SATURATION 2005

(mini-review)

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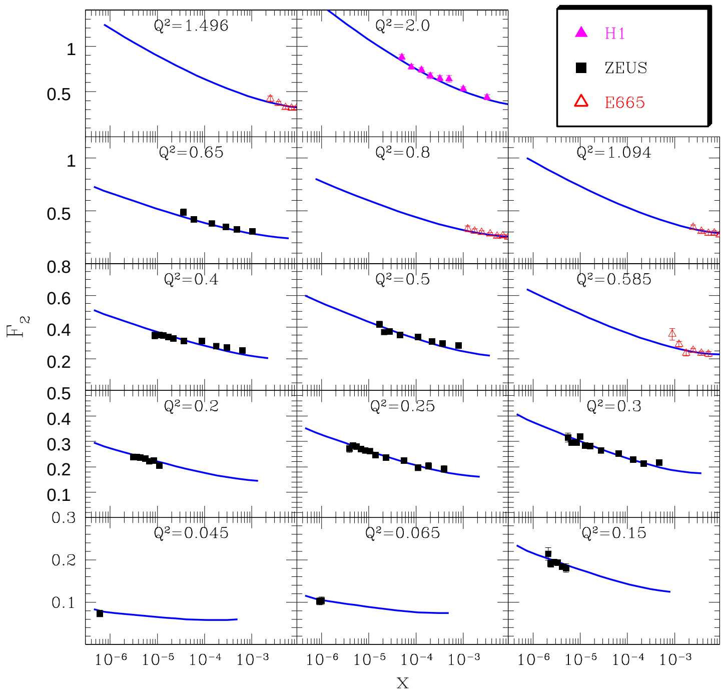
The Goal: Brief review of ups and downs of high density QCD approach in the saturation domain - my personal selection

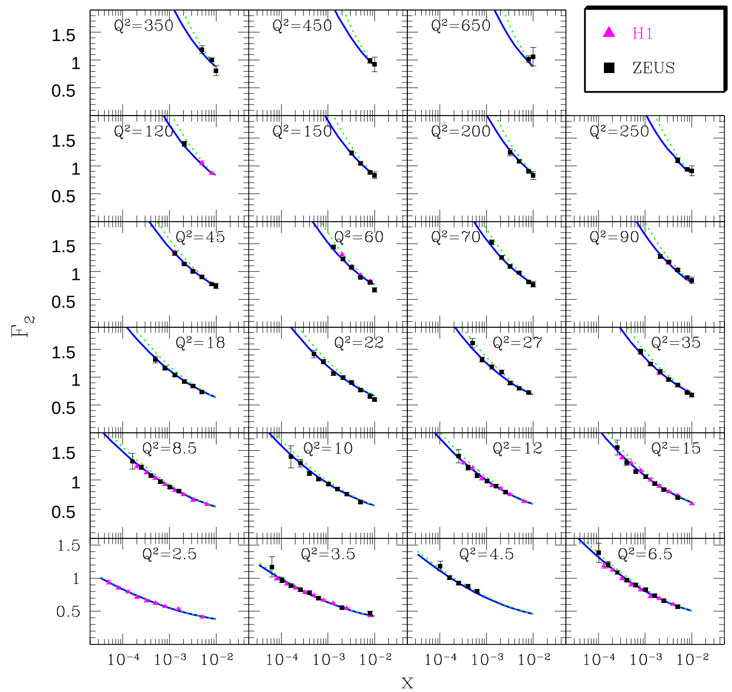
Outline:

- Saturation at HERA and RHIC?
- Reliable predictions for LHC:
  - Saturation scale;
  - Modified Balitsky-Kovchegov equation;
  - Antishadowing effect;
  - Excess for heavy quark and di-jet production at LHC ;
- Theory developments:
  - Colour dipoles versus B-JIMWLK approach;
  - Hunt for Pomeron loops;
  - Saturation and stochastic processes;
- Problems, ideas, solutions ... $\equiv$ bright future ;

What we have learned about saturation at HERA and RHIC

Low Q^2: **Large Q^2:**

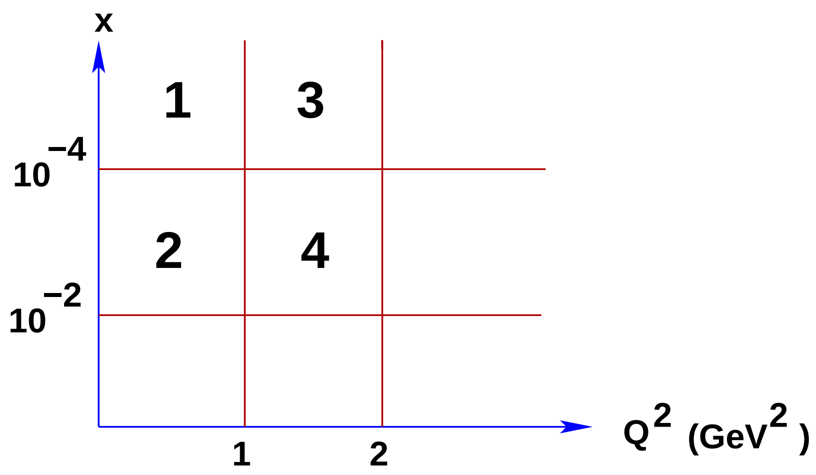




HERA

- **E. Gotsman, E. Levin, M. Lublinsky and U. Maor (2002);**
- **E. Iancu, K. Itakura and S. Munier (2003)**

χ^2 :



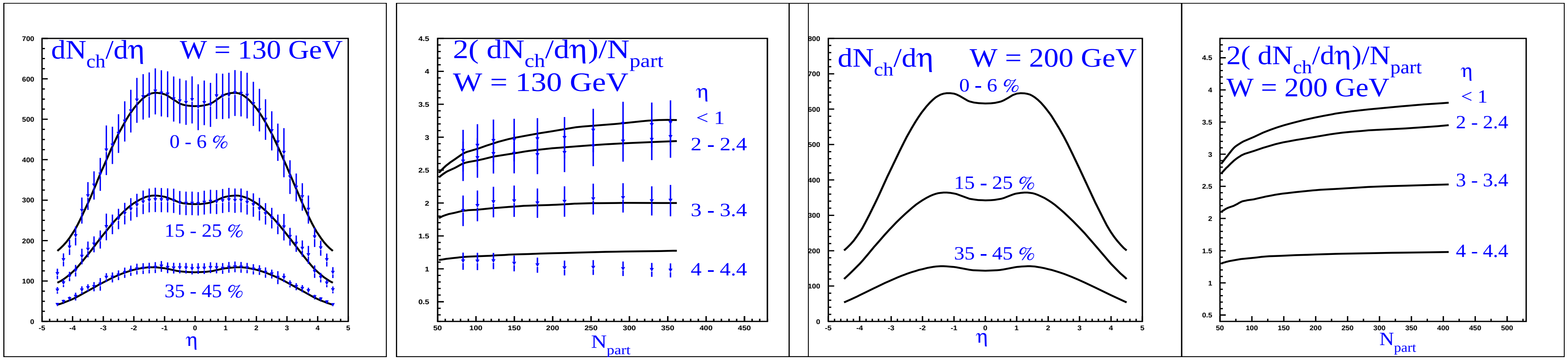
A : 1;

B : 1 + 3;

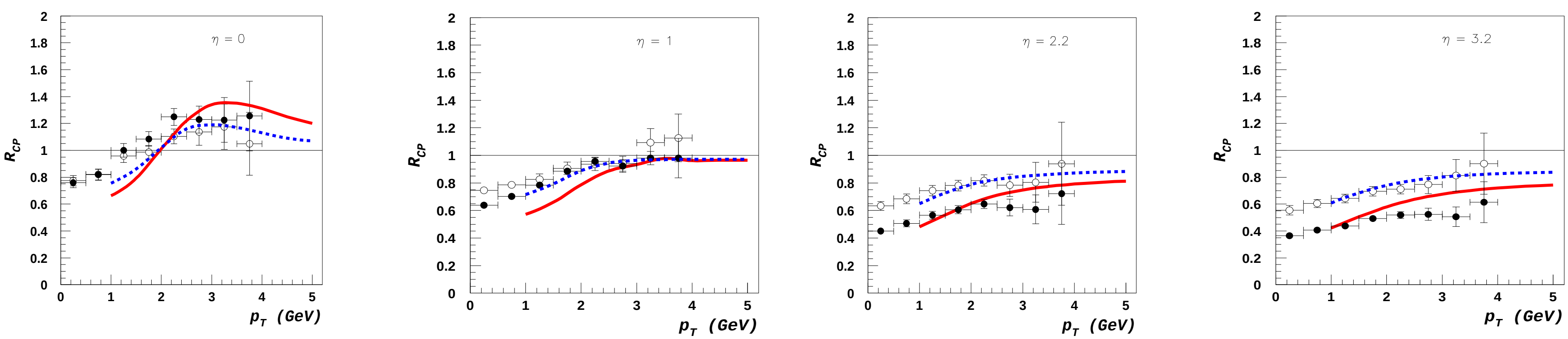
C : 1 + 2;

D: 1 + 2 + 3 + 4 ;

Region	2 σ		2.7 σ	
	χ^2/d.o.f.	points	χ^2/d.o.f.	points
A	0.55	51	0.75	53
B	0.55	55	0.8	58
C	0.75	84	1.2	91
D	0.7	103	1.1	112



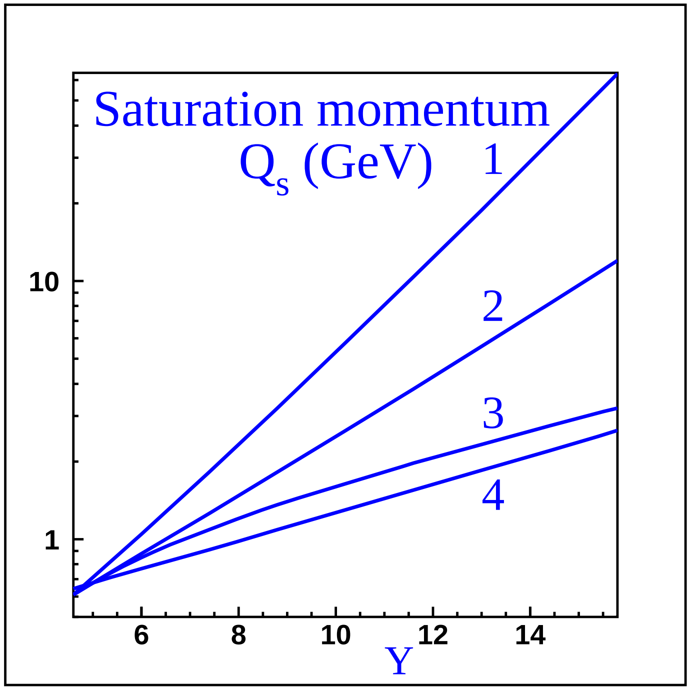
- **RHIC** **Multiplicity** **(Kharzeev, Levin & Nardi (2001))**



- **RHIC** **d-A dN/dy** **(Kharzeev, E.L. and McLerran (2002); Kharzeev, Kovchegov & Tuchin (2004))**

Prediction for LHC:

Saturation scale



1 - LO BFKL

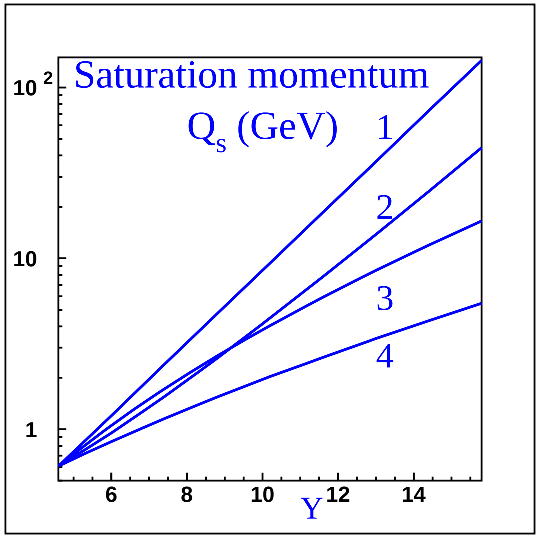
2 - Our kernel;

3 - NLO BFKL (Durham);

4 - BGW model;

Theory progress:

Energy dependence (Munier & Peschanski (2000))



1 - High energy behaviour (fixed α_S);

2 - Low energy corrections (fixed α_S);

3 - High energy behaviour (running α_S);

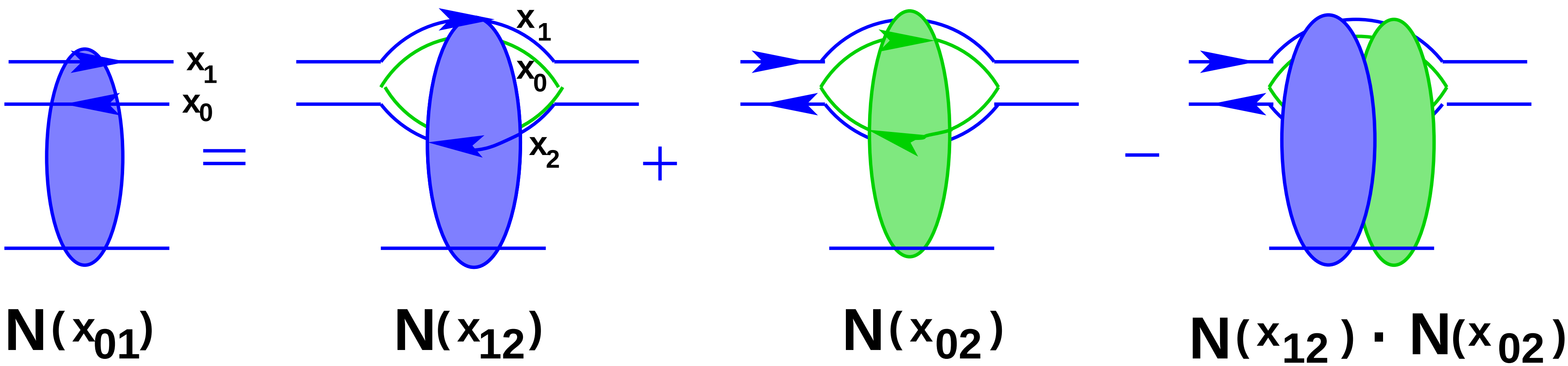
4 - Low energy corrections (running α_S);

Transferred momentum dependence

- **Phenomenology and models :** (Munier,Stasto & Mueller (2001), Munier & Wallon (2003), Kowalski & Teaney (2003))
- **Semiclassical approach:** (Bondarenko, Kozlov & Levin (2003))
- **Numeric solution to BK equation:** (Golec-Biernat & Stasto (2003), Gotsman, Kozlov, Levin, Maor & Naftali (2004))
- **Analytic approach:** (Ikeda and McLerran (2002), Marquet, Peschanski & Soyez (2005))

$$Q_s^2(Y) \;=\; q^2 \exp\left(\bar{\alpha}_S \frac{\chi(\gamma_{cr})}{1-\gamma_{cr}} Y - \frac{3}{2(1-\gamma_{cr})} ln Y\right) \;=\; (q^2/Q_0^2)\; Q_s^2(Y, q=0)$$

B-K non-linear equation:



$$\frac{\partial N(y,\vec{x}_{01},\vec{b})}{\partial y} = \frac{C_F\alpha_S}{2\pi^2}\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\;x_{12}^2}\left(2N(y,\vec{x}_{12},\vec{b}-\frac{1}{2}\vec{x}_{02})\right.$$

$$\left. - N(y,\vec{x}_{01},\vec{b}) - N(y,\vec{x}_{12},\vec{b}-\frac{1}{2}\vec{x}_{02})\,N(y,\vec{x}_{02},\vec{b}-\frac{1}{2}\vec{x}_{12})\right)$$

Deficiencies of B-K equation:

- **Correct only in LLA approximation of pQCD with BFKL kernel in LO;**
- **The mean field approximation to the JIMWLK equation;**
- **It is not correct in the saturation region;**
- **The region where we can neglect the non-linear corrections should be specified by conditions beyond the BK equation;**

B-K equation versus NLO BFKL:

$N_{\text{non-linear term}} \propto \alpha_S^4\, s^{2\Delta_{BFKL}}$; $\quad N_{\text{linear term}} \propto \alpha_S^2\, s^{\Delta_{BFKL}}$;

with $\Delta_{BFKL} = \alpha_S\,\chi_{LO\,BFKL} + \alpha_S^2\,\chi_{NLO\,BFKL}$

Correct strategy (theory point of view):

For $1/\alpha_S > y = \ln s > 1$ $\qquad N^{LOBFKL}_{\text{linear term}}$

For $y = \ln s > (2/\alpha_S)\,\ln(1/\alpha_S)$ $\qquad N^{LOBFKL}_{\text{linear term}} + N_{\text{n-l term}}$

For $y = \ln s > (1/\alpha_S^2)$ $\qquad N^{NLOBFKL}_{\text{linear term}} + N_{\text{n-l term}}$

Our suggestion: (Ellis, Kunszt and Levin 1994)

$$\bar{\alpha}_S\,\chi_{NLO}(\gamma) \;=\; -\,\omega\,\bar{\alpha}_S\,\chi_{LO}(\gamma)\;;$$

$$\omega(\gamma) \;=\; \bar{\alpha}_S\;(1\;-\;\omega)\;\bar{\alpha}_S\,\chi_{LO}(\gamma)\;;\;\;\gamma^{DGLAP} = \bar{\alpha}_S\left(\frac{1}{\omega} - 1\right);$$

Modified B-K equation:

- $$\frac{\partial N(r,Y;b)}{\partial Y} = \frac{C_F\,\alpha_S}{\pi^2}\int \frac{d^2r'\,r^2}{(\vec{r}-\vec{r}\,')^2\,r'^2}$$

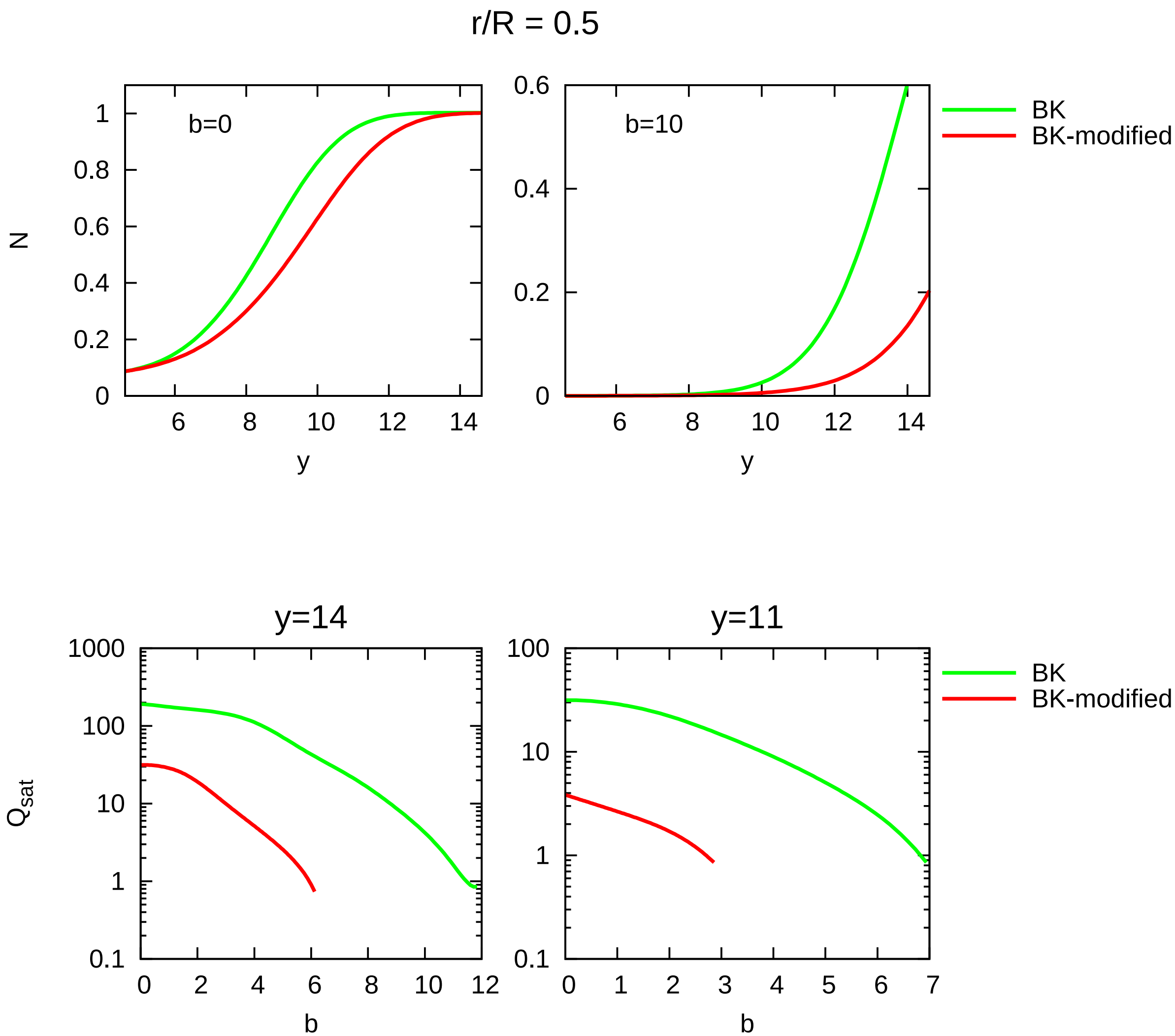
$$\left(2N\left(r',Y;\vec{b}-\frac{1}{2}(\vec{r}-\vec{r}\,')\right) - N\left(r',Y;\vec{b}-\frac{1}{2}(\vec{r}-\vec{r}\,')\right)N\left(\vec{r}-\vec{r}\,',Y;b-\frac{1}{2}\vec{r}\,'\right)\right)_{B-Kterm} -$$

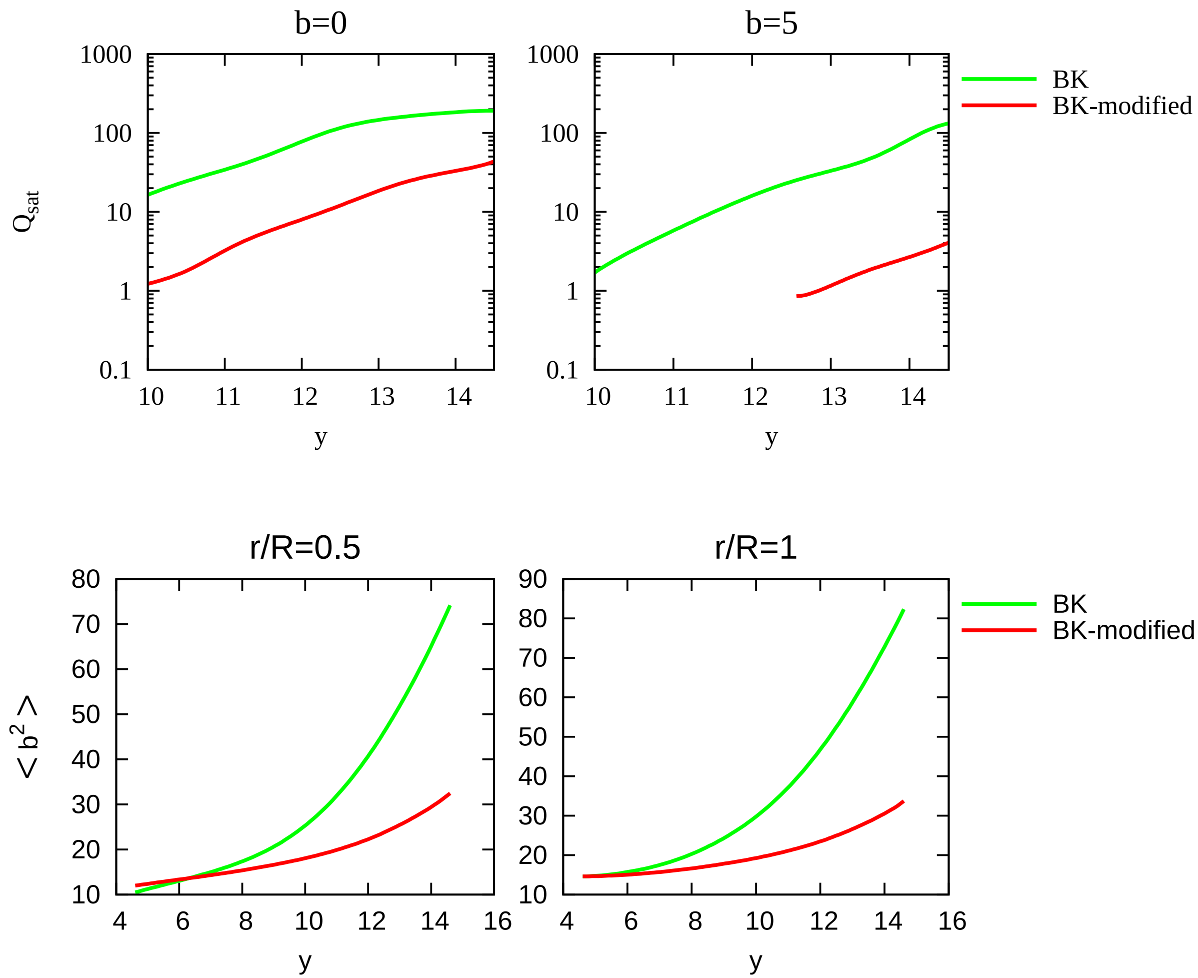
$$-\frac{\partial}{\partial Y}\left(2N\left(r',Y;\vec{b}-\frac{1}{2}(\vec{r}-\vec{r}\,')\right) - N\left(r',Y;\vec{b}-\frac{1}{2}(\vec{r}-\vec{r}\,')\right)N\left(\vec{r}-\vec{r}\,',Y;b-\frac{1}{2}\vec{r}\,'\right)\right)_{new}$$

$\bar{\alpha}_S\;\omega\;\chi_{LO}(\gamma)$ **has the following form in Y, r representation**

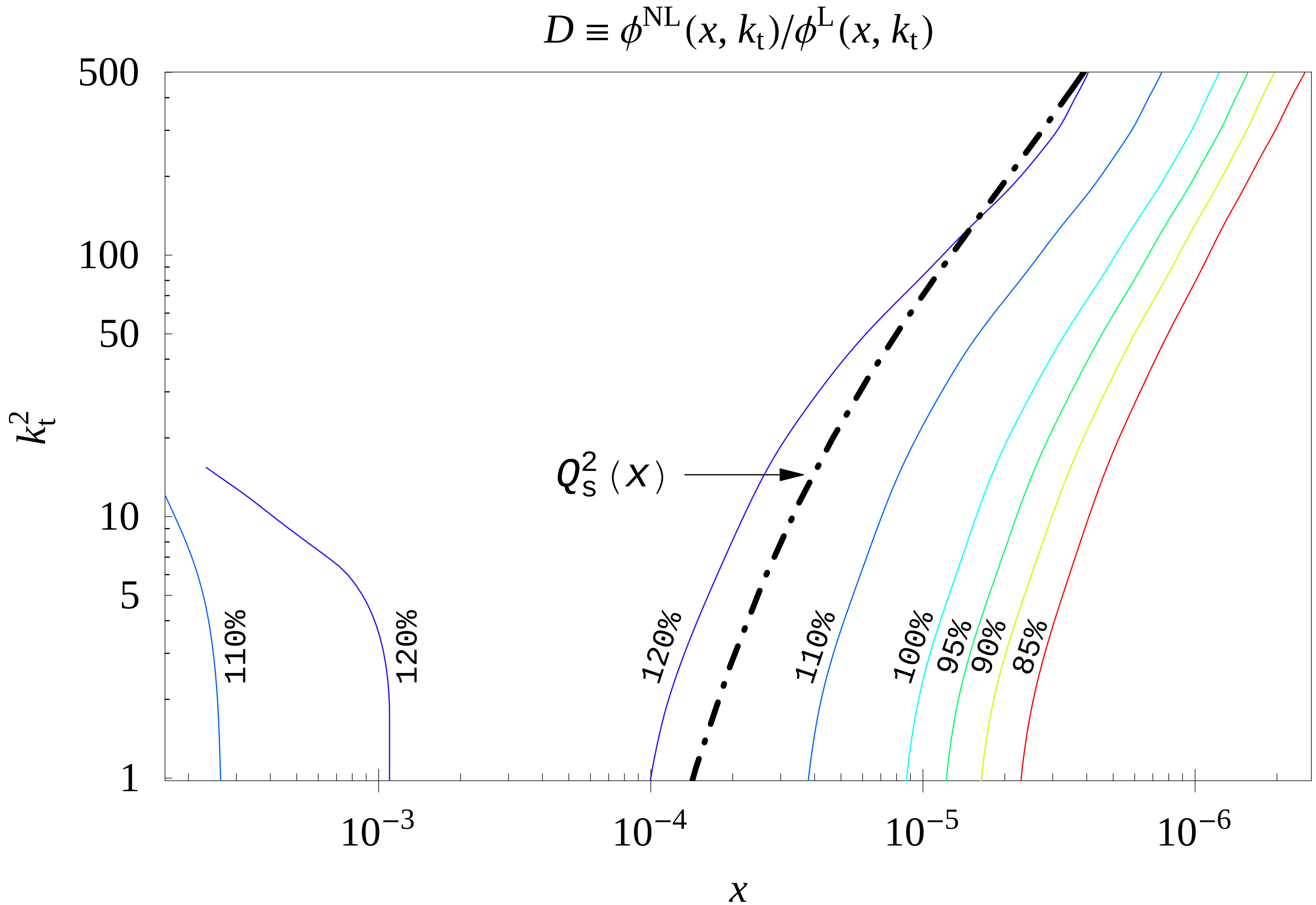
$$\bar{\alpha}_S\,\omega\;\chi_{LO}(\gamma) \;\rightarrow\; \bar{\alpha}_S\int K_{LO}(r,r')\;d^2r'\,\frac{\partial N(Y,r')}{\partial Y}$$

Modified B-K equation versus B-K equation:




b=0
b=5
1000
100
10
1
0.1
10
11
12
13
14
y
Q_{sat}
BK
BK-modified
r/R=0.5
r/R=1
80
70
60
50
40
30
20
10
90
4
6
8
10
12
14
16
y
$<b^2>$
BK
BK-modified

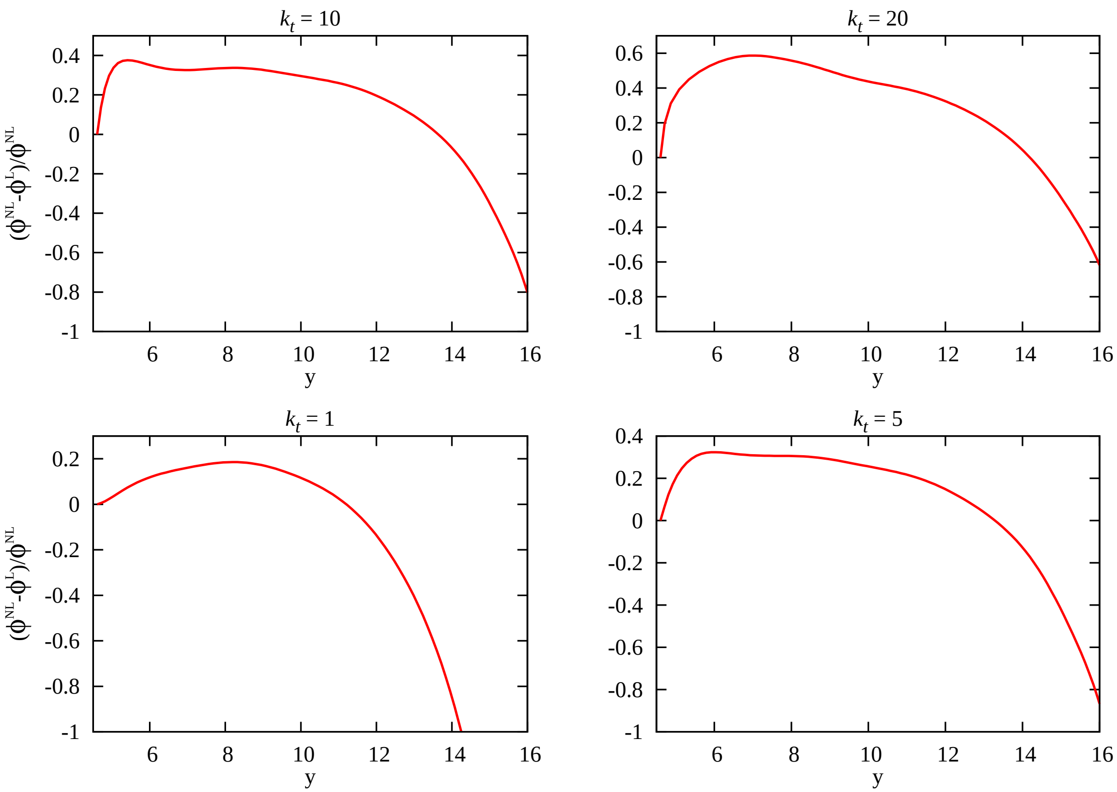
$$\frac{d\sigma}{dy\,d^2p_t} \propto \frac{\alpha_S}{p_t^2} \int d^2k_t\,\phi(k_t^2)\;\phi((\vec{p}-\vec{k})_t^2)$$



Antishadowing effects:

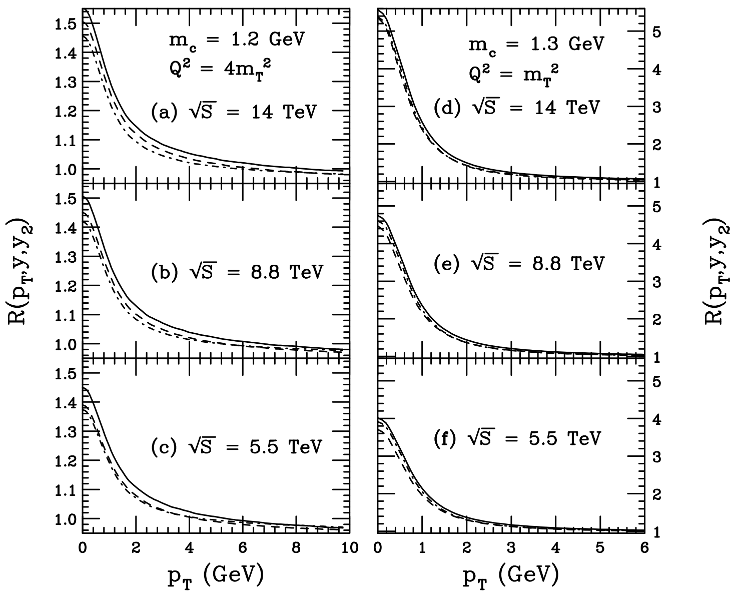
$$\int dx\, \phi^{NL}(k^2, Y) \;=\; Const(k^2) \;=\; \int dx\, \phi^{L}(k^2, Y)$$

Plot of $R - 1 = (\phi^{NL} - \phi^{L})/\phi^{L}$



Excess of heavy quarks:

(Eskola, Honkanen, Kolhinen, J-W Qiu & Salgado (2002-))



In GLR-MQ approach:

$$\frac{\partial^2 xG}{\partial \ln Q^2\, d\partial y} = \frac{\partial^2 xG^{DGLAP}}{\partial \ln Q^2\, \partial y} - \bar{\alpha}_S^2 C\, \frac{xG^2}{Q^2}$$

- **Eskola, Kolhinen & Vogt (2004)**

Theory development: Dipoles versus B-JIMWLK

Dipoles (Mueller (94))

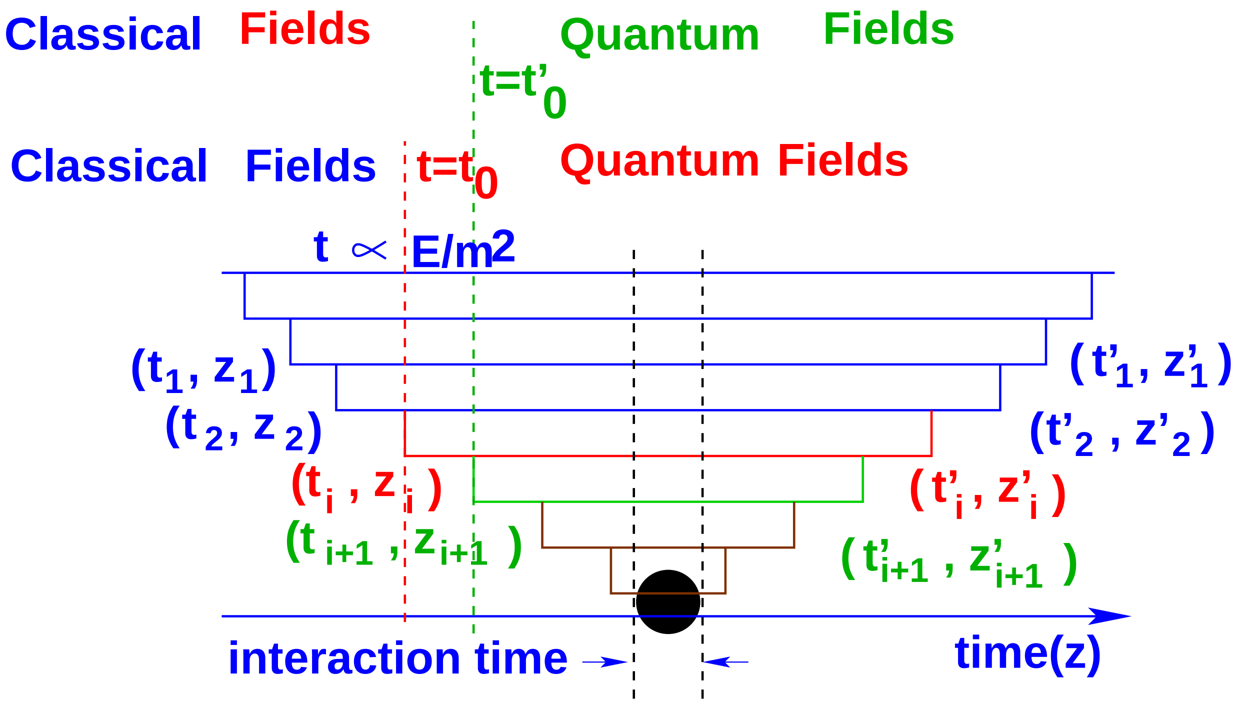
- $Z(Y;[u]) = \sum_i \int P_n(\ldots x_i, y_i \ldots) \prod_{i=1}^{n} u(x_i, y_i)\, dV_i$
- $\frac{\partial P_n}{\partial Y} = -\sum_i \Gamma(1 \to 2) \bigotimes \left(P_n(\ldots x_i, y_i \ldots) - P_{n-1}(\ldots x_i, y_i \ldots)\right)$
- $\frac{\partial Z(Y,[u])}{\partial Y} = \Gamma(1 \to 2) \bigotimes u(1-u) \frac{\delta}{\delta u} Z(Y,[u])$

(E.L. & Lublinsky (2002))

Initial conditions:

- $Z(Y,[u=1]) = 1$;
- $Z(Y = Y_0,[u]) = u$;

CGC and JIMWLK equation:



$t_1 - t'_1 \gg t_2 - t'_2 \gg \ldots \gg t_i - t'_i \gg t_{i+1} - t'_{i+1}$

At $t = t_0$: $L(\rho) + j_\mu \cdot A_\mu + L(A)$

At $t = t'_0$: $L(\rho) + j_\mu \cdot A_\mu + L(A)$

B- JIMWLK $\longleftrightarrow$ BFKL Pomeron Calculus:

(Iancu & Mueller, Mueller & Shoshi, Iancu & Triantafyllopoulos, McLerran & . . . . , ... , ... , Kovner & Lublinsky)

Good news:

- **B- JIMWLK $=$ BFKL Pomeron Calculus;**
- **B- JIMWLK $\longrightarrow$ all BFKL Pomeron vertices;**
- **B- JIMWLK $\longrightarrow$ the first operator proof of Pomeron Calculus;**
- **B- JIMWLK does not lead to merging of Pomerons ;**
- **B- JIMWLK $\equiv$ dipole approach;**

Bad news:

- **No progress has been achieved in Pomeron calculus;**
- **My nightmare: Lipatov is correct with his effective Lagrangian;**

Probabilistic interpretation: a hope?!

" Reggeon field theory is equivalent to a chemical process where a radical can undergo diffusion, absorption, recombination, and autocatalytic production. Physically, these "radicals" are wee partons (colour dipoles)."

(P. Grassberger & K. Sundermeyer: "Reggeon Field Theory and Markov processes" (1978))

(Grassberger & Sundermeyer (1978), E.L. (1992), Boreskov (2004))

Probabilistic interpretation:

Colour dipoles:

1. **the wee partons of the BFKL Pomeron;**

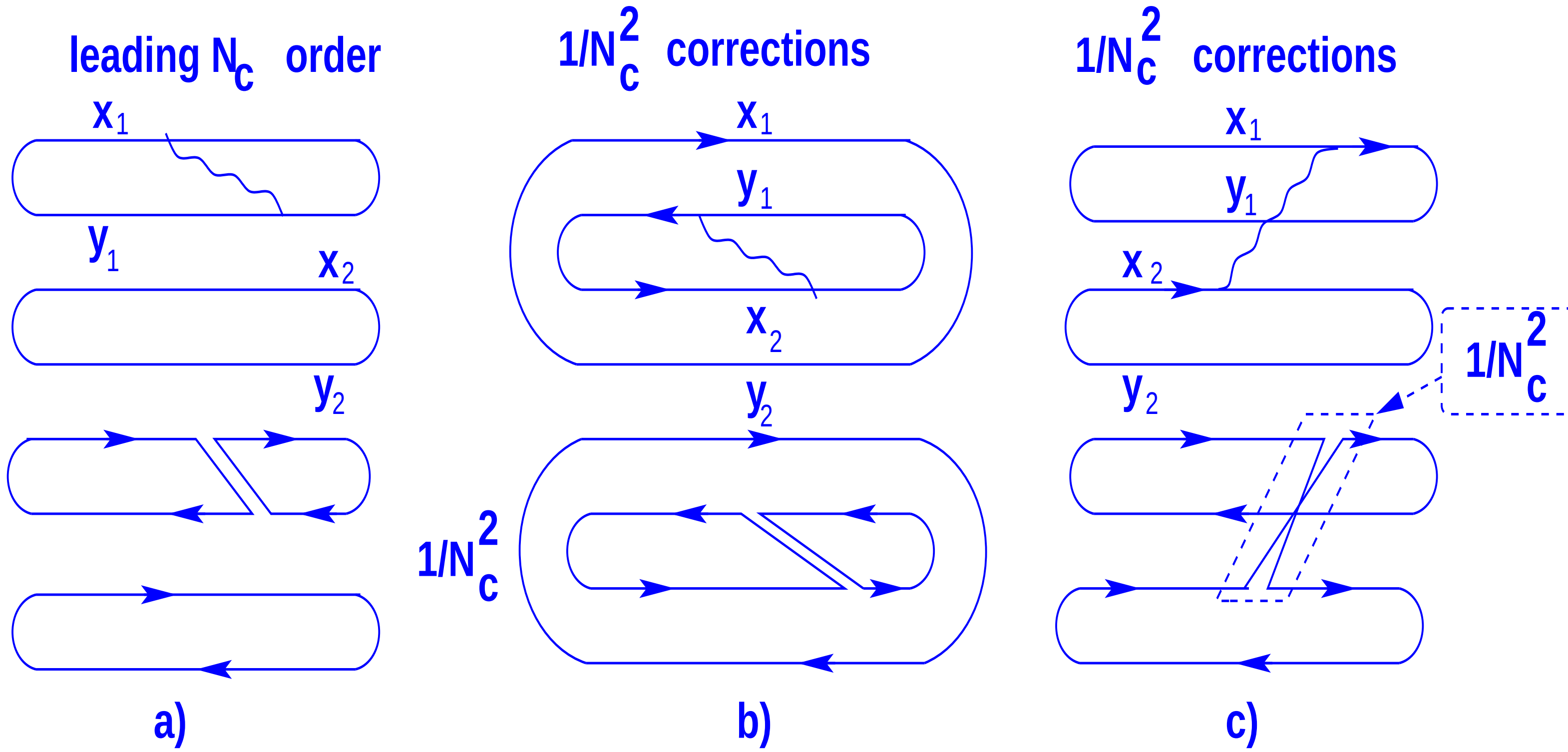
2. **the correct degrees of freedom at high energy;**

The typical death-birth process (Markov chain):

- $\frac{\partial P_n}{\partial Y} = -\sum_i \Gamma(1 \to 2) \bigotimes \left(P_n(\ldots x_i, y_i \ldots) - P_{n-1}(\ldots x_i, y_i \ldots) \right)$

DOF:

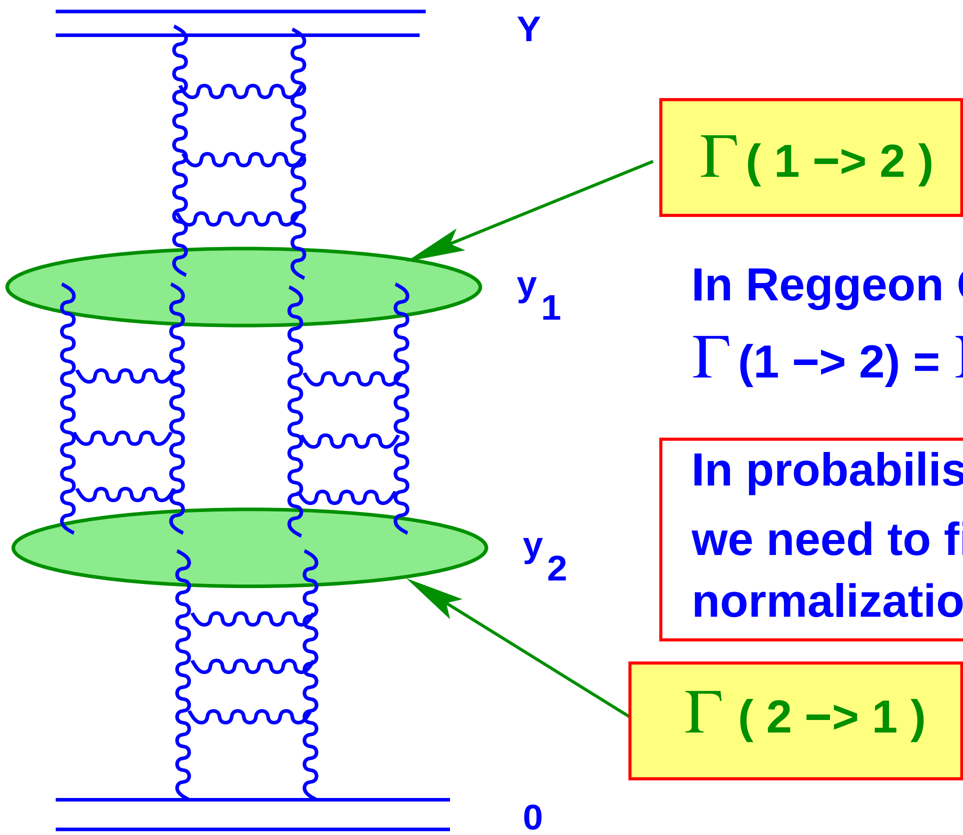
P -> 2 P　　　2 P -> 3 P　　　2P -> MRP



1 BFKL Pomeron topology　　**2 BFKL Pomeron topology**　　**Multi Reggeon Pomeron**

$$\Delta_{2} \gg \Delta_{MRP}$$

Hunt for Pomeron loops:



In Reggeon Calculus

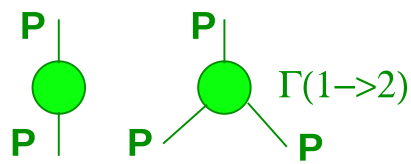
Γ(1 –> 2) = Γ(2–> 1)

In probabilistic interpretation we need to find a correct normalization for Γ(2–> 1)

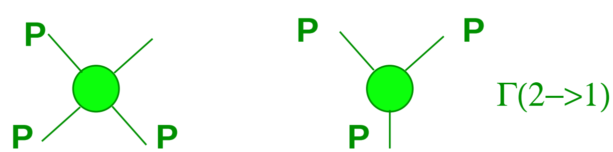
(Salam, Mueller & Salam,
Iancu & Mueller,
Mueller, Munier & Shoshi,
Iancy & Triantafyllopoulos,
Mueller, Shoshi & Wong,
E.L & Lublinsky,
Kovner & Lublinsky,
Rembiesta & Stasto,
....................................)

Markov process with Pomeron loops:

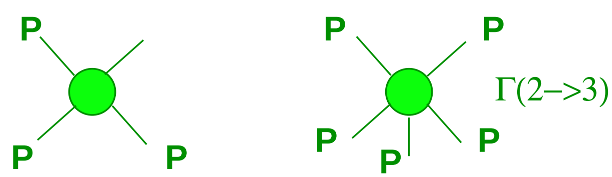
- $\frac{\partial P_n}{\partial Y} = -\sum_i \Gamma(1 \longrightarrow 2) \otimes \left(P_n(\ldots x_i, y_i \ldots) - P_{n-1}(\ldots x_i, y_i \ldots)\right)$



$-\sum_{i,k} \Gamma(2 \longrightarrow 1) \otimes \left(P_n(\ldots x_i, y_i, \ldots, x_k, y_k, \ldots) - P_{n+1}(\ldots x_i, y_i, \ldots, x_k, y_k, \ldots)\right)$



$-\sum_{i,k} \Gamma(2 \longrightarrow 3) \otimes \left(P_n(\ldots x_i, y_i, \ldots, x_k, y_k, \ldots) - P_{n-1}(\ldots x_i, y_i, \ldots, x_k, y_k, \ldots)\right)$



Toy model:

$$\frac{\partial Z}{\partial Y} = -\Gamma(1\to 2)\, u(1-u)\,\frac{\partial Z}{\partial u} +$$

$$\Gamma(2\to 1)\, u(1-u)\,\frac{\partial^2 Z}{(\partial u)^2} + \Gamma(2\to 3) u\,(1-u)^2 \frac{\partial^2 Z}{(\partial u)^2}$$

$$\Gamma(1\to 2) \propto \bar{\alpha}_S\,; \quad \Gamma(2\to 1) \propto \bar{\alpha}_S^3/N_c^2; \quad \Gamma(2\to 3) \propto \bar{\alpha}_S/N_c^2;$$

$$\frac{\Gamma(1\to 2)}{\Gamma(2\to 1)} \propto \frac{N_c^2}{\bar{\alpha}_S^2} \gg 1\,; \quad \frac{\Gamma(1\to 2)}{\Gamma(2\to 3)} \propto N_c^2 \gg 1$$

Asymptotic solution for $\Gamma(2 \to 3) = 0$:

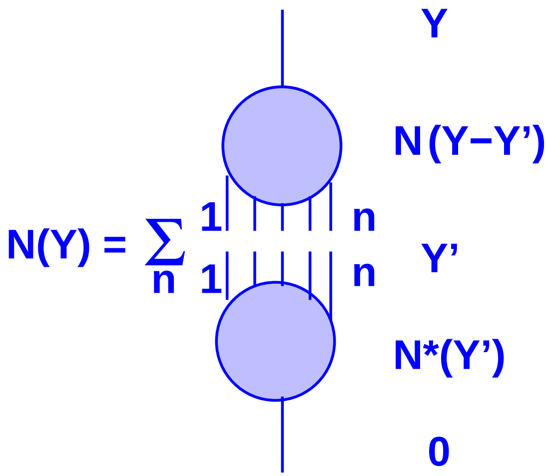
Equation:

- $$0 = -\Gamma(1 \to 2)\; u(1-u)\,\frac{\partial\, Z(Y=\infty, u)}{\partial u} + \Gamma(2 \to 1)\; u(1-u)\;\frac{\partial^2\, Z(Y=\infty, u)}{(\partial u)^2}$$

Solution:

$$Z(u; Y \to \infty) = 1 - B + B e^{\kappa(u-1)}$$

$B = 1$ from unitarity constraints



Unitarity constraint:

$$N(\infty) = \sum_{n=1}^{\infty} \left(\frac{-1}{\kappa}\right)^n n!\, \rho^p_n(\infty)\ \rho^t_n(\infty)$$

$$\rho_n \equiv \partial^n Z / \partial u^n |_{u=1}$$

Answer (**Boreskov (2004),E.L. (2005), Rembiesa & Stasto (2005))** :

$$N(Y = \infty) = 1 - e^{-\kappa} < but \neq 1$$

Lessons:

- **Asymptotic solution leads to a grey disc (not black!!!);**

- **Using the large parameters of our theory, the semiclassical approach can be developed for searching for both the asymptotic solution and the correction to this solution;**

- **The corrections to the asymptotic solution decrease at large values of Y and can be found from the Liouville-type linear equation;**

- **The important region of u are $u \rightarrow 1$ which should be specified by using the unitarity constraint;**

Saturation and stochastic processes:

(Weigert, Blaizot,Iancu & Weigert, Iancu & Triantafyllopoulos, Mueller, Shoshi & Wong, E.L., . . .)

Poisson representation:

- $$P_n(Y) = \int d\alpha\, F(\alpha, Y) \left(\frac{\alpha^n}{n!} e^{-\alpha} \right) \equiv < P_n(\alpha) >$$

- $$Z(Y; u) = \int d\alpha\, F(\alpha, Y)\, e^{(u-1)\alpha}$$

- $$\frac{\partial\, F(\alpha, Y)}{\partial\, Y} = -\frac{\partial}{\partial\, \alpha} \left(A(\alpha F(\alpha, Y)) + \frac{1}{2} \frac{\partial^2}{\partial\, \alpha^2} (B(\alpha F(\alpha, Y)) \right.$$

$$d\alpha = A(\alpha) + \sqrt{B(\alpha)}\, dW(Y)$$

where $dW(Y)$ is a stochastic differential for the Wiener process

Could lead to a computation of the high energy amplitude using direct methods !!!

My conclusions:

- We have a good chance to make reliable estimates of the non-linear effects at LHC energies;
- These effects lead not only to a suppression but also to an increase of the inclusive cross sections for $k \geq Q_s$;
- Theory becomes dangerously complicated but very interesting;
- We still have a hope to develop a reliable computer procedure to calculate the main properties of the new phase of QCD Colour Glass Condensate;