

# I : A Unified Model for inelastic e-N and neutrino-N cross sections at all $Q^2$

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# II : Add- Duality and QCD based fits to Nucleon Form Factors

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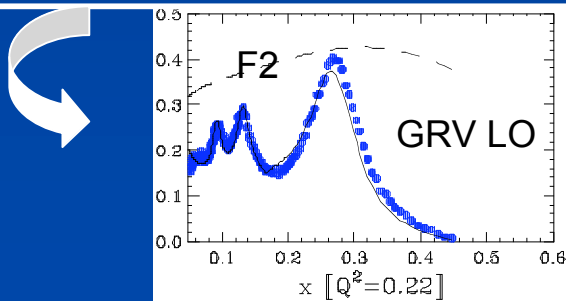
John Arrington- Argonne National Lab

DIS 2005 Madison , April, 2005

# Modeling on neutrino cross sections

- Describe DIS, resonance, even photo-production ( $Q^2=0$ ) in terms of **quark-parton model**. With PDFs, it is straightforward to convert charged-lepton scattering cross sections into neutrino cross section.
- **Challenge:**
  - Understanding of high  $x$  PDFs at very low  $Q^2$ ?
  - Understanding of resonance scattering in terms of quark-parton model?

- **NNLO QCD+TM approach**  
good to explain non-pert. QCD effects at low  $Q^2$
- **Effective LO approach** (pseudo NNLO: for MC)  
Use effective LO PDFs with a new scaling variable,  $\xi_W$  to absorb target mass, higher twist, missing higher orders



$q$  Resonance, higher twist, and TM  
 $P=M$   $m_f=M^*$   
 (final state interaction)

$$\xi W = \frac{Q^2 + m_f^2 + O(m_f^2 - m_i^2) + A}{M_N (1 + (1 + Q^2/\nu^2))^{1/2} + B} \rightarrow X_{bj} = Q^2 / 2 M_N$$

# Effective LO model - 2003

1. Start with GRV98 LO ( $Q^2_{\min}=0.80 \text{ GeV}^2$ )  
 - dashed line- describe  $F_2$  data at high  $Q^2$

2. Replace the  $Xb_j$  with a new scaling,  $\xi_w$

3. Multiply all PDFs by K factors for photo prod. limit and higher twist

$$[\sigma(\gamma) = 4\pi\alpha/Q^2 * F_2(x, Q^2)]$$

$$K_{\text{sea}} = Q^2/[Q^2 + C_{\text{sea}}]$$

$$K_{\text{val}} = [1 - G_D^2(Q^2)]$$

$$*[Q^2 + C_{2V}] / [Q^2 + C_{1V}] \text{ motivated by Adler}$$

Sum rule

$$\text{where } G_D^2(Q^2) = 1/[1 + Q^2/0.71]^4$$

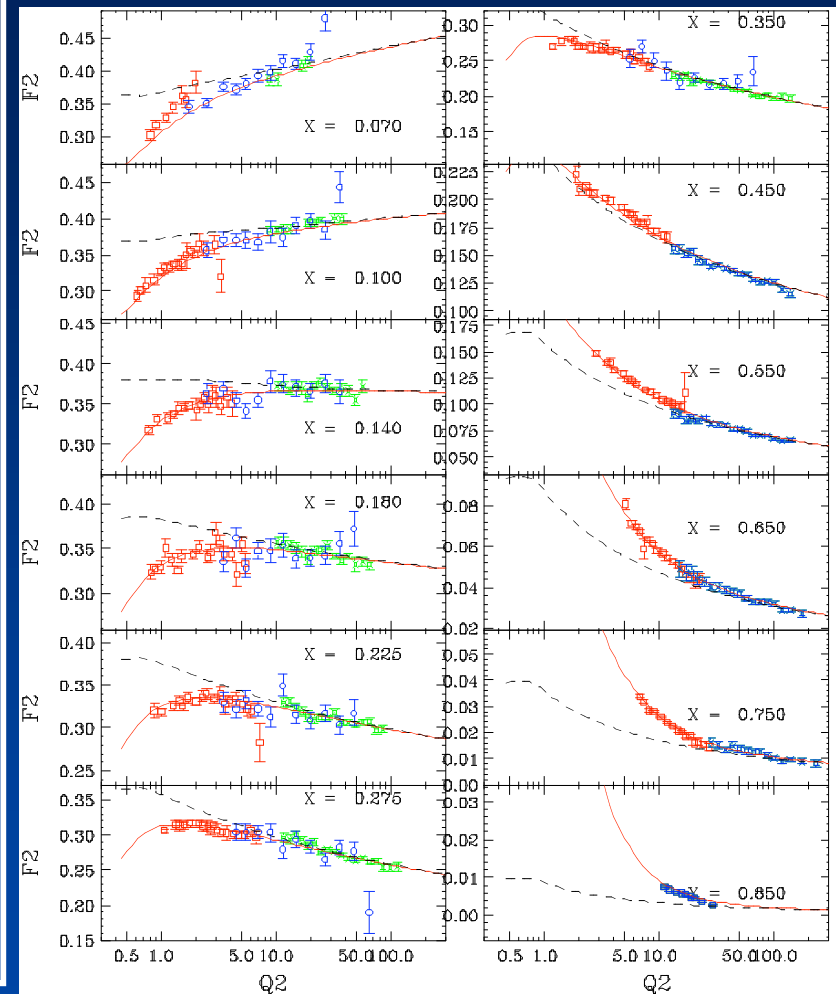
4. Freeze the evolution at  $Q^2 = Q^2_{\min}$

$$- F_2(x, Q^2 < 0.8) = K(Q^2) * F_2(Xw, Q^2=0.8)$$

➤ Fit to all DIS  $F_2$  P/D (with low x HERA data)  
 $A=0.418, B=0.222$

$$C_{\text{sea}} = 0.381, C_{1V} = 0.604, C_{2V} = 0.485$$

$$\chi^2/\text{DOF} = 1268 / 1200 \text{ Solid Line}$$



**A** : initial binding/TM effect+ higher order

**B** : final state mass  $m_f^2, \Delta m^2$ .

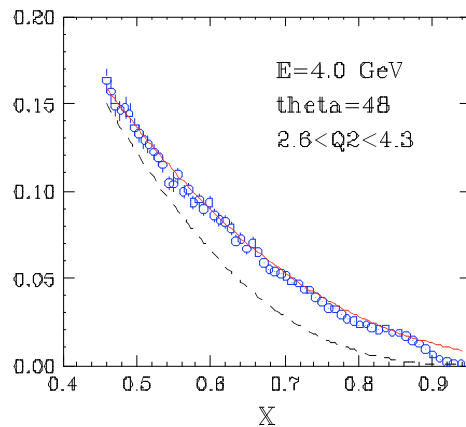
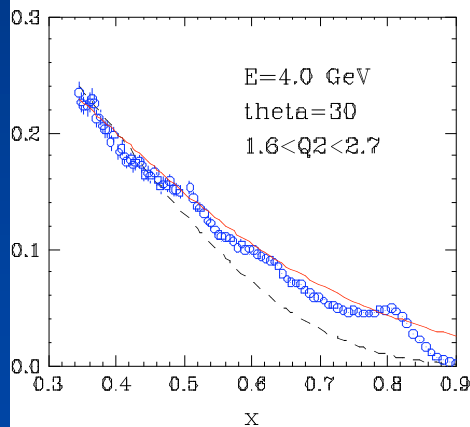
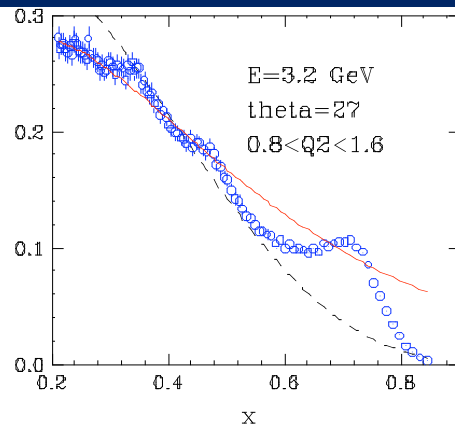
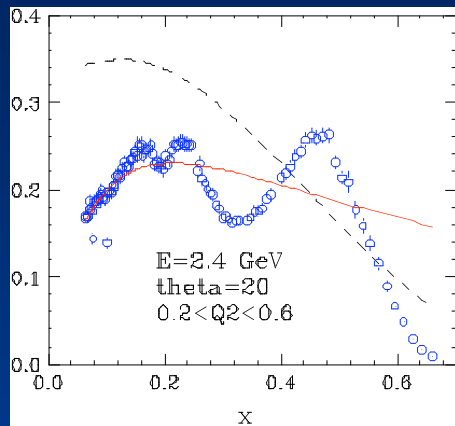
**K Factor**: Photo-prod limit ( $Q^2 = 0$ ), Adler sum rule

**F2 e-Proton**

Solid- GRV98 PDFs

Dashed -Modified GRV98 PDFs

# Comparison with effective LO model



$F_2(d)$  resonance  
low  $Q^2$

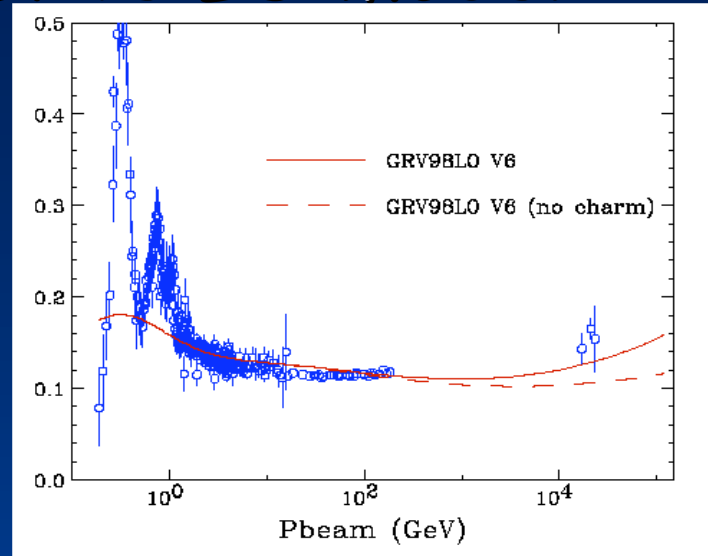


Photo-production (P)

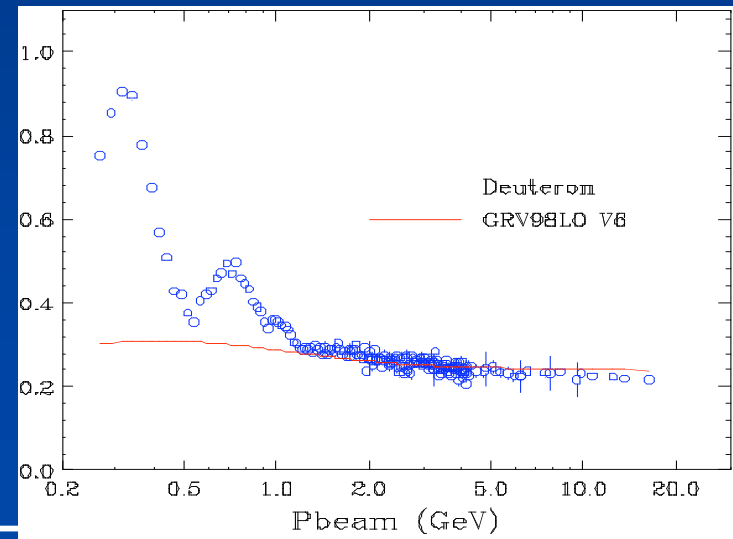
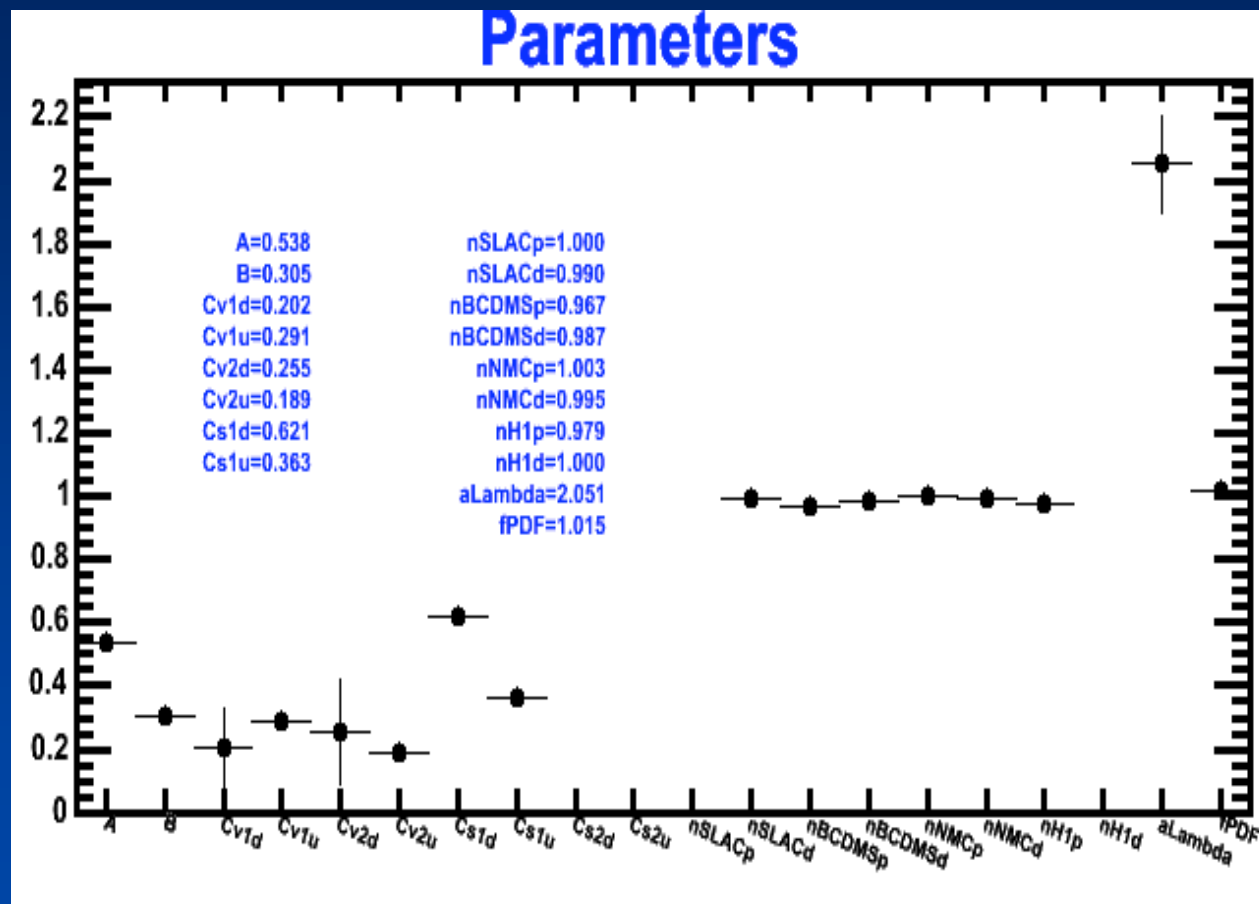


Photo-production (d)

# 2004 Updates on effective LO model

- Improvements in our model
  - Separate low  $Q^2$  corrections to d and u valence quarks, and sea quarks
  - Include all inelastic F2 proton/deuterium (SLAC/NMC/BCDMC/HERA), photo-production on proton/deuterium in the fits (the c-cbar photon-gluon fusion contribution is included, important at high energy)
- Toward axial PDFs ( vector PDFs vs axial PDFs)
  - Compare to neutrino data (assume  $V=A$ )  
CCFR-Fe, CDHS-Fe, CHORUS-Pb differential cross section (without c-cbar boson-fusion in yet - to be added next since it is high energy data)
  - We have a model for axial low  $Q^2$  PDFs, but need to compare to low energy neutrino data to get exact parameters - next.  
 $K_{vec} = Q^2/[Q^2+C1] \rightarrow K_{ax} = \sqrt{[Q^2+C2]}/[Q^2+C1]$

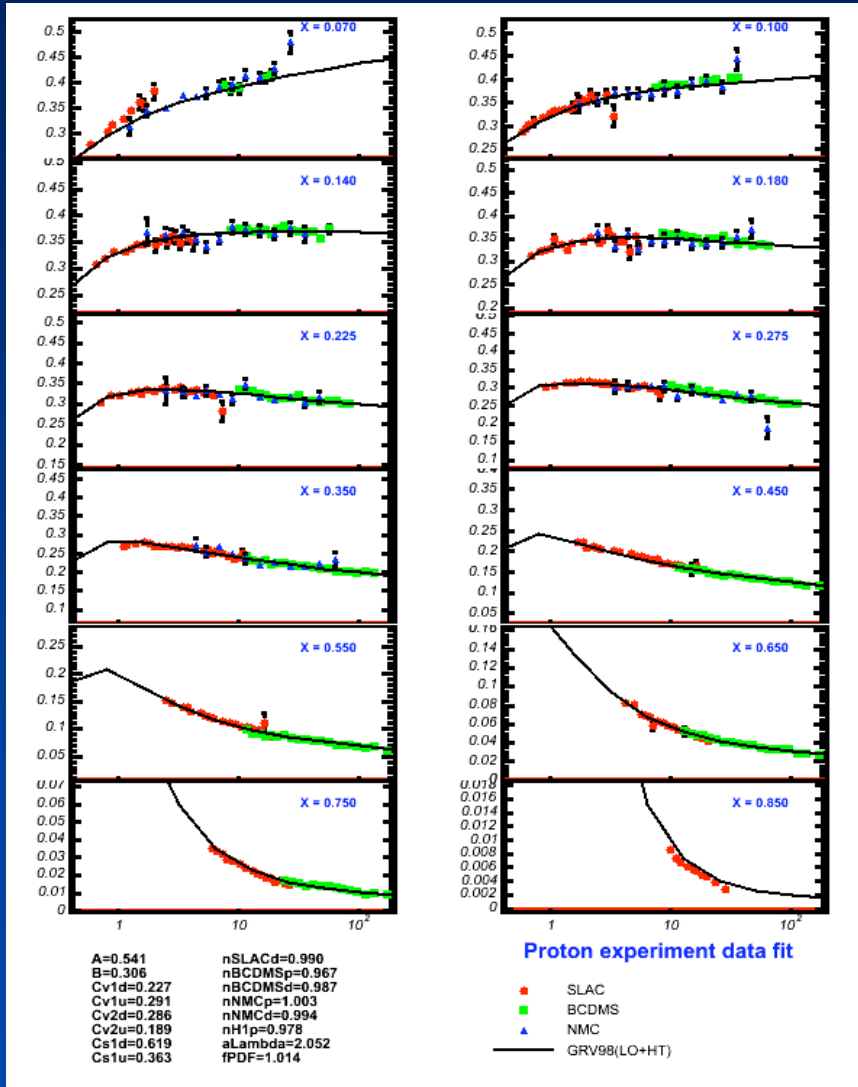
# Fit results using the updated model



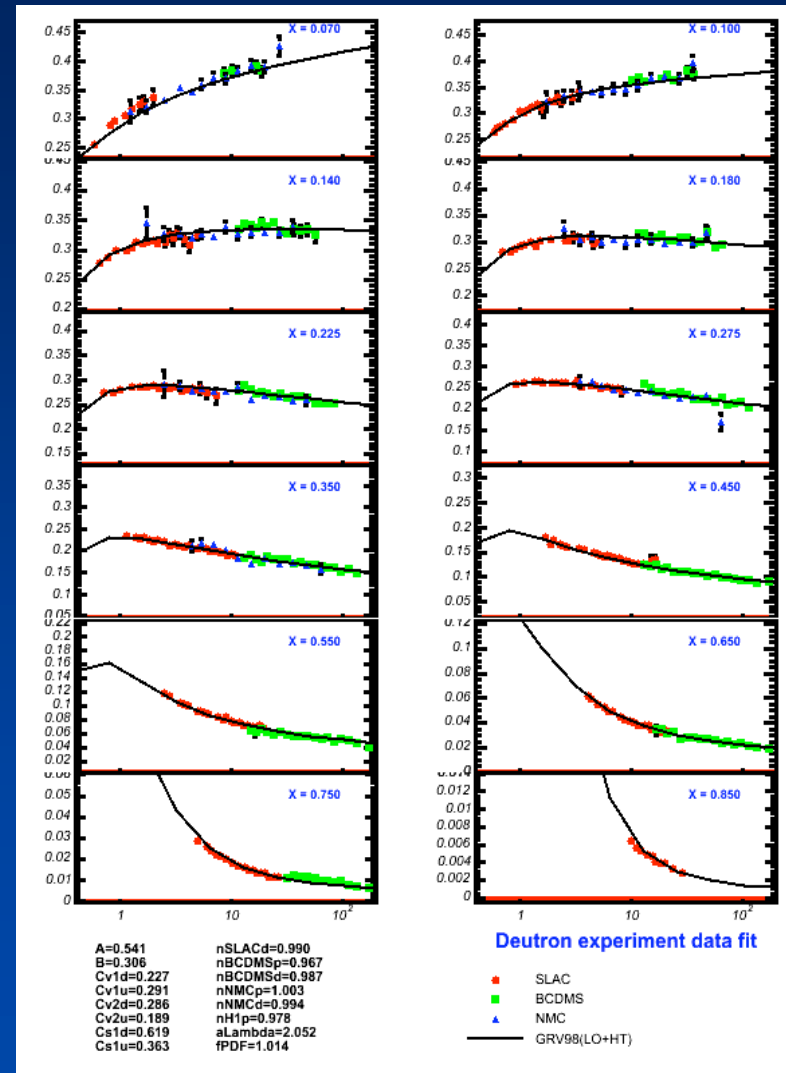
Separate K factors  
for  $uv$ ,  $dv$ ,  $us$ ,  $ds$

<http://web.pas.rochester.edu/~icpark/MINERvA/>

# Fit results

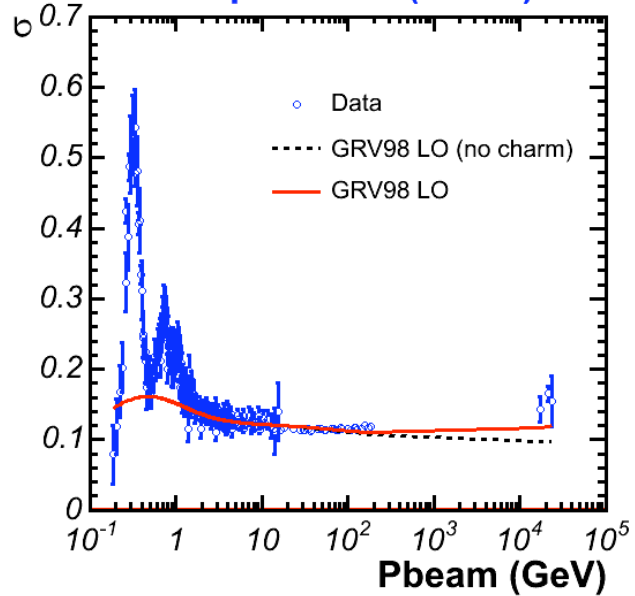


F2 proton

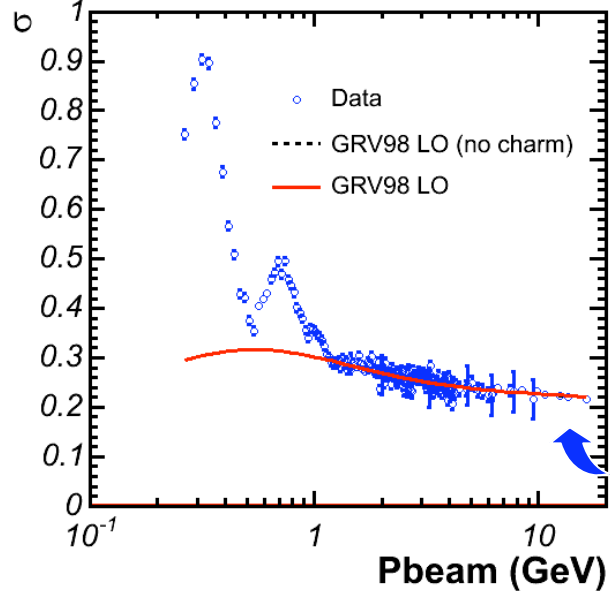


F2 deuterium

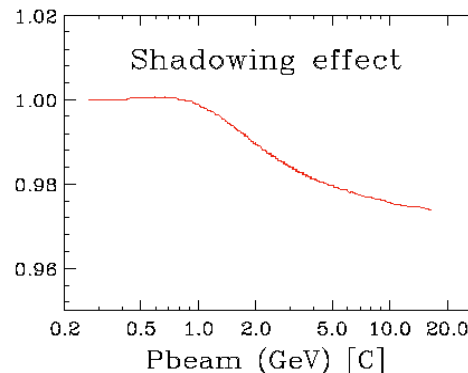
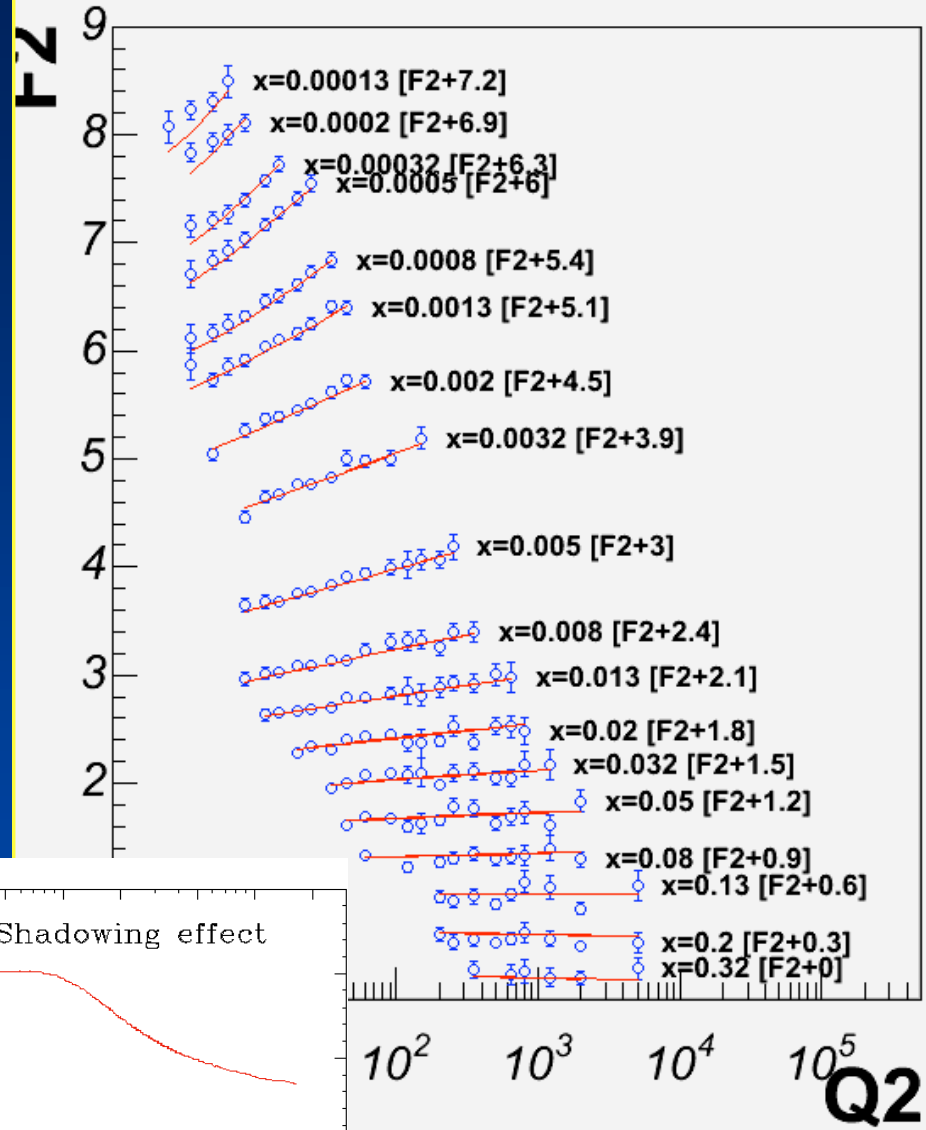
### Photo-production (Proton)



### Photo-production (Deuteron)

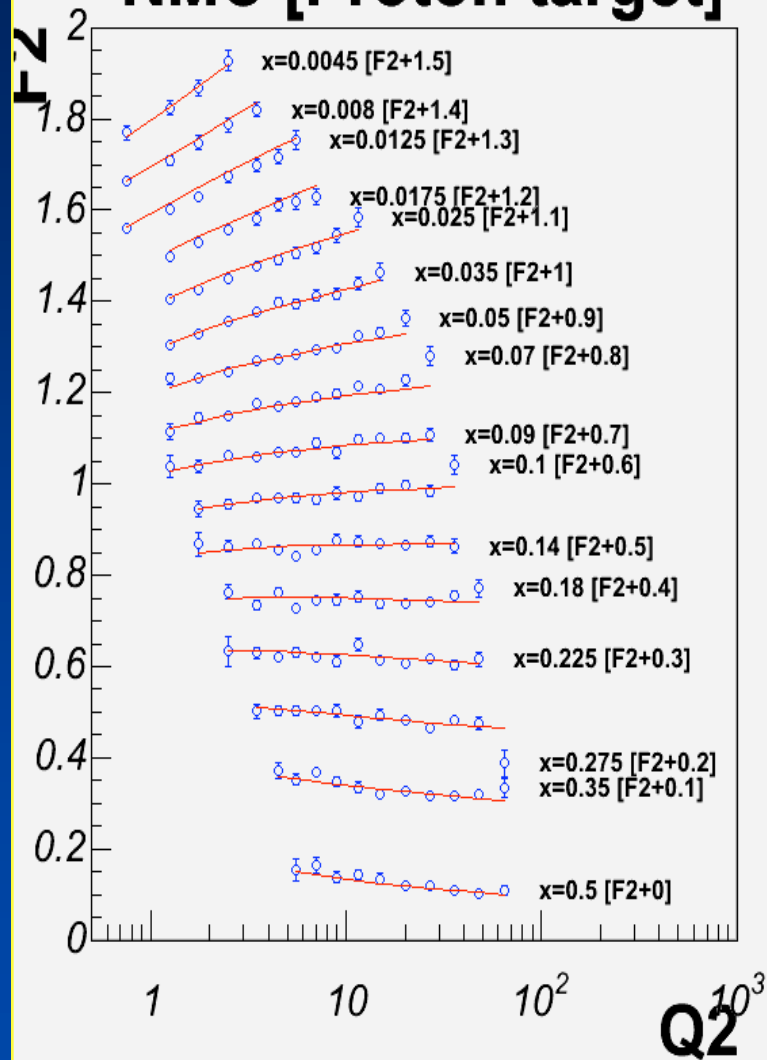


# H1

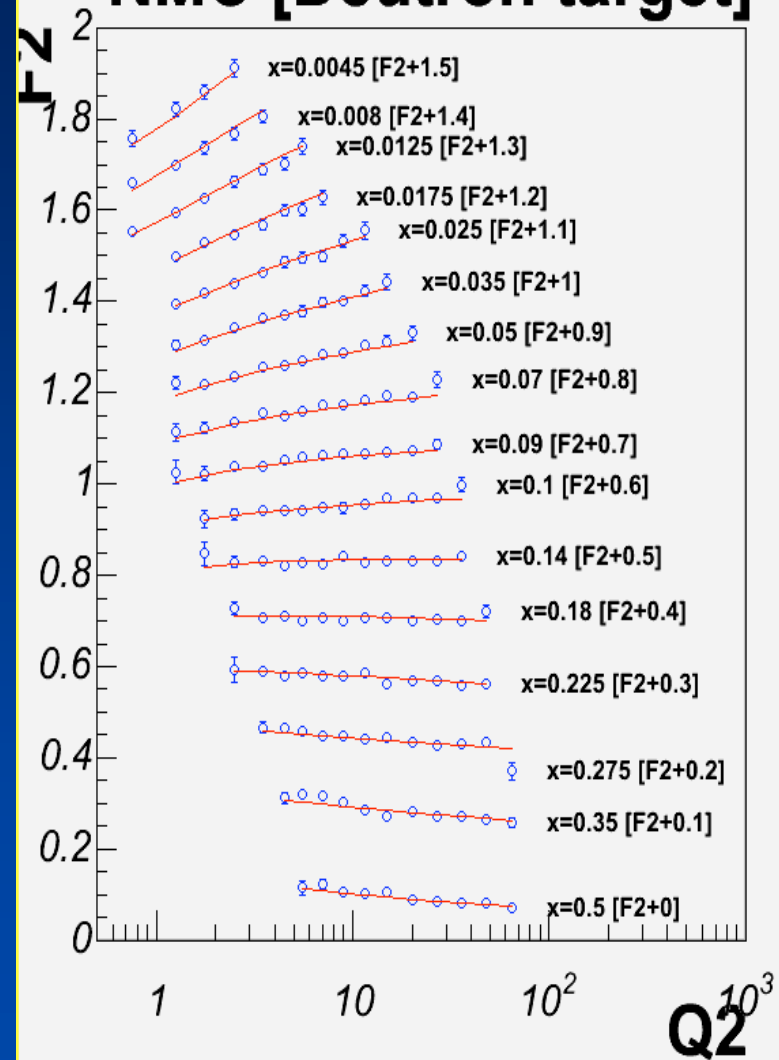




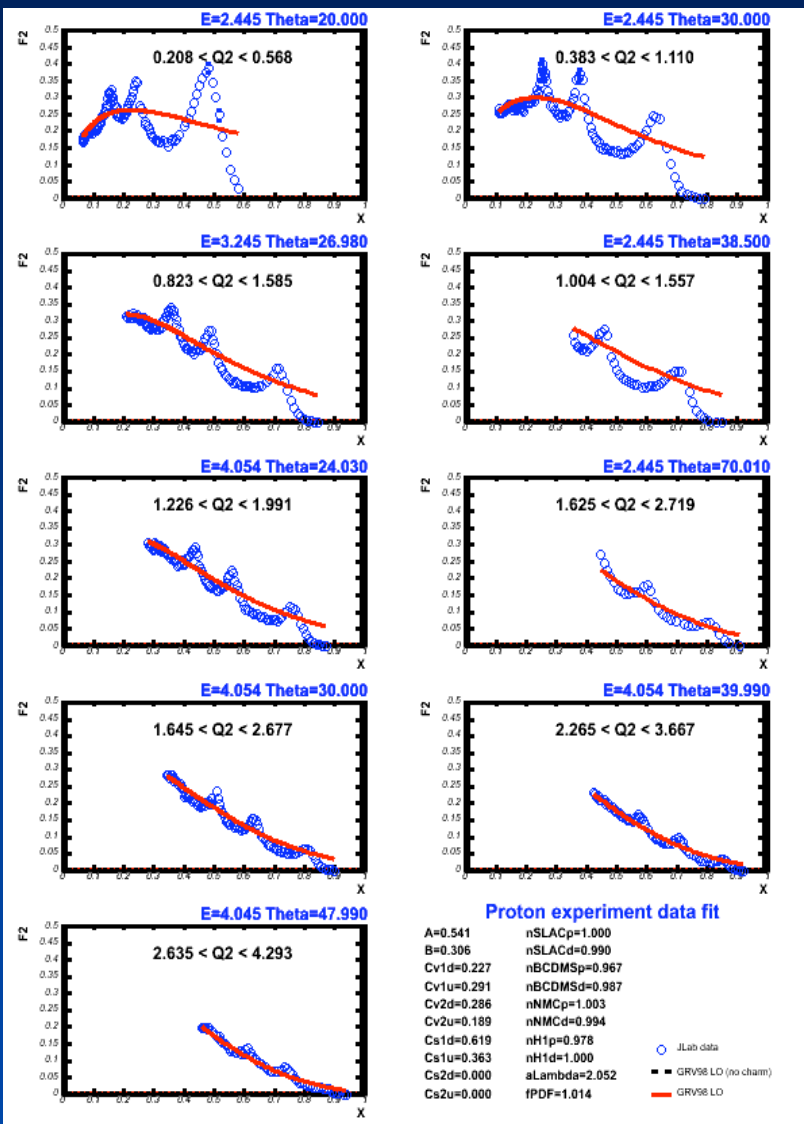
# NMC [Proton target]



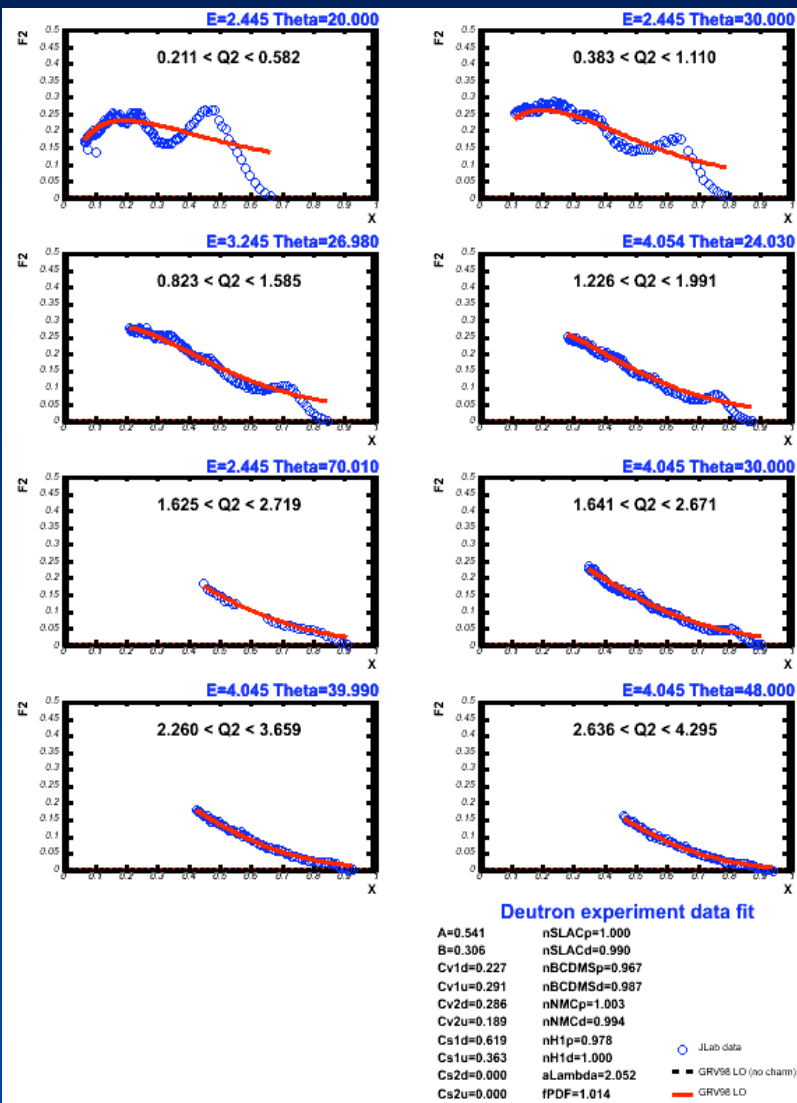
# NMC [Deuteron target]



# Resonance F2 proton

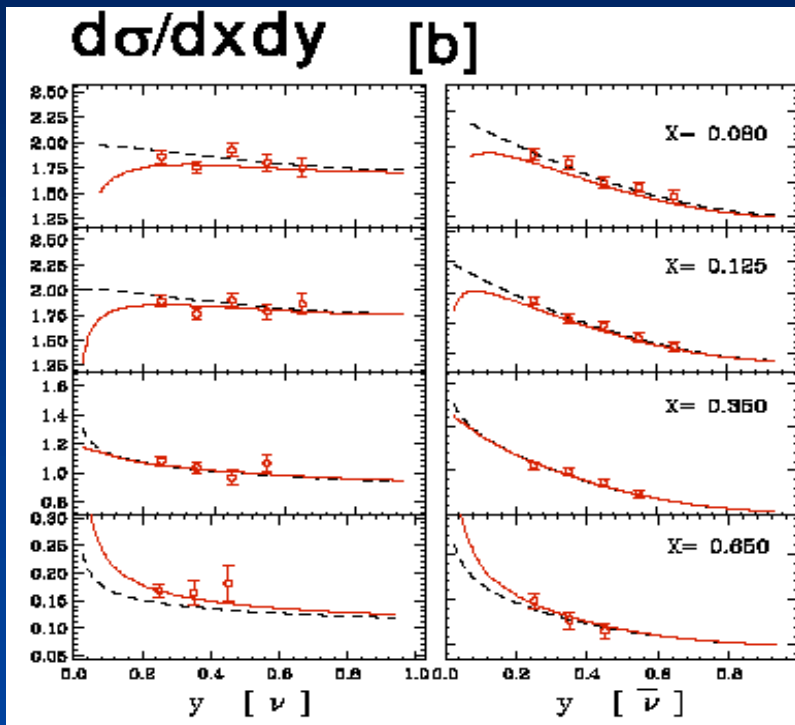


# Resonance F2 deuterium



Resonance data are not included in the fit!!!

# Comparison with neutrino data (assume $V=A$ )



— $\xi w$  PDFs GRV98 modified

---- GRV98 ( $x, Q^2$ ) unmodified

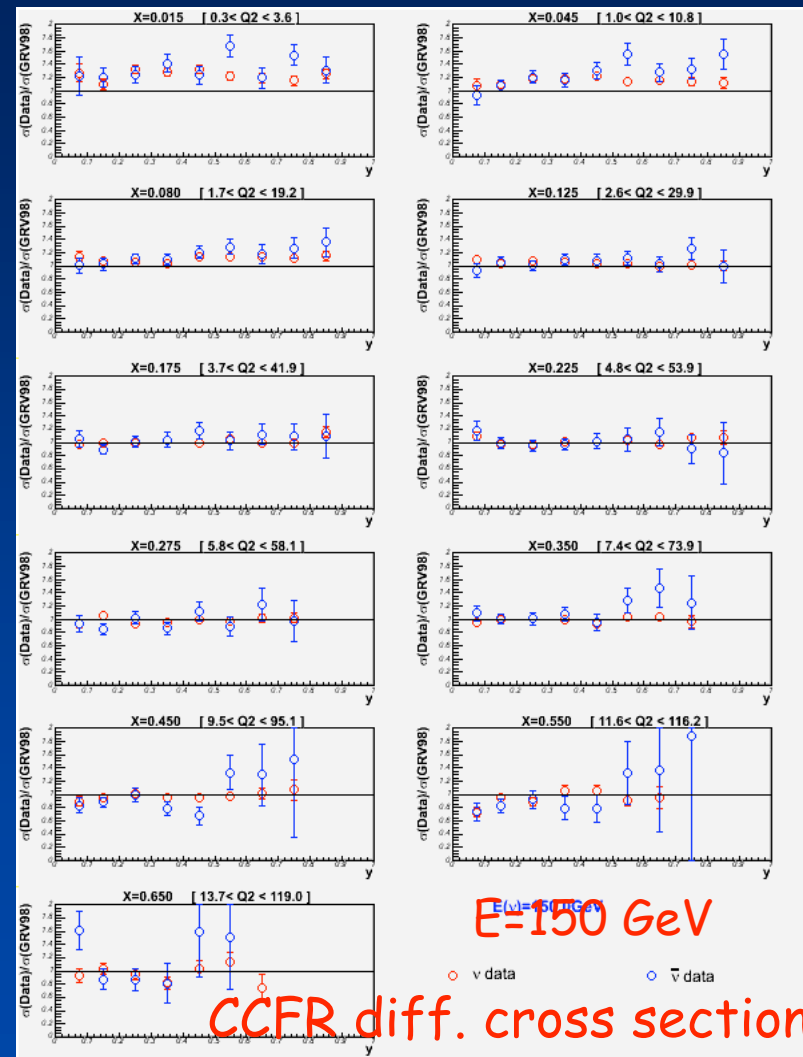
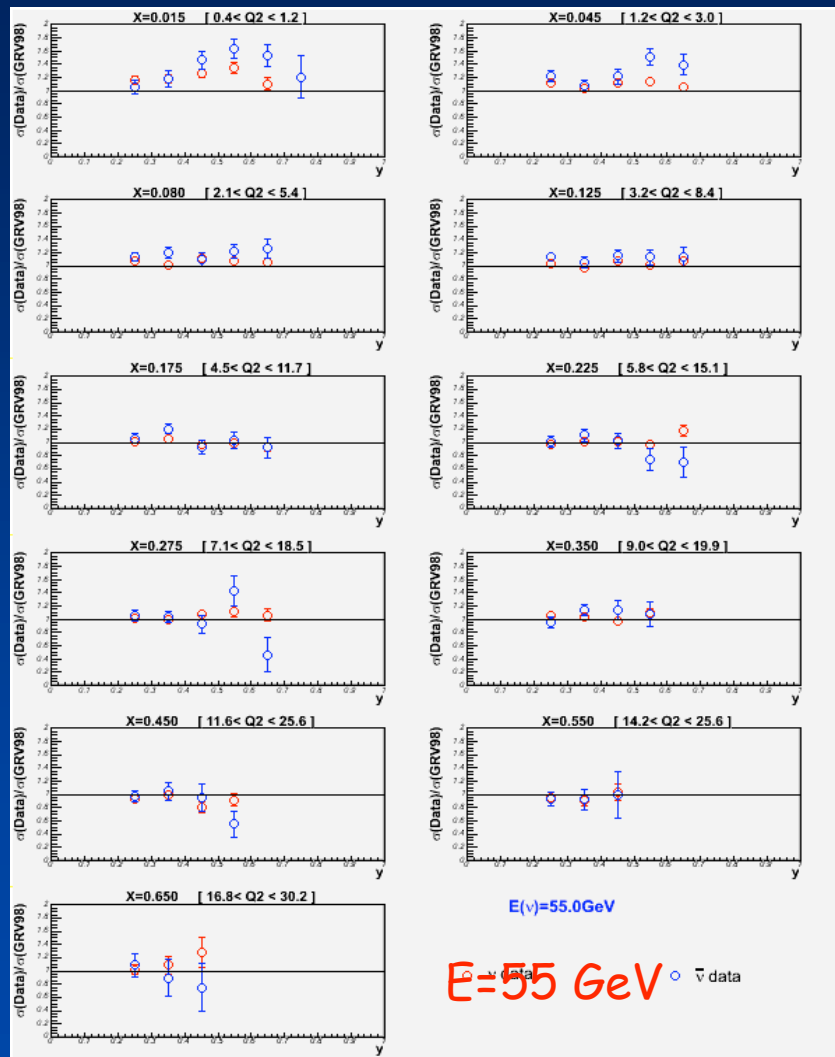
Left: (neutrino), right anti-neu

(NuFact03 version)

- Apply nuclear corrections using e/m scattering data.
- Calculate  $F_2$  and  $x F_3$  from the modified PDFs with  $\xi w$
- Use  $R=R_{\text{world}}$  fit to get  $2x F_1$  from  $F_2$
- Implement charm mass effect through  $\xi w$  slow rescaling algorithm, for  $F_2$ ,  $2x F_1$ , and  $x F_3$

Our model describe CCFR diff. cross sect. ( $E_n=30-300$  GeV) well (except at the lowest  $x$ )

# Comparison with updated model (assume $V=A$ )

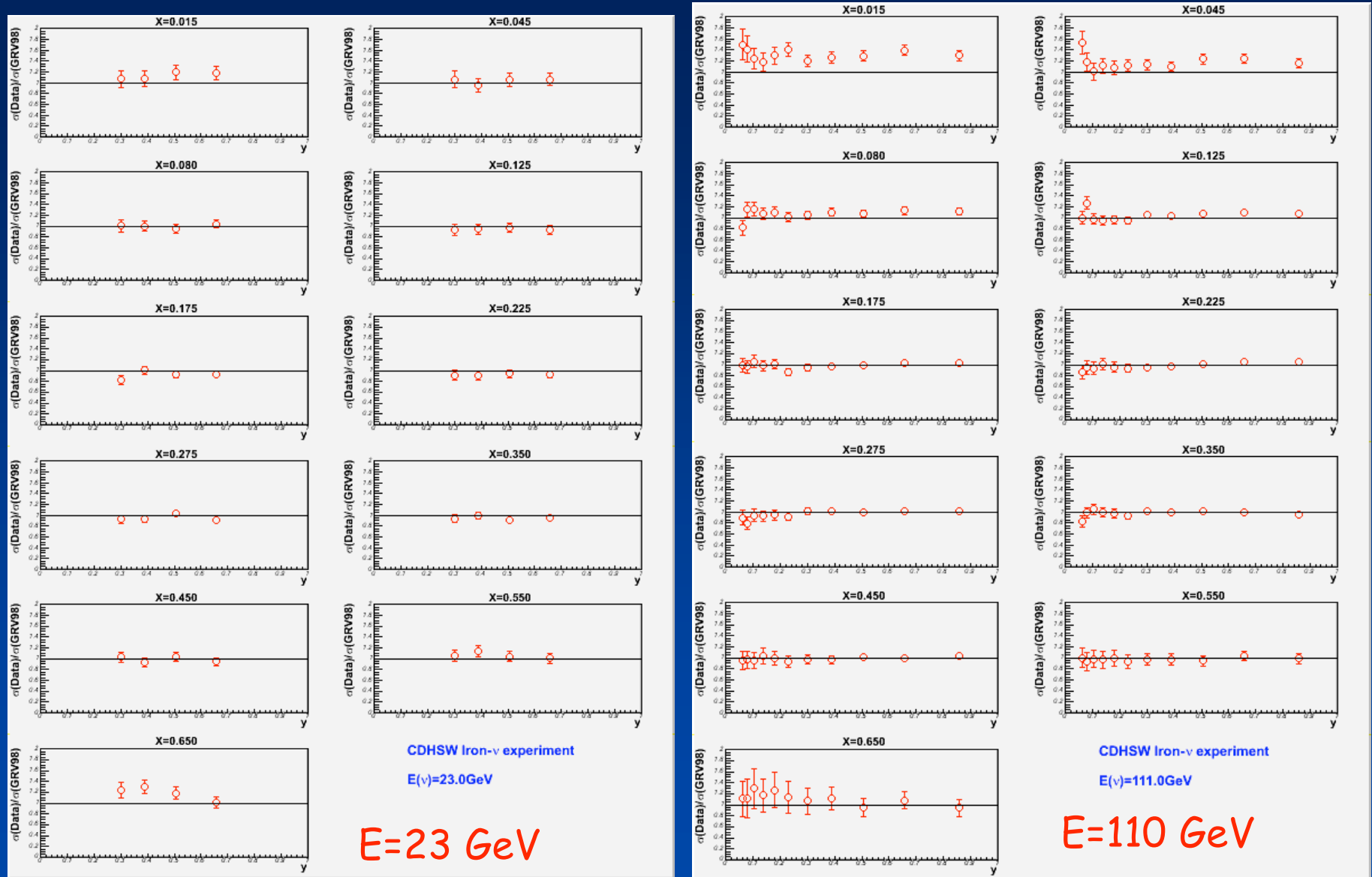


CCFR diff. cross section data

Plots for all energy regions:

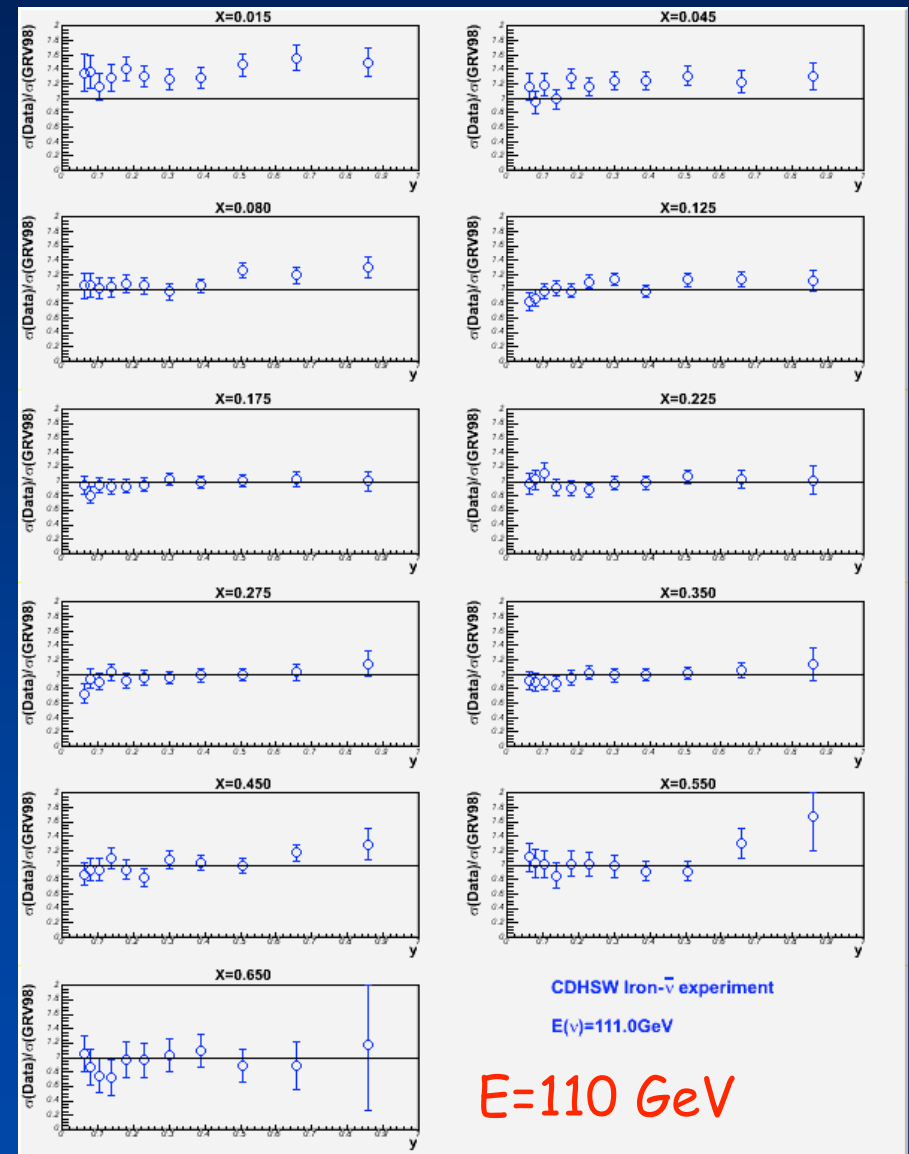
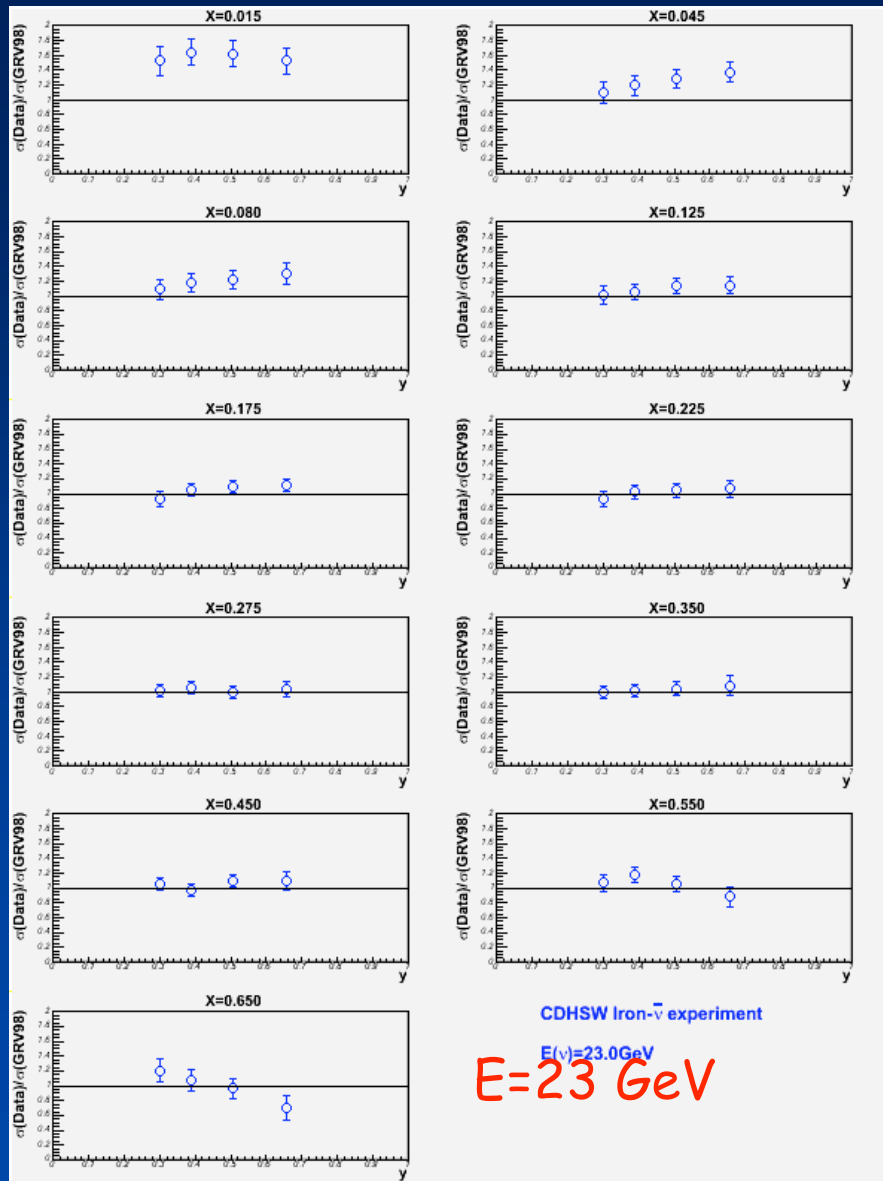
<http://web.pas.rochester.edu/~icpark/MINERvA/>

# Comparison with CDHSW neutrino data



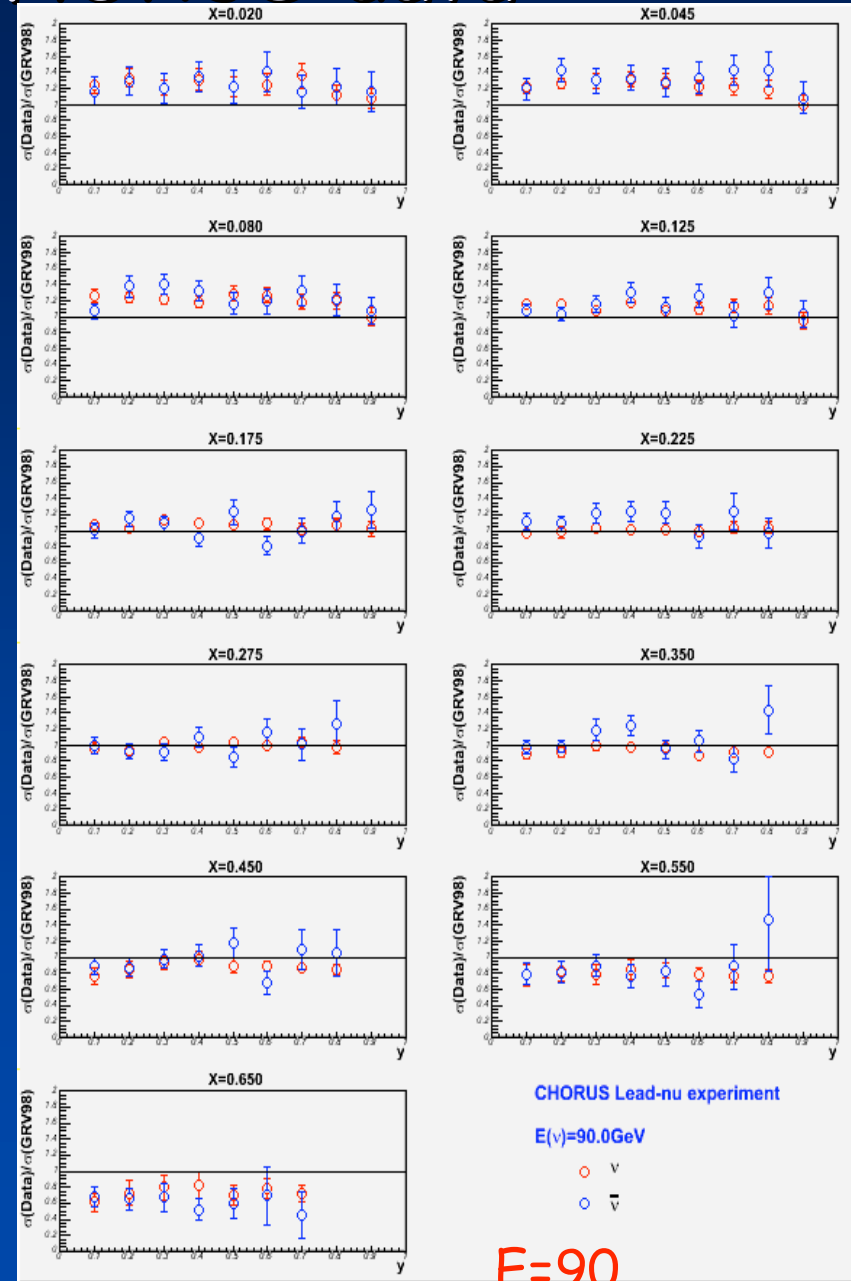
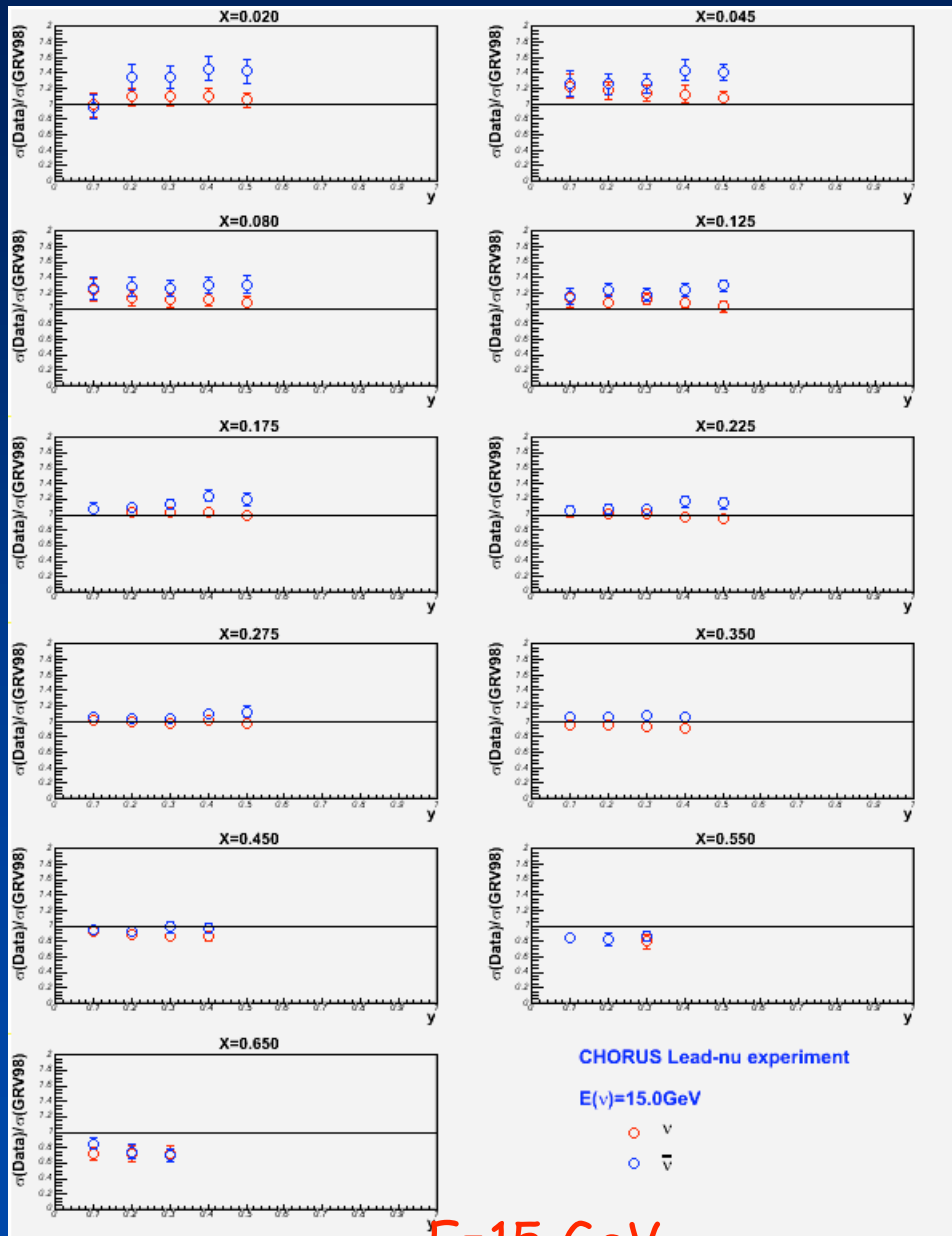
Radiative correction,  $c\bar{c}$  contribution at low  $x$

# Comparison with CDHSW anti-neutrino data



Radiative correction,  $c\bar{c}$  contribution at low  $x$

# Comparison with CHORUS data



# Correct for Nuclear Effects measured in e/muon expt.

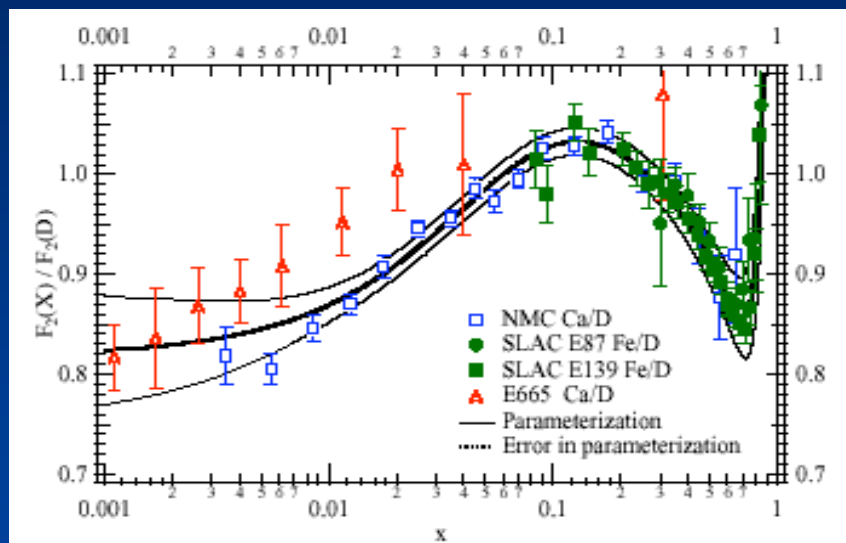
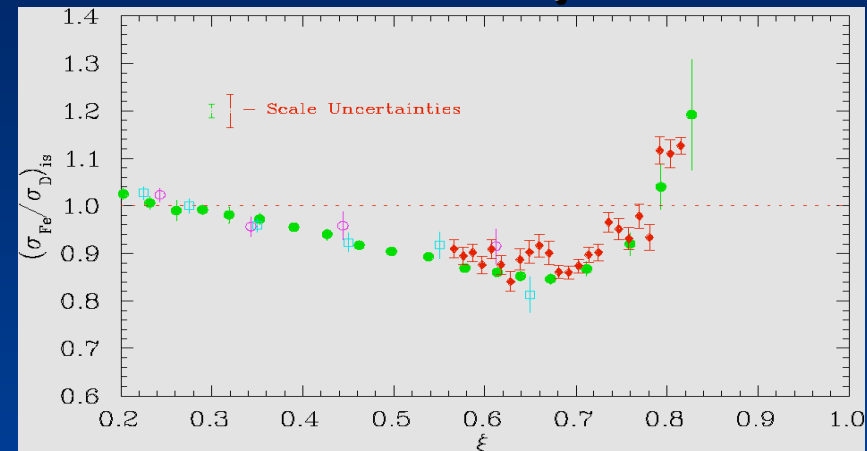


Figure 5. The ratio of  $F_2$  data for heavy nuclear targets and deuterium as measured in charged lepton scattering experiments (SLAC, NMC, E665). The band shows the uncertainty of the parametrized curve from the statistical and systematic errors in the experimental data [16].



Comparison of Fe/D  $F_2$  data  
In resonance region (JLAB)  
Versus DIS SLAC/NMC data  
In  $\tau_M$  (C. Keppel 2002).



# I. Summary and Plans

- Our effective LO model describe all F2 DIS, resonance, and photo-production data well.
- This model provide a good description on the neutrino cross section data (except axial vector contribution).
- Now working on the axial structure functions and next plan to work on resonance fits.
- JUPITER at Jlab (Bodek, Keppel) taken January 05 - will provide electron-Carbon (also e-H and e-D and other nuclei such as e-Fe) in resonance region (summer 05)
- Future: MINERvA at FNAL (McFarland, Morfin) will provide Neutrino-Carbon data at low energies.

# II : Duality and QCD based fits to Nucleon Form Factors

Arie Bodek, R. Bradford, H. Budd -U. of Rochester

John Arrington- Argonne National Lab

$$2xF_1^{inel}(x, Q^2) = x^2 G_M^2(Q^2) \delta(x-1)$$

$$F_2^{inel}(x, Q^2) = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} \delta(x-1)$$

$$R(x=1, Q^2) = \frac{4M^2}{Q^2} \left( \frac{G_E^2}{G_M^2} \right)$$

$$G_D(Q^2) \equiv \frac{1}{(1 + Q^2 r_0^2)^2}$$

$$r_0^2 = (0.24 \text{ fm})^2 = 1/0.71$$

(GeV)<sup>-2</sup> DIS 2005 Madison

Duality:  $R_p = R_n$  (inelastic) at  
high  $Q^2$  near  $x=1$ .

QCD and Duality: At High  $Q^2$  near  
 $x=1$   $F_{1n}/F_{1p} = (G_{Mn}/G_{Mp})^2$

Use  $F_{1n}/F_{1p}$  predicted with  
 $d/u = 0.2$  at  $x=1$  (From QCD)

Use a form proposed by J. J. Kelly Phys. Rev. C 70, 068202 (2004). This form satisfies QCD constraints at High  $Q^2$  with 4 parameters for  $G_{ep}$ ,  $G_{mp}$ ,  $G_{mn}$ .

$$G(Q^2) \propto \frac{\sum_{k=0}^n a_k \tau^k}{1 + \sum_{k=1}^{n+2} b_k \tau^k}, \quad (1)$$

where both numerator and denominator are polynomials in  $\tau = Q^2/4m_p^2$  and where the degree of the denominator is larger than that of the numerator to ensure that  $G \propto Q^{-4}$  for large  $Q^2$ . For magnetic form factors we include a factor of  $\mu$  on the right-hand side, such that  $a_0 \approx 1$  if the data for low  $Q^2$  are normalized accurately. With  $n=1$  and  $a_0=1$ , this parametrization provides excellent fits to  $G_{Ep}$ ,  $G_{Mp}/\mu_p$ , and  $G_{Mn}/\mu_n$  using only four parameters each. However, this approach is less successful for  $G_{En}$  because the existing data are still too limited. Therefore, for  $G_{En}$  I continue to use the Galster parametrization [8],

$$G_{En}(Q^2) = \frac{A\tau}{1 + B\tau} G_D(Q^2),$$

where  $G_D = (1 + Q^2/\Lambda^2)^{-2}$  with  $\Lambda^2 = 0.71 \text{ (GeV}/c)^2$

For Gen Kelly  
uses the Galster  
Parametrization

TABLE I. Parameters fitted to data for nucleon electromagnetic form factors. The normalization parameter  $a_0=1$  was held constant. The second column lists chi-square per datum.

Quantity	$\chi^2/N$	$a_1$	$b_1$	$b_2$	$b_3$	$r_{\text{rms}}$ (fm)	$A$	$B$	$\langle r_n^2 \rangle$ (fm <sup>2</sup> )
$G_{Ep}$	0.78	$-0.24 \pm 0.12$	$10.98 \pm 0.19$	$12.82 \pm 1.1$	$21.97 \pm 6.8$	$0.863 \pm 0.004$			
$G_{Mp}/\mu_p$	1.06	$0.12 \pm 0.04$	$10.97 \pm 0.11$	$18.86 \pm 0.28$	$6.55 \pm 1.2$	$0.848 \pm 0.003$			
$G_{Mn}/\mu_n$	0.51	$2.33 \pm 1.4$	$14.72 \pm 1.7$	$24.20 \pm 9.8$	$84.1 \pm 41$	$0.907 \pm 0.016$			
$G_{En}$	0.80						$1.70 \pm 0.04$	$3.30 \pm 0.32$	$-0.112 \pm 0.003$

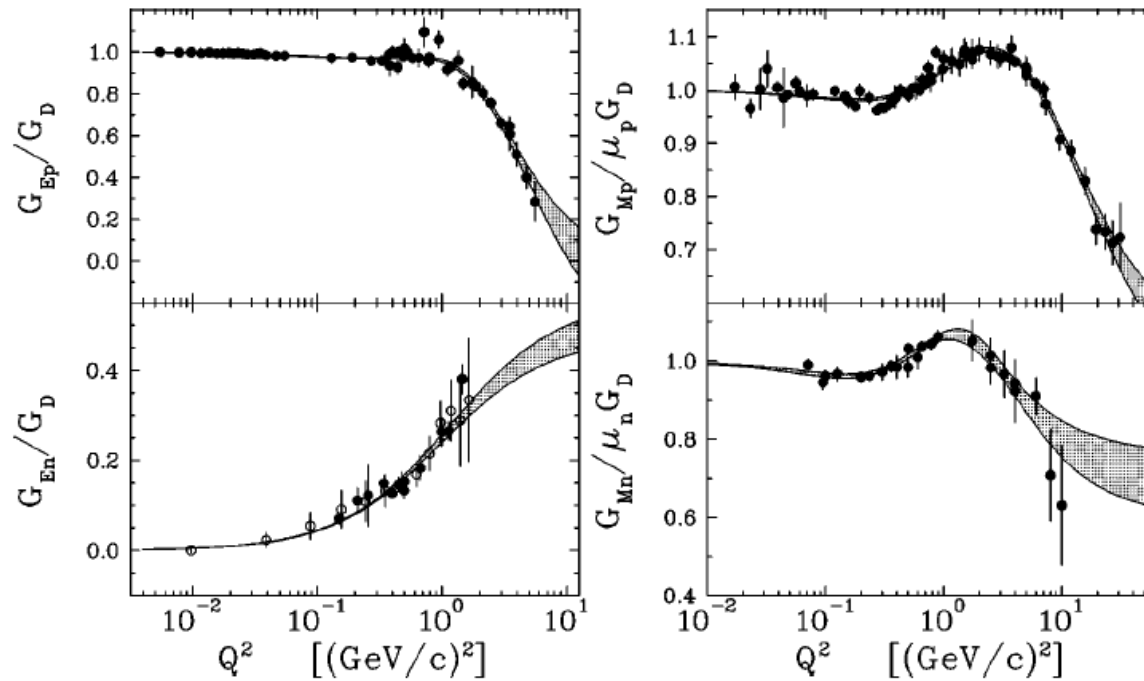


FIG. 1. Fits to nucleon electromagnetic form factors. For  $G_{En}$ , data using recoil or target polarization [16–22] are shown as filled circles while data obtained from the deuteron quadrupole form factor [23] are shown as open circles.

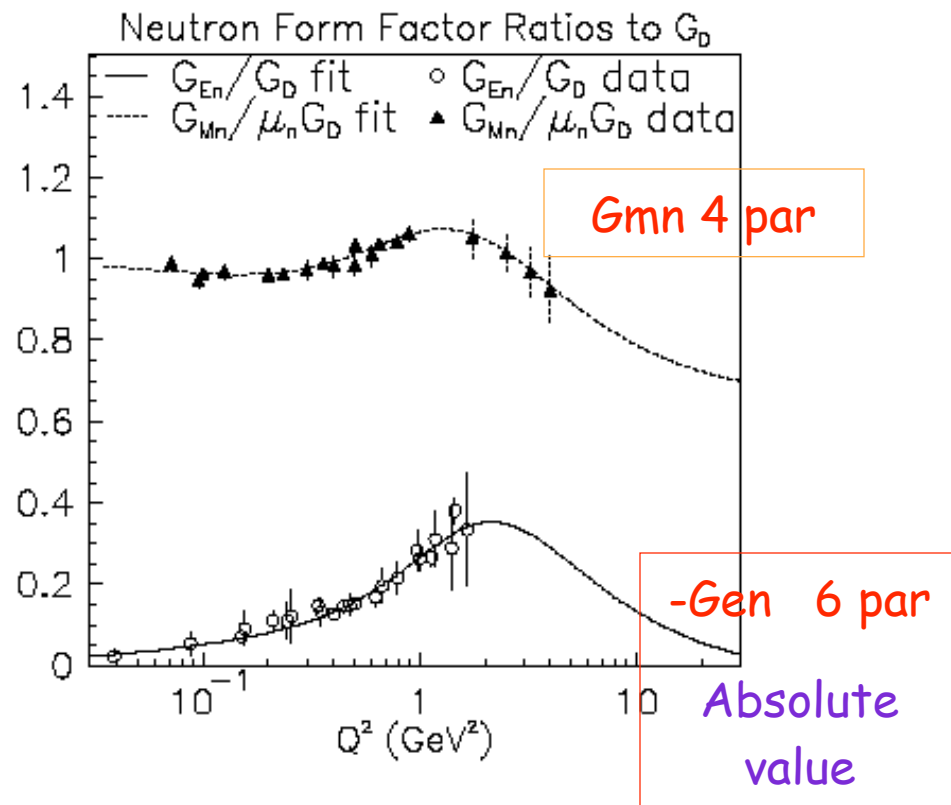
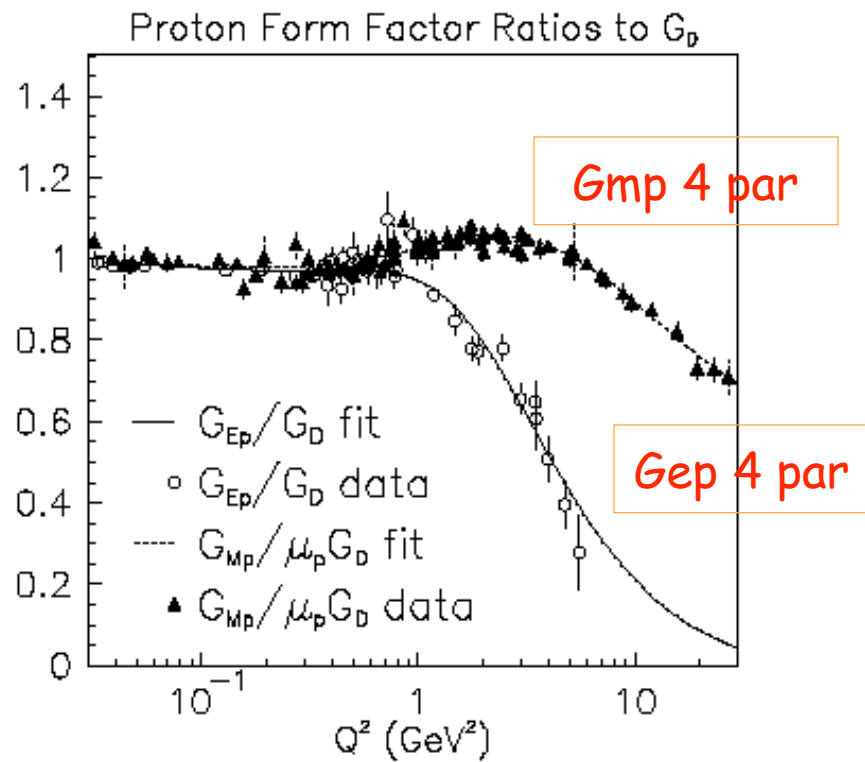
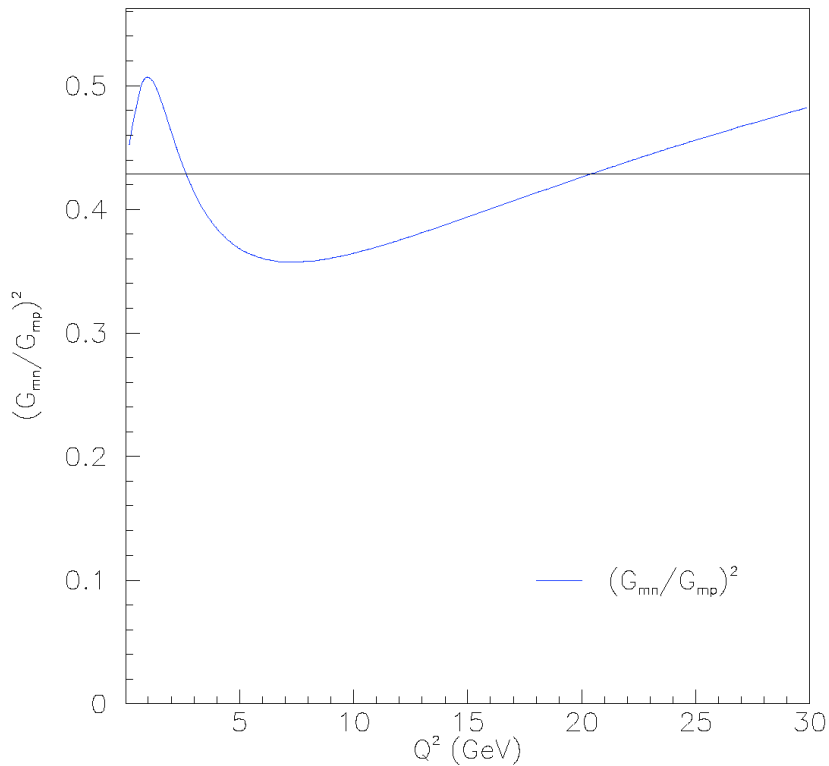


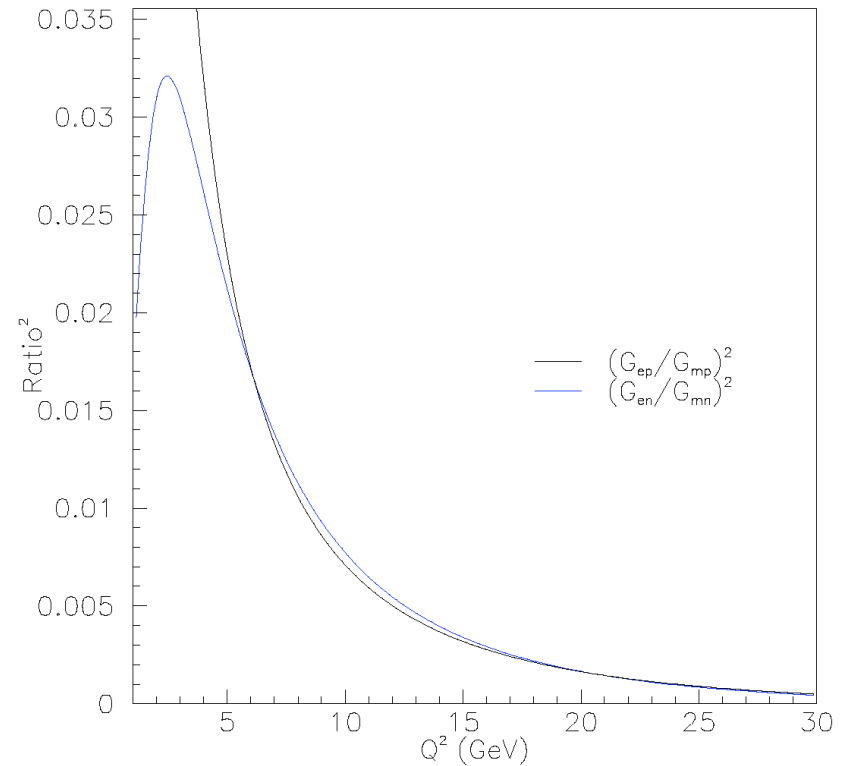
Figure 1: Our recent fits to  $G_E^p/G_{Dipole}$ ,  $G_M^p/\mu_p G_{Dipole}$ ,  $G_E^n/G_{Dipole}$ , and  $G_M^n/\mu_p G_{Dipole}$ . The data used to extract these fits are shown. These fits use constraints from QCD and duality at high  $Q^2$ , where there is no data.

BBBA- 2005 - Bodek, Bradford, Budd, Arrington QCD-Duality Constraint Form Factors-----We refit the Form Factors using the Kelly Parametrization for Gep, Gmp, Gmn, In addition, we use this parametrization (with 6 parameters) to also fit Gen. All parameters are varied such as to satisfy QCD duality constraints at high Q<sup>2</sup>.

$$(G_{mn}/G_{mp})^2 \rightarrow .42857$$



$$(G_{ep}/G_{mp})^2 = (G_{en}/G_{mn})^2 \text{ as } Q^2 \rightarrow \infty$$



QCD and Duality: At High  $Q^2$  near  $x=1$

$$F1n/F1p = (G_{mn}/G_{mp})^2 = 0.43$$

From  $F1n/F1p$  DIS prediction with

$d/u = 0.2$  at  $x=1$  (From QCD)

Duality:  $R_p=R_n$  (inelastic) at high  $Q^2$  near  $x=1$ .

Implies

$$|G_{en}/G_{mn}| = |G_{ep}/G_{mp}|$$

•BBBA2005 Form Factors satisfy QCD and Duality constraints

BBBA- 2005 - Bodek, Bradford, Budd, Arrington QCD-Duality  
 Constraint Form Factors - should work both at low and High Q<sup>2</sup>

a1	b1	b2	b3	GEP/GD
-0.5972961E-01	11.17692	13.62536	32.96109	parameter
0.1852694	0.2370240	1.435371	9.888144	error

a1	b1	b2	b3	GMP/GD
0.1500081	11.05341	19.60742	7.536569	parameter
0.3059846E-01	0.1020370	0.2822719	0.9482885	error

a1	a2	b1	b2	b3	b4	GEN/GD
3.488283	-.2175027	51.54248	16.33405	146.7034	159.0527	param.
0.5096608	0.4067491E-01	9.908267	31.33876	78.36866	25.36869	error

a1	b1	b2	b3	GMN/GD
1.815929	14.09349	20.69266	68.58896	parameter
0.4214175	0.6181164	2.643576	14.75946	error

Next (summer 2004)

- (1) Working on getting better description of axial form factor  $F_a(Q^2)$  using High Q<sup>2</sup> QCD Duality Constraints for both vector and axial form factors and the Adler Sum rule.
- (2) Compare to absolute value predictions from Duality using standard PDFs at high Q<sup>2</sup>.

## Summary of Unified LO Approach works from $Q^2=0$ to high $Q^2$

*For applications to Neutrino Oscillations at Low Energy (down to  $Q^2=0$ ) the best approach is to use a LO PDF analysis (including a more sophisticated target mass analysis) and **modify** to include the missing QCD higher order terms via Empirical Higher Twist Corrections.*

### Reason:

For  $Q^2 > 5$  both Current Algebra exact sum rules (e.g. Adler sum rule) and QCD sum rules (e.g. momentum sum rule) are satisfied. This is why duality works in the resonance region (Here we can also use NNLO QCD analysis or a **modified leading order analysis**): **Use duality + Adler to constrain elastic vector and axial form factors.**

For  $Q^2 < 1$ , QCD corrections diverge, and all QCD sum rules (e.g. momentum sum rule) break down, and duality breaks down in the resonance region. In contrast, Current Algebra Sum rules e.g. Adler sum rule which is related to the Number of (U minus D) Valence quarks) are valid. Our unified approach works for both inelastic and elastic.