Diffractive deep inelastic scattering

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Abstract. A new approach to the analysis of diffractive deep inelastic data is presented. We show that the collinear factorisation theorem, which holds for diffractive DIS, has important modifications in the sub-asymptotic HERA regime, which can be quantified by using perturbative QCD. In fact the diffractive parton densities are shown to satisfy an inhomogeneous evolution equation. Moreover it is necessary to include both the gluonic and sea-quark t-channel components of the perturbative Pomeron.

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INTRODUCTION

A notable feature of deep-inelastic scattering is that some $10-20\%$ of the events are diffractive, $\gamma^* p \to X + p$, in which the slightly deflected proton and the cluster X of outgoing hadrons are well-separated in rapidity. At high energies, the large rapidity gap is believed to be associated with Pomeron, or vacuum-quantum-number, exchange. Some secondary Reggeons also have vacuum quantum numbers, but these contributions are exponentially suppressed as a function of the gap size, and are negligible at small $x_{I\!P}$.

It is common to perform analyses of diffractive DIS (DDIS) data based on two levels of factorisation. First the diffractive structure function $F_2^{\rm D}$ may be written as the convolution of the usual coefficient functions as in DIS with diffractive parton distribution functions $a^{\rm D}$ [1]:

$$F_2^{\rm D} = \sum_{a=q,g} C_{2,a} \otimes a^{\rm D}, \tag{1}$$

with factorisation scale μ_F, where $a^{\rm D} = \beta q^{\rm D}$ or $\beta g^{\rm D}$ satisfy DGLAP evolution in μ_F. The *collinear factorisation* theorem (1) applies when μ_F is made large, therefore it is correct up to power-suppressed corrections. In a second stage, *Regge factorisation* is usually assumed [2], such that the diffractive parton densities $a^{\rm D}$ are written as a product of the Pomeron flux factor $f_{I\!P}(x_{I\!P}, t)$ and the Pomeron parton densities $a^{I\!P} = \beta q^{I\!P}$ or $\beta g^{I\!P}$. Taking $\mu_F = Q$, the t-integrated form is

$$a^{\rm D}(x_{I\!P}, \beta, Q^2) = f_{I\!P}(x_{I\!P})\, a^{I\!P}(\beta, Q^2), \tag{2}$$

where the Pomeron flux factor is taken from Regge phenomenology,

$$f_{I\!P}(x_{I\!P}) = \int_{t_{\rm cut}}^{t_{\rm min}} {\rm d}t \frac{{\rm e}^{B_{I\!P} t}}{x_{I\!P}^{2\alpha_{I\!P}(t)-1}}, \tag{3}$$

with $\alpha_{I\!P}(t) = \alpha_{I\!P}(0) + \alpha'_{I\!P} t$. For simplicity of presentation we omit the contribution of secondary Reggeons to the right-hand side of (2). Strictly speaking, the parameters $\alpha_{I\!P}(0)$, $\alpha'_{I\!P}$, and $B_{I\!P}$ must be taken from fits to soft hadron data. However, then one fails to describe the $x_{I\!P}$ dependence of the $F_2^{\mathrm{D}(3)}$ diffractive data. Therefore, as a rule (see [3]), $\alpha'_{I\!P}$ and $B_{I\!P}$ are fixed by the analyses of soft hadron data, while $\alpha_{I\!P}(0)$, and the parameters describing the input Pomeron parton distributions at some scale μ_0, are determined from the fit to the DDIS data. In these analyses, the Pomeron is treated as an effective pole in the complex angular momentum plane, and regarded as a hadron-like object of more or less fixed size. This Regge factorisation takes place in the non-perturbative region at some low scale μ, with $\mu < \mu_0$, of the order of the (inverse) size of the hadron. However, in such fits, the value of $\alpha_{I\!P}(0)$ extracted from DDIS data (for example, 1.17 in Ref. [3]) lies significantly above the value of 1.08 obtained from soft hadron data [4]. This indicates that there is a contribution coming from the small-size component of the Pomeron, that is, coming from the perturbative QCD region where the vacuum singularity has a larger intercept.

The Regge factorisation approach is a simplified phenomenological model. We shall not assume Regge factorisation for the whole a^{D}, but instead study the impact of applying perturbative QCD to the analysis of DDIS data. Moreover, although collinear factorisation holds in DDIS in the asymptotic limit, we will show that at the relevant HERA energies there exist important modifications.

EVOLUTION OF THE DIFFRACTIVE PARTON DENSITIES

The LO Feynman graph for the DDIS amplitude is shown by (the left-hand side of) Fig. 1, where Pomeron exchange is described by a gluon ladder. The kinematics of the process are fixed by the momentum $\tilde{k}$ of the first emitted parton at the end of the rapidity gap. Indeed, the virtuality of the first t-channel parton in the upper ladder fixes the scale $\mu^2 = k_t^2/\tilde{\beta}$, where $\tilde{\beta}$ is the (light-cone) momentum fraction of the Pomeron carried by parton $\tilde{k}$ and k_t is its transverse momentum. This scale μ plays the rôle of the lowest factorisation scale for the usual DGLAP evolution in the upper part of the diagram. *Simultaneously*, it is the upper scale for the lower part of the diagram. Indeed, the integral over the transverse momentum l_t of the t-channel gluon has the logarithmic form for $l_t^2 \ll \mu^2$, whereas for $l_t^2 \gg \mu^2$ it converges as $\mathrm{d}l_t^2/l_t^4$ and its contribution may be neglected as usual. After accounting for the evolution in the lower part of the diagram, and integrating over l_t, the probability amplitude to find the appropriate t-channel gluons is given, at LO, by the conventional integrated gluon distribution of the proton, $x_{I\!P} g(x_{I\!P}, \mu^2)$, which is known from global analyses of DIS and related hard-scattering data. To be precise, the skewing effect is incorporated using Ref. [5].

The evolution of a^{D} is a little subtle. Before we integrate over the momentum k_t — that is, selecting events with a fixed transverse momentum of the lowest parton — we can indeed see that the scale μ is the lowest possible factorisation scale for the diffractive parton densities a^{D}. However, in inclusive DDIS, we must integrate over k_t. The integral

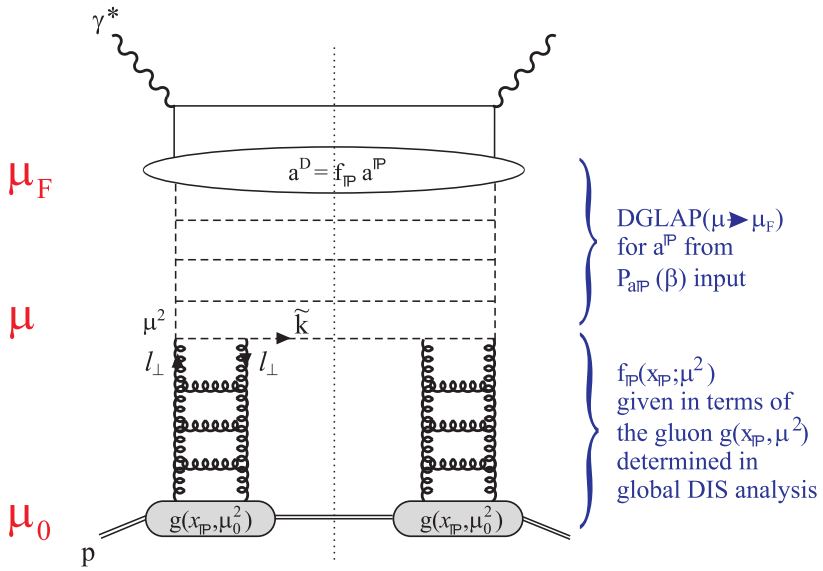

FIGURE 1. A ladder-type diagram showing a contribution to the diffractive parton densities $a^{\rm D}(x_{I\!P},\beta,\mu_F^2)$ in the perturbative region, $\mu > \mu_0 \sim 1$ GeV. DGLAP evolution for the Pomeron densities ($a^{I\!P} = \beta q^{I\!P}, \beta g^{I\!P}$) is performed from μ to μ_F for each component μ of the Pomeron in the perturbative interval $\mu_0 < \mu < \mu_F$ and then the sum is taken. The factorisation scale μ_F^2 is usually taken to be Q^2.

over k_t translates into an integral over μ, which is of the form

$$a^{\rm D} \;=\; \int_{\mu_0^2}^{Q^2} \frac{{\rm d}\mu^2}{\mu^2}\frac{1}{x_{I\!P}}\left[\frac{\alpha_S}{\mu}x_{I\!P}g(x_{I\!P},\mu^2)\right]^2 a^{I\!P} \;\equiv\; \int_{\mu_0^2}^{Q^2}\frac{{\rm d}\mu^2}{\mu^2}\, f_{I\!P}\, a^{I\!P}. \tag{4}$$

where the Pomeron flux, $f_{I\!P}$, is now given in terms of the integrated gluon, unlike (3).

Differentiating (4) with respect to $\ln Q^2$, we see that the evolution equations for the diffractive parton densities ($a^{\rm D} = \beta q^{\rm D}$, $\beta g^{\rm D}$) are

$$\begin{aligned}\frac{\partial a^{\rm D}}{\partial \ln Q^2} &= \int_{\mu_0^2}^{Q^2}\frac{{\rm d}\mu^2}{\mu^2}\, f_{I\!P}(x_{I\!P};\mu^2)\frac{\partial a^{I\!P}}{\partial \ln Q^2} \;+\; f_{I\!P}(x_{I\!P};\mu^2)\, a^{I\!P}(\beta,Q^2;\mu^2)\big|_{\mu^2=Q^2} \\ &= \frac{\alpha_S}{2\pi}\sum_{a'=q,g} P_{aa'}\otimes a'^{\rm D} \;+\; f_{I\!P}(x_{I\!P};Q^2)\,P_{a I\!P}(\beta).\end{aligned} \tag{5}$$

Here, $a^{I\!P}(\beta,Q^2;\mu^2)$ are the Pomeron parton densities DGLAP-evolved from a starting scale μ^2 up to Q^2, from input distributions $a^{I\!P}(\beta,\mu^2;\mu^2) = P_{a I\!P}(\beta)$. The quantities $P_{a I\!P}(\beta)$, with $a = q, g$, are the splitting functions of the perturbative Pomeron into quarks and gluons, whose LO forms are known from perturbative QCD [6, 7]. To allow for higher-order effects, we multiply the LO splitting functions by constant K factors, which are taken as free parameters in a fit to the DDIS data [8].

If we were to assume that $f_{I\!P}$ were independent of μ^2, as is the case for BFKL asymptotics where $g(x_{I\!P},\mu^2) \sim (\mu^2)^{0.5}$, then (5) would be an inhomogeneous DGLAP equation exactly analogous to that for the evolution of the parton densities of the photon [9]. At first sight this appears strange. The Pomeron, unlike the photon, is an extended object, and we might have anticipated a form factor dependence $f_{I\!P} \sim 1/(\mu^2 R_{I\!P}^2)$. For such an extreme behaviour, the inhomogeneous term would be a power correction, which conventionally is neglected in the evolution. However, as $x_{I\!P} \to 0$, the proton gluon density $g(x_{I\!P},\mu^2)$ grows as $(\mu^2)^{0.5}$, and compensates the form factor power-like suppression. The HERA domain is an intermediate region: the anomalous dimension γ is not small, but is less than 0.5, with $g \sim (\mu^2)^{\gamma}$. Thus, although the integral in (4) is convergent at large μ^2, we still cannot neglect the inhomogeneous term in (5).

In practice, rather than to solve the inhomogeneous equation (5) directly, it is more convenient to use a separate standard homogeneous DGLAP equation for each component μ of the Pomeron. Then we can sum up (that is, integrate over μ) the different μ^2 contributions, in which the standard DGLAP evolution of each component starts from its own scale μ, provided that μ is in the pQCD domain ($\mu > \mu_0$), and continues up to the (collinear) factorisation scale μ_F. The contribution coming from $\mu < \mu_0$ must be treated non-perturbatively. This component of the diffractive densities $a^{\rm D}$ is included in the starting distributions at μ_0 whose parameters are obtained by fitting to data.

SUB-ASYMPTOTIC MODIFICATION OF DDIS FACTORISATION

From the formal viewpoint, for any fixed $x_{I\!P}$, the inhomogeneous term in the evolution equation (5) for $a^{\rm D}$ dies out as $Q^2 \to \infty$. Thus if we evolve from a large enough starting scale, μ_S, we are entirely in the perturbative regime and the diffractive parton densities, $a^{\rm D}$, are described by the usual (homogeneous) DGLAP equations and satisfy the collinear factorisation theorem [1].

However, this is not true in the HERA domain. For a low starting scale μ_S, but still satisfying $\mu_S \gg \Lambda_{\rm QCD}$, we can no longer omit the inhomogeneous term in the evolution (5) of the diffractive parton densities, $a^{\rm D}$. One consequence is that we will obtain a *smaller* diffractive gluon density $g^{\rm D}$. Indeed, it is known from the global analyses of DIS data that the gluon is mainly driven by $\partial F_2/\partial \ln Q^2 \sim \alpha_S\, g$. However, for DDIS, the perturbative Pomeron contribution to $F_2^{\rm D(3)}$ is [7, 8]

$$F_{2,\rm pert.}^{\rm D(3)}(x_{I\!P},\beta,Q^2) = \int_{\mu_0^2}^{Q^2} \frac{{\rm d}\mu^2}{\mu^2}\, f_{I\!P}(x_{I\!P};\mu^2)\, F_2^{I\!P}(\beta,Q^2;\mu^2) \tag{6}$$

$$\Longrightarrow \quad \frac{\partial F_{2,\rm pert.}^{\rm D(3)}}{\partial \ln Q^2} = \int_{\mu_0^2}^{Q^2} \frac{{\rm d}\mu^2}{\mu^2}\, f_{I\!P}(x_{I\!P};\mu^2) \frac{\partial F_2^{I\!P}(\beta,Q^2;\mu^2)}{\partial \ln Q^2} + f_{I\!P}(x_{I\!P};Q^2) F_2^{I\!P}(\beta,Q^2;Q^2). \tag{7}$$

Here, $F_2^{I\!P}(\beta,Q^2;\mu^2)$ is the Pomeron structure function, evaluated from the DGLAP-evolved Pomeron parton densities $a^{I\!P}(\beta,Q^2;\mu^2)$. The first term of (7) is roughly $\alpha_S\, g^{\rm D}$. Therefore, since part of the derivative $\partial F_2^{\rm D(3)}/\partial \ln Q^2$ comes from the upper limit Q^2 of

the integral over the Pomeron scale μ, this results in a smaller diffractive gluon density than if the second term of (7) was neglected.

In addition, contributions associated with the charm quark and F_L^D violate factorisation. For moderate Q^2 it is convenient to use the fixed flavour number scheme, where the charm contribution arises from photon-gluon fusion $\gamma^* g^{I\!P} \to c\bar{c}$. However, in this case we will miss the diagram where the Pomeron directly produces the c quark, that is, when c is the lowest parton $\tilde{k}$ in the upper ladder of Fig. 1. In this case there is no evolution and no factorisation. This contribution should be added separately. Also, for large β and the Q^2 values typical at HERA, we cannot neglect the twist-four $F_L^{\mathrm{D}(3)}$ contribution [6, 7].

THE SEA-QUARK COMPONENT OF THE POMERON

So far we have taken the Pomeron to be a parton ladder where the two uppermost t-channel partons are gluons. However, at small scales μ^2, the gluon densities have a valence-like structure. They decrease with decreasing x already from $x \sim 0.01$ for $\mu^2 \sim 2$ GeV^2. On the other hand, the sea quark density $S = 2(\bar{u} + \bar{d} + \bar{s})$ increases as $xS \sim x^{-0.2}$ with decreasing x. As a consequence we have to include another contribution to the Pomeron in which the two uppermost t-channel partons in the lower ladders in Fig. 1 are a sea quark–antiquark pair. Thus we must introduce a sea-quark Pomeron flux $f_{I\!P=S}$ given by (4) with $x_{I\!P} g$ replaced by $x_{I\!P} S$. To avoid confusion we denote the flux in (4) by $f_{I\!P=G}$. In addition, we must include the $I\!P = GS$ interference term. In general, the two lower ladders in Fig. 1, shown as gluon ladders, contain both quarks and gluons.

PQCD ANALYSIS OF DDIS DATA

The pQCD approach described above has been used to analyse the new HERA DDIS data [8]. This was accomplished by summing up the parton densities given by a series of homogeneous DGLAP equations with different values of the starting scale μ. Suppressing the $x_{I\!P}$ and β dependence, the general structure is

$$a^{\mathrm{D}}(\mu_F;\mu_0) = a^{\mathrm{D}}_{\mathrm{non-pert.}}(\mu_F;\mu_0) \; + \; \int_{\mu_0^2}^{\mu_F^2} \frac{\mathrm{d}\mu^2}{\mu^2} \, f_{I\!P}(\mu^2) \, a^{I\!P}(\mu_F;\mu), \tag{8}$$

where $a^{I\!P}(\mu_F;\mu)$ results from DGLAP evolution up to μ_F starting from the input $a^{I\!P}(\mu;\mu) = P_{a I\!P}(\beta)$. The non-perturbative term, $a^{\mathrm{D}}_{\mathrm{non-pert.}}$, is exactly analogous to the procedure used in the Regge factorisation analysis described in the Introduction. The important new ingredient is the pQCD contribution given by the second term on the right-hand side of (8), which has contributions from the gluonic and sea-quark Pomeron ($I\!P = G,S$), and their interference ($I\!P = GS$). Setting $\mu_F = Q$, it is clear that (8) satisfies the inhomogeneous evolution equation (5).

A good description of both the ZEUS [10] and H1 [3] data was obtained. Diffractive parton densities resulting from the fit [8] to these combined data are shown in Fig. 2.

In summary, we have shown how to obtain *universal* diffractive parton densities a^{D} which can be used in the description of different diffractive processes. Of course, for

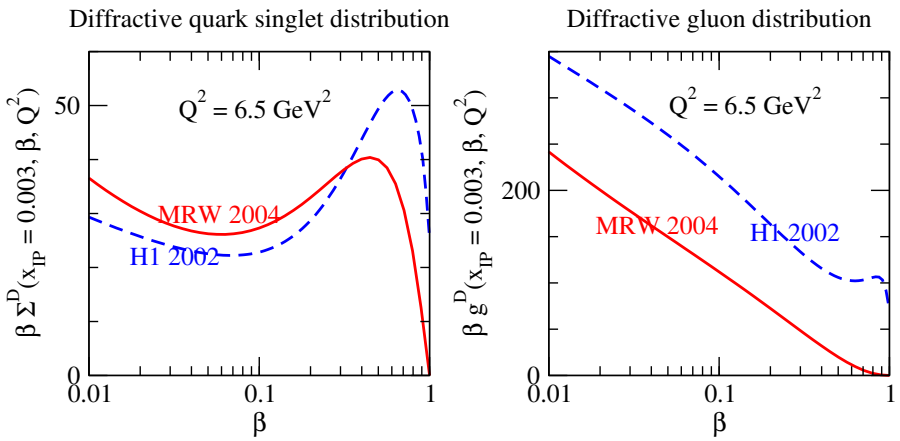

FIGURE 2. The diffractive parton densities at $Q^2 = 6.5$ GeV^2 obtained [8] from a fit to ZEUS [10] and H1 [3] DDIS data. The dashed curves are the densities obtained in the H1 fit [3].

diffractive production in high energy hadron-hadron collisions, we have to take care that the rapidity gaps are not populated by secondaries produced during the soft interaction of spectators. This well-known rapidity gap survival factor, usually denoted by S^2, may be calculated from phenomenological models tuned to describe elastic and related 'soft' hadron-hadron processes [11]. Also, in addition to the hard subprocesses originating from the collision of quarks and gluons from the *resolved* Pomeron, we must include the matrix element where the perturbative Pomeron *directly* participates in the hard interaction.

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