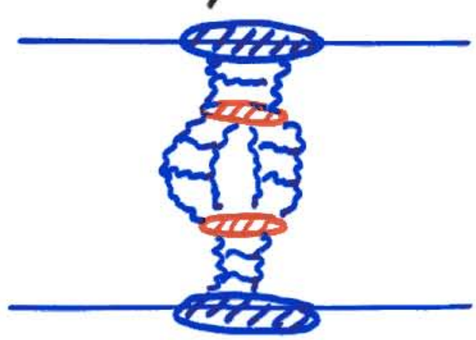


$n \to n$

conserving number
of particles

Bartels, Kwieciński,
Praszałowicz

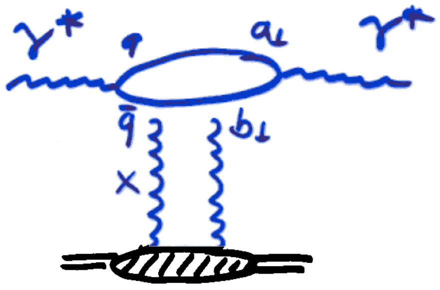
$n \to n$ process



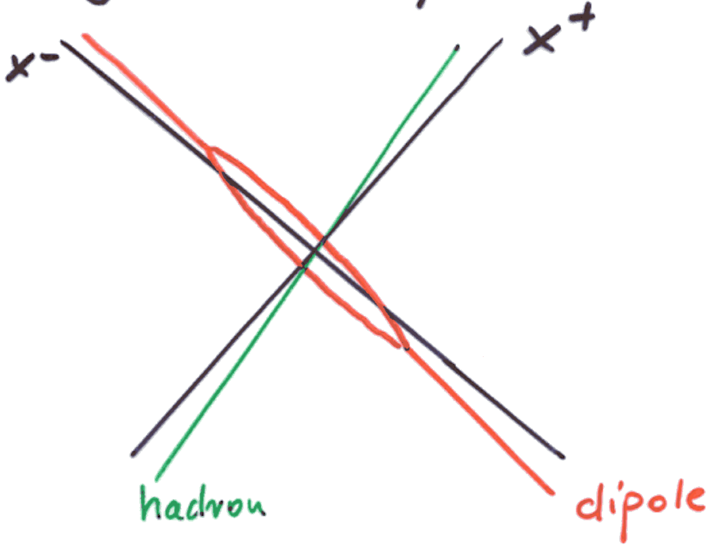
Changing number of
particles in the intermediate
steps → Pomeron loops

Color Glass Condensate

DIS at small x



Light cone picture



Hadron $\Rightarrow$ large P_H^+

Partons of the hadron $\Rightarrow$ large p^+

They are the color sources for the soft gluons

$$x \approx \frac{q^+}{p^+} \ll 1$$

The amplitude

$$N =$$

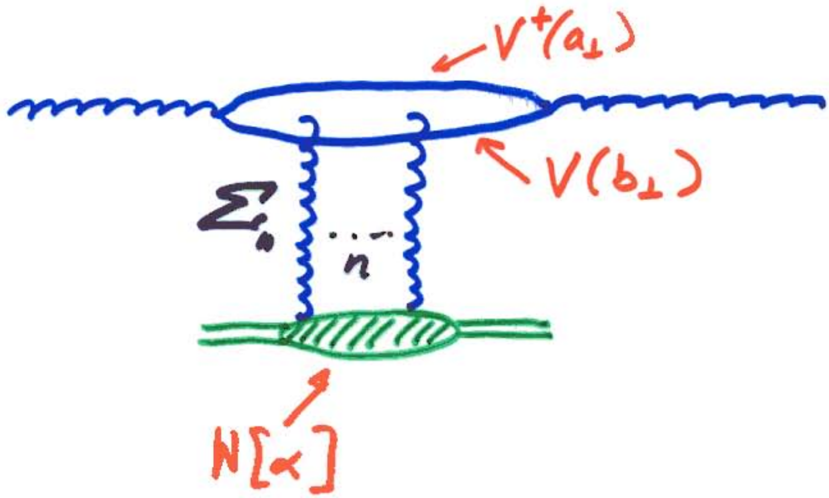
$$S(a_\perp\ b_\perp) = \frac{1}{N_c}\left\langle tr\left(V^+(a_\perp)\, V\, b_\perp)\right)\right\rangle_W$$

Wilson line $\quad V^+(a_\perp) = P\exp\left(ig\int dx\ \alpha(x, a_\perp)\right)$

α field generated by color sources in the hadron

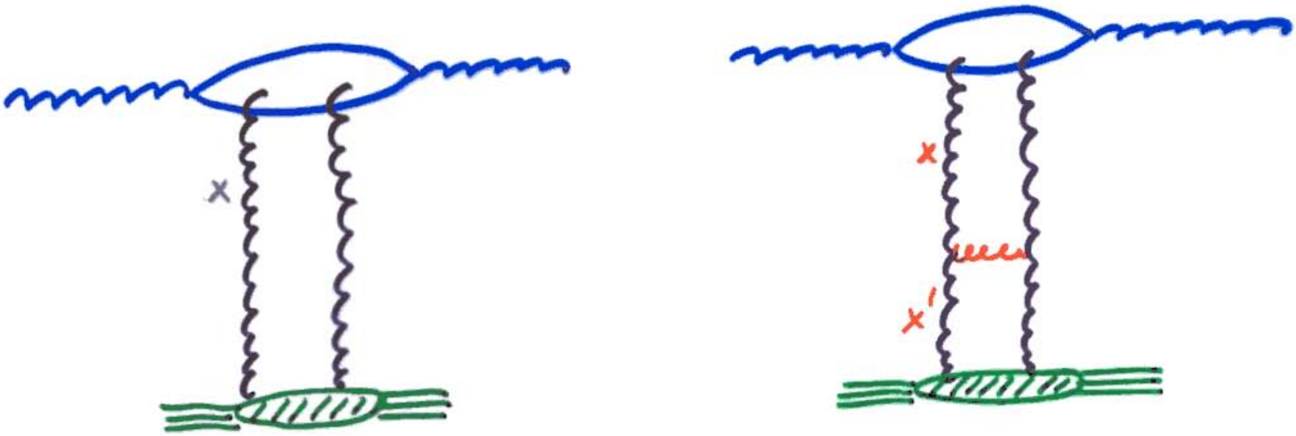
$$\frac{1}{N_c}\langle tr(V^+(a_\perp)V(b_\perp))\rangle_{W[\alpha]} \equiv$$

$$\int D[\alpha]\, W[\alpha]\, \frac{1}{N}\, tr\left(V^+(a_\perp)V(b_\perp)\right)$$



One cannot calculate $W[\alpha]$ exactly

but one can calculate its evolution with x



$$W_x - W_x = \ln\frac{x}{x'}\, \mathcal{H}_{BFKL} \otimes W_x$$

$\mathcal{H}_{BFKL}$ describes one rung of evolution in the BFKL limit

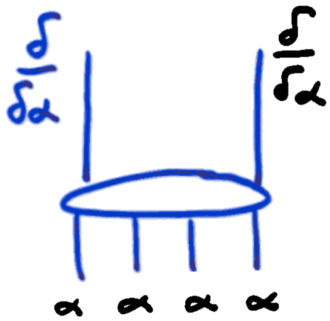
JIMWLK equation

Jalilian-Marian
Iancu
McLerran
Weigert
Leonidov
Kovner

Nonlinear, functional evolution equation in the high energy limit

$$\frac{dW_Y}{dY} \quad \mathcal{H}_{JIMWLK}[\;,\alpha]\,W_Y \qquad Y$$

$$\mathcal{H}_{JIMWLK} = \int_{x_\perp, y_\perp, z_\perp} K_{xyz} \frac{\delta}{\delta\alpha_x}\left(+ V_x^\dagger V_y - V_z^\dagger V_x - V_y^\dagger V_z \right) \frac{\delta}{\delta\alpha_y}$$

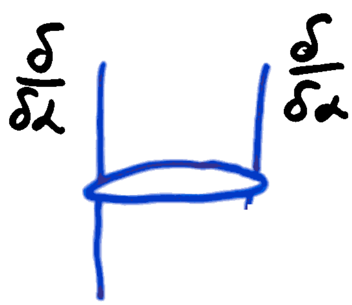
$\mathcal{H} \rightarrow$ 

$$V_x = \mathcal{P}\; ig\int dx^- \alpha^a(x)\, T^a$$

$\mathcal{H}$ contains transiti vertices for

BFKL expand Wilson lines in α's

$$\mathcal{H}_{BFKL} \quad \int_{xyz} \frac{\delta}{\delta\alpha_x}\; \alpha_x - \alpha_z)(\alpha_y - \alpha_z) \frac{\delta}{\delta\alpha_y}$$



$$K_{xyz} \sim \frac{(x_\perp - z_\perp)\cdot(y_\perp - z_\perp)}{(x_\perp - z_\perp)^2 (y_\perp - z_\perp)^2}$$

JIMWLK and JIMWLK dual limits

JIMWLK expand $V_+ \to \frac{\delta}{\delta\alpha}$ and keep V

JIMWLK dual expand $V_- \to \alpha$ and keep V_+

The form of S_{eff} very similar to model by Verlinde & Verlinde 93

VV derive action from scaling argument

$x^{\pm} \to \lambda x^{\pm}$ $\quad s \quad \lambda^{e} s \quad$ $\lambda \to 0$

$$S[g_A\ h_B] \Rightarrow \log s \int_x \mathcal{H}^{AB}\, tr\left(g_A^{-1} D_i^{+} g_A\ h_B^{-1} D_i^{+} h_B\right)$$

where

$$\mathcal{H}^{AB} \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \qquad B, A = 2$$

$$g_A \longleftrightarrow V(+\infty)$$

$$h_A \longleftrightarrow V_+(+\infty)$$

JIMWLK describes the evolution which is not symmetric

$$\mathcal{H}\left[\frac{\delta}{\delta\alpha}\quad\alpha\right]$$

quadratic

all orders

It is not suitable for description of scattering of two identical probes

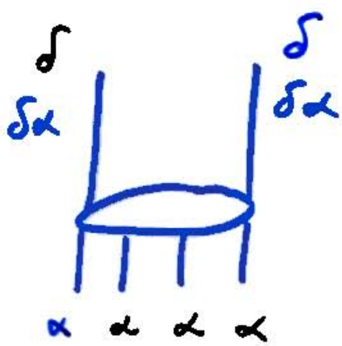
Evolution should be symmetric

Kovner
Lublinsky

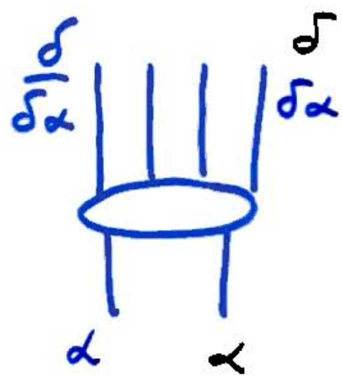
Balitsky

$$\alpha \longleftrightarrow \frac{\delta}{\delta\alpha}$$

$\Rightarrow$ talk Alex Kovner



JMWLK



Dua JMWLK

Try to find the full (selfdual) Hamiltonian that reduces to H_{JIMWLK} and H_{DUAL} in appropriate limits.

Need to include a second Wilson line along x^+ direction

$$V_- \quad \text{and} \quad V_+$$

Dipole - dipole amplitude is (ex. $\gamma^*\gamma^*$)

$$\int Da^+ Da^- \, e^{iS_{eff}[V_-, V_+]} \, \mathrm{tr}\left[V_-^\dagger(x_1) V_-(y_1)\right] \mathrm{tr}\left[V_+^\dagger(x_2) V_+(y_2)\right]$$

$$\begin{aligned} S_{eff} = \frac{\ln s}{g^2 N_c} \int_{x_\perp} \mathrm{tr}\Big[& V_+^\dagger(-\infty)\partial^i V_+(-\infty)\, \partial^i V_-(-\infty) V_-^\dagger(-\infty) \\ & + V_-^\dagger(-\infty)\partial^i V_-(-\infty)\, \partial^i V_+^\dagger(\infty) V_+(\infty) \\ & + V_+(\infty)\partial^i V_+^\dagger(\infty)\, \partial^i V_-^\dagger(\infty) V_-(\infty) \\ & + V_-(\infty)\partial^i V_-^\dagger(\infty)\, \partial^i V_+(-\infty) V_+^\dagger(-\infty) \Big] \end{aligned}$$

Hatta
Iancu
McLerran
[illegible]
A.S.

$$V_- \longleftrightarrow V_+$$

Summary & final remarks

- Effective action that involves Wilson lines (VV-like nonliner σ model)
- It reproduces $\mathcal{H}_{JIMWLK}$ and its dual in appropriate limits
- Relation to other approaches

  Lipatov, Kirschner, Szymanowski action

  Balitsky action

- Reduction to the dipde model

(Eqs. generating Pomeron loops are now known in dipole model

↳ talk by Genya Levin

- Other corrections, NLL ..