

---

## Diffraction Production of Vector Mesons & the gluon at small $x$

---

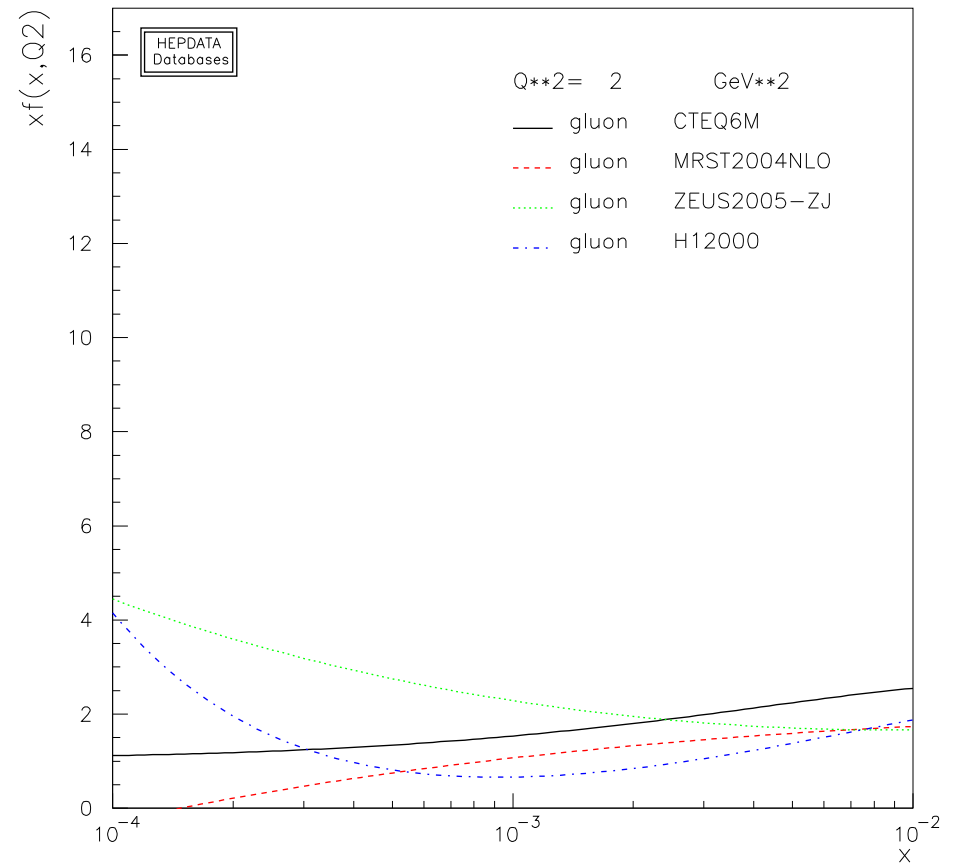
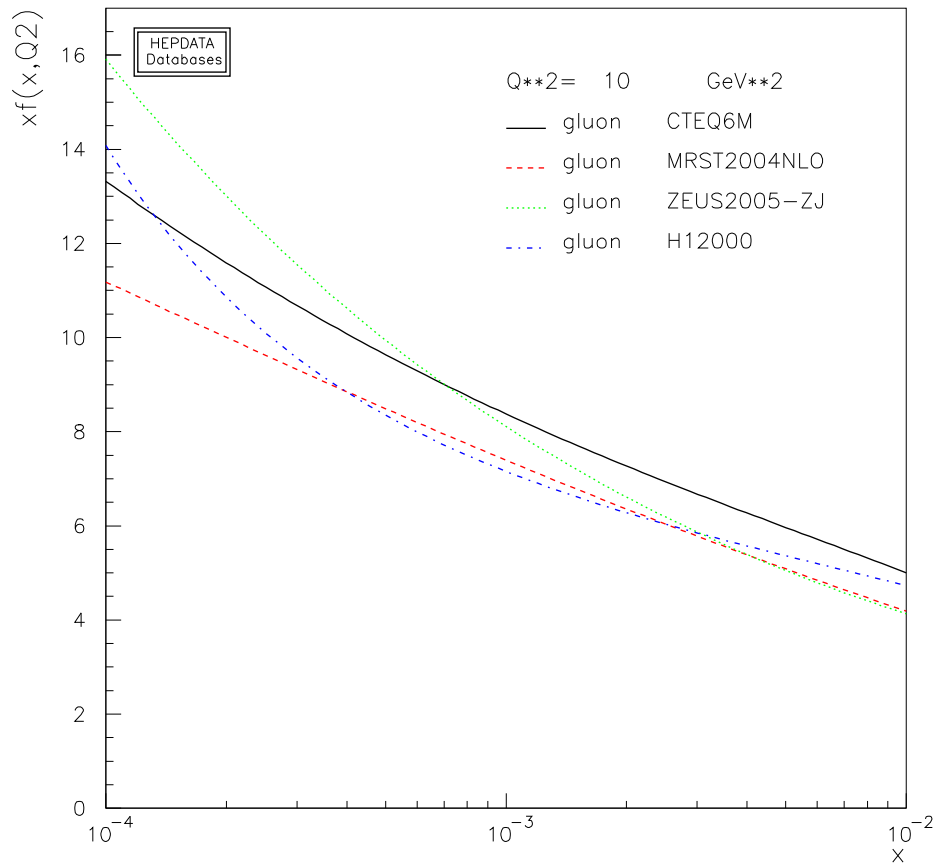
Thomas Teubner (The University of Liverpool)

- I. Why study  $\gamma^* p \rightarrow V p$  ?
- II. Theoretical Description in QCD (à la MRT using Parton Hadron Duality)
  - unintegrated gluon  $f(x, k_T^2)$  ( $k_T$  factorization)
  - skewing corrections, IR regime,  $K$  factor
- III. Results à la MRT for  $J/\psi$ :
  - how relevant are the Skewing corrections?
  - which scales; IR ‘pollution’?
- IV. Data vs TH: constraining  $f(x, k_T^2)$  via diffr. VM production. Conclusions

## I. Why study $\gamma^* p \rightarrow V p$

- Lots of data from HERA with increased accuracy and in a wider range of phase space ( $W, Q^2, M_V^2, t$ ); diffraction at hadron colliders, ILC, ...
- Challenge to understand diffractive scattering *quantitatively*
- Chance to learn about *QCD dynamics in the semi-hard regime*
- $\sigma(\gamma^* p \xrightarrow{IP \sim 2g} V p) \sim [xg(x, scale)]^2$ : *constrain the gluon distribution* at small  $x$  and small-to-intermediate scales
  - ↪ regime for diffractive Higgs at LHC!
  - ↪ relevant for description of Underlying Events at LHC, ...
  - Currently the (global) fits are only poorly constrained in this regime:

# CTEQ6M, MRST2004, ZEUS-2005 and H1-2000 gluon fits:



→ Sizeable differences even at moderately small scales.

→ At small scales even the shapes are very different!

→ Negative gluon at small  $x$  ?!

## II. Theoretical description of $\gamma^* p \rightarrow V p$ in QCD

- Recent TH-predictions go beyond the original LO formula

$$\frac{d\sigma}{dt} (\gamma^* p \rightarrow V p) \Big|_{t=0} = \frac{\Gamma_{ee}^V M_V^3 \pi^3 \alpha_s (\overline{Q}^2)^2}{48\alpha \overline{Q}^8} \left[ x g(x, \overline{Q}^2) \right]^2 \left( 1 + \frac{Q^2}{M_V^2} \right)$$

with the *effective scale*  $\overline{Q}^2 = (Q^2 + M_V^2) / 4$ . (Non-relativistic limit.)

## II. Theoretical description of $\gamma^* p \rightarrow V p$ in QCD

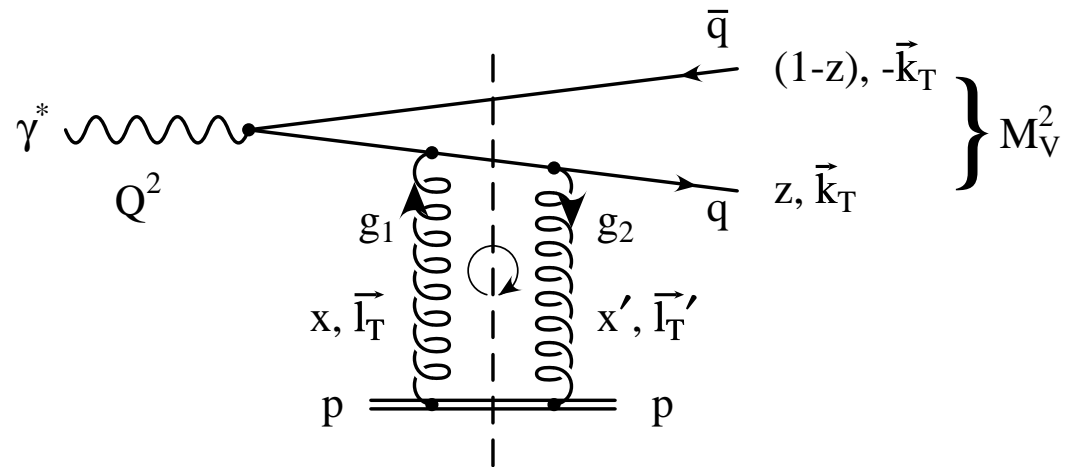
- Recent TH-predictions go beyond the original LO formula

$$\frac{d\sigma}{dt} (\gamma^* p \rightarrow V p) \Big|_{t=0} = \frac{\Gamma_{ee}^V M_V^3 \pi^3 \alpha_s (\bar{Q}^2)^2}{48\alpha \bar{Q}^8} \left[ x g(x, \bar{Q}^2) \right]^2 \left( 1 + \frac{Q^2}{M_V^2} \right)$$

with the *effective scale*  $\bar{Q}^2 = (Q^2 + M_V^2) / 4$ . (Non-relativistic limit.)

- Allow for transverse momentum  $\mathbf{k}_T$  of  $q, \bar{q}$ , avoiding the non-rel. limit.

- Take into account  $x \neq x' \rightsquigarrow$  **skewing** effects (see below),  
 $\mathcal{A} \sim$  **generalized PDF**.



- Allow for transverse momentum  $\mathbf{l}_T$  of the gluons  $\rightsquigarrow$  ' $\mathbf{l}_T$ ' factorization:

$k_T$  ( $l_T$ ) factorization formula with unintegrated gluon:

$$\mathcal{A}(\gamma_{L,T}^* p \rightarrow q\bar{q}p) = \int_0^\infty \frac{dl_T^2}{l_T^4} \alpha_s(l_T^2) f(x, x', l_T^2) \phi^{L,T}(Q^2, m^2, k_T^2, z, l_T^2)$$

- In the LLA formula the  $l_T$  of the gluons is neglected ( $l_T^2 \ll \bar{Q}^2 + k_T^2$ ):

$$\mathcal{A}^{LLA} \sim \frac{\alpha_s(K^2)}{K^2} \int^{K^2} \frac{dl_T^2}{l_T^2} f(x, l_T^2) = \frac{\alpha_s(K^2)}{K^2} xg(x, K^2)$$

$$(K^2 = z(1-z)Q^2 + k_T^2 + m^2)$$

→ Numerically this is a poor approximation!

$k_T$  ( $\ell_T$ ) factorization formula with unintegrated gluon:

$$\mathcal{A}(\gamma_{L,T}^* p \rightarrow q\bar{q}p) = \int_0^\infty \frac{d\ell_T^2}{\ell_T^4} \alpha_s(\ell_T^2) f(x, x', \ell_T^2) \phi^{L,T}(Q^2, m^2, k_T^2, z, \ell_T^2)$$

- **Unintegrated** from integrated gluon:  $f(x, \ell_T^2) = \left. \frac{\partial [xg(x, q_0^2)T(q_0^2, \mu^2)]}{\partial \ln q_0^2} \right|_{q_0^2 = \ell_T^2}$

[The *Sudakov* factor  $T = \exp\left[\frac{-C_A\alpha_s(\mu^2)}{4\pi} \ln^2 \frac{\mu^2}{q_0^2}\right]$  resums virtual corrections;

*probability for no gluon emission* in the interval  $q_0^2 < q_T^2 < \mu^2 \sim (Q^2 + M^2)/4$ .]

- At small  $\ell_T^2 < \ell_0^2 \sim 2 \text{ GeV}^2$  (**IR regime**) MRT use the 'linear' appr.:

$$\alpha_s(\ell_T^2)g(x, \ell_T^2) = (\ell_T^2/\ell_0^2)\alpha_s(\ell_0^2)g(x, \ell_0^2)$$

Alternatively: neglect  $\ell_T$  dep. in  $\phi \rightsquigarrow$  similar IR contribution, see below.

## Skewing (or off-diagonal) effects:

- Momentum fractions satisfy:  $x \simeq \frac{Q^2+M^2}{W^2+Q^2} \gg x' \simeq \frac{\ell_T^2}{W^2+Q^2}$
- In this regime the skewed (integrated) gluon  $H_g(x, x')$  is enhanced through the *off-diagonal evolution* by → Shuvaev et al.

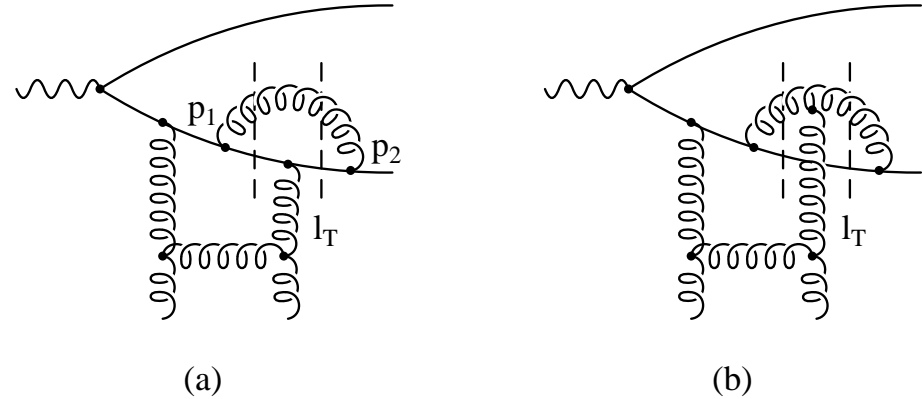
$$R_g = \frac{H_g(x, x' \ll x)}{H_g(x, x)} = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + \frac{5}{2})}{\Gamma(\lambda + 4)}$$

- The *effective power*  $\lambda(Q^2)$  of the gluon [assuming  $xg \sim x^{-\lambda}$ ] is calculated numerically for all amplitudes  $\mathcal{A}^{L,T}$ :  $\lambda = \frac{\partial \log \mathcal{A}^{L,T}}{\partial \log(1/x)}$ .
- Note:  $R_g$  is a leading  $\ln Q^2$  effect and can be sizeable when the gluon is sampled at 'large' scales (for large  $Q^2$  or  $M^2$ ), e.g.  $R_g^2 \sim 2$  for  $\Upsilon$  photoproduction at HERA.



## *K* factor:

Important missing ingredient for a full NLO prediction: **One loop corrections to the  $[(q\bar{q})(2g)]$  vertex**



- Typically lead to a significant enhancement in the normalization of QCD processes  $\rightarrow$  *K factor* (may also be fitted from data)
- Up to now no full calculation within  $k_T$  factorization
- MRT estimate the *K* factor from  $\pi^2$  enhanced terms, analogous to the well known corrections in Drell-Yan  $\rightsquigarrow \sigma = \sigma^0 \exp[\pi^2 C_F \alpha_s(\dots)/\pi]$ .\*
- First results for diagrammatic calculation for  $\mathcal{A}^L$  by **D.Yu. Ivanov et al.**

\* Exp. of the double logarithmic Sudakov form factor  $\sim \ln^2(-M^2)$ ,  $\ln(-M^2) = \ln M^2 + i\pi$

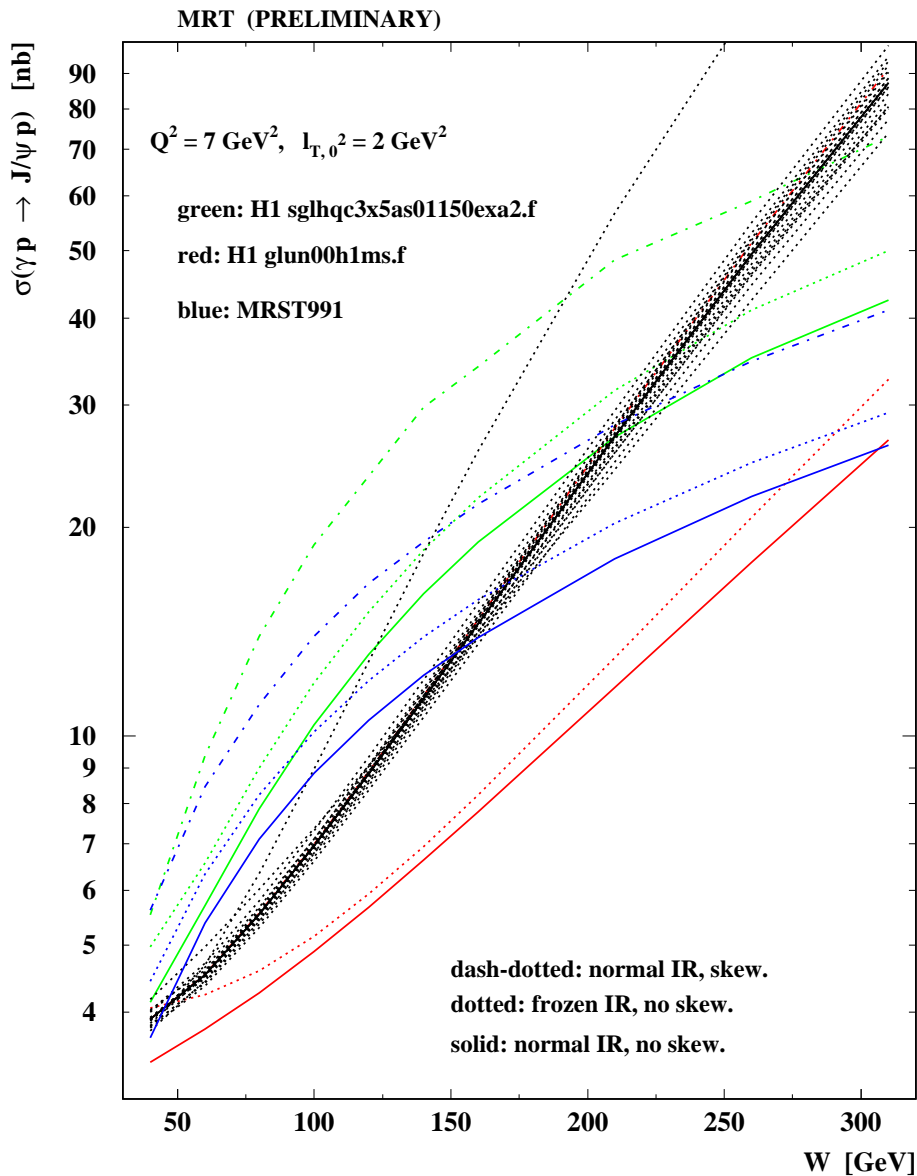
### III. Results à la MRT for $J/\psi$

How well is the TH pred. under control? Quantify different contributions!

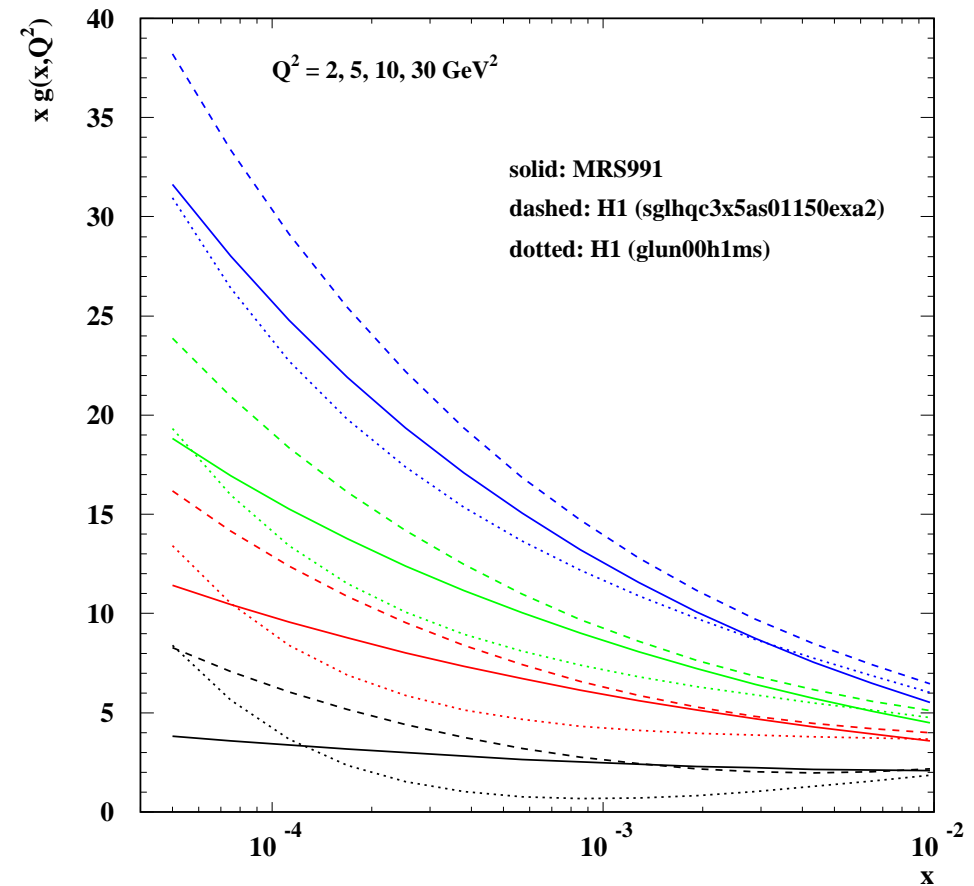
We'll see:

- **Skewing** corrections large but calculable.
- Uncertainty due to 'error band' of input gluon small compared to spread when using different fits.
- **IR** contribution non-negligible but under control.
- **High sensitivity to input gluon where it is poorly constrained!**

# Skewing corr. PDF uncertainty.



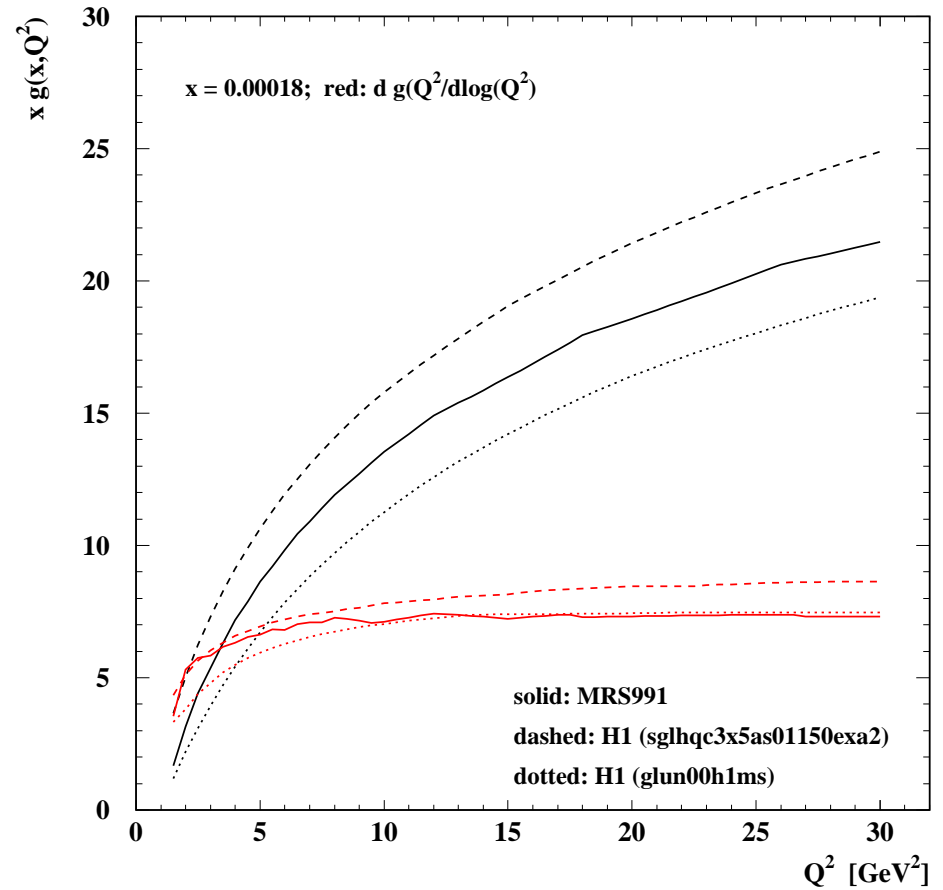
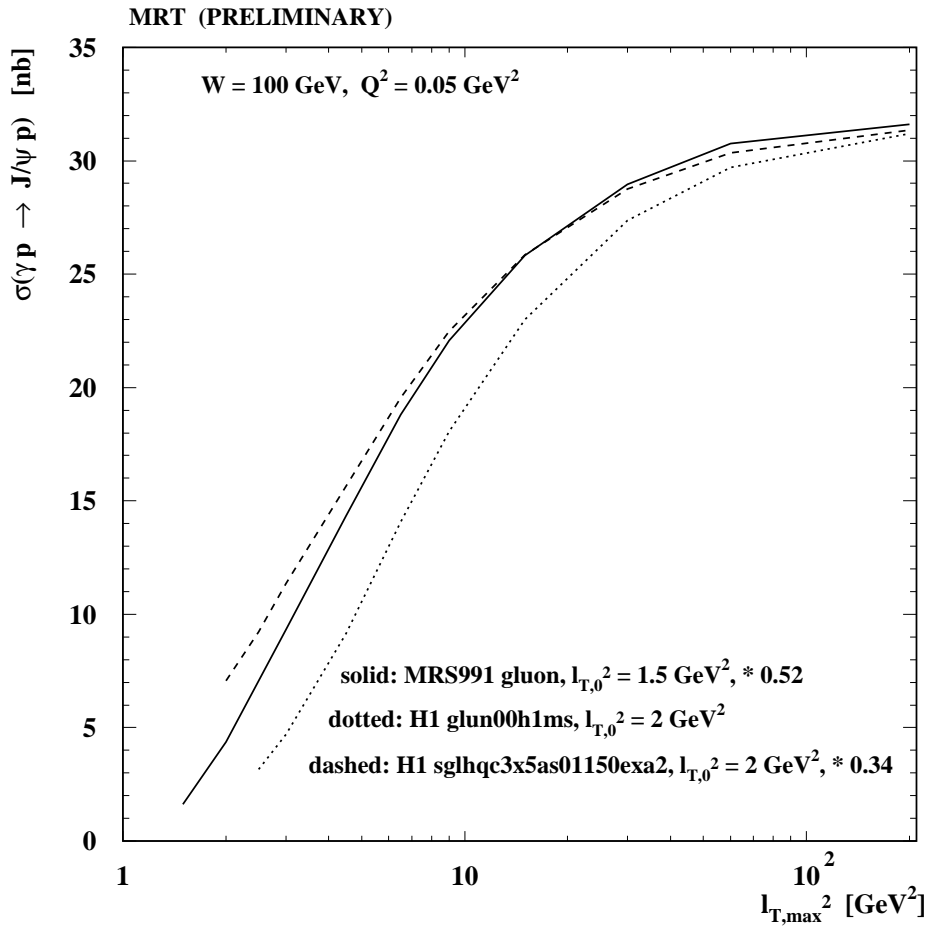
- Different energy dep.  $\sigma(W)$  reflects different functional form of gluons. Effect enhanced through skewing corrections.
- 'Error band' from H1 gluon narrow compared to spread using different gluons.



# Which scales are relevant?

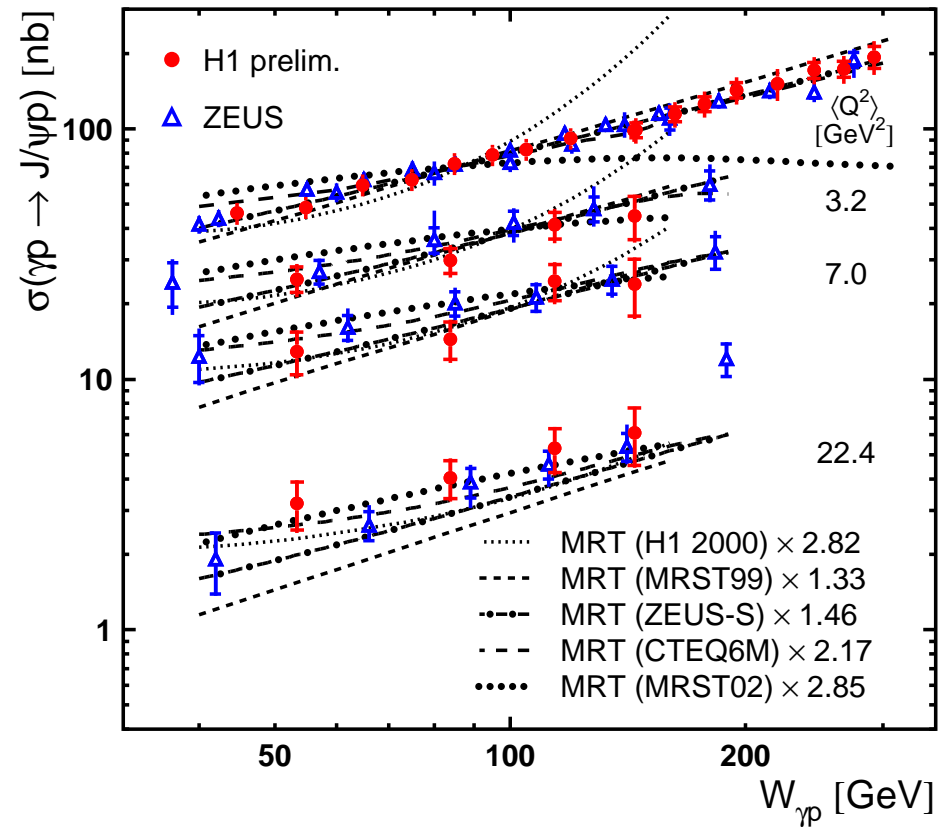
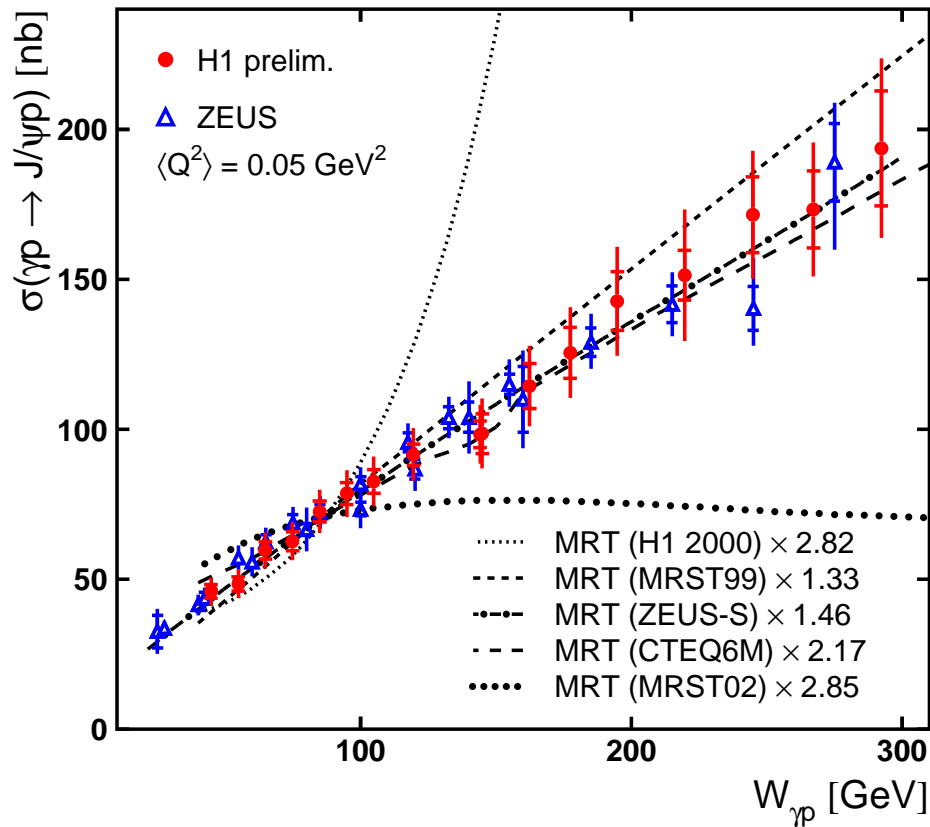
$$A \sim xg(x, \ell_0^2) \dots + \int_{\ell_0^2}^{\infty} \frac{d\ell_T^2}{\ell_T^4} f(x, x', \ell_T^2) \phi \dots$$

Distribution  $\sigma(\ell_{max}^2)$  [ $A \sim \int_0^{\ell_{max}} \frac{d\ell_T^2}{\ell_T^4} \dots$ ]    Input gluons, int. & unintegrated



# IV. Data vs TH: constraining $f(x, k_T^2)$ via diffr. VM production

Preliminary H1 and ZEUS data compared to MRT predictions:



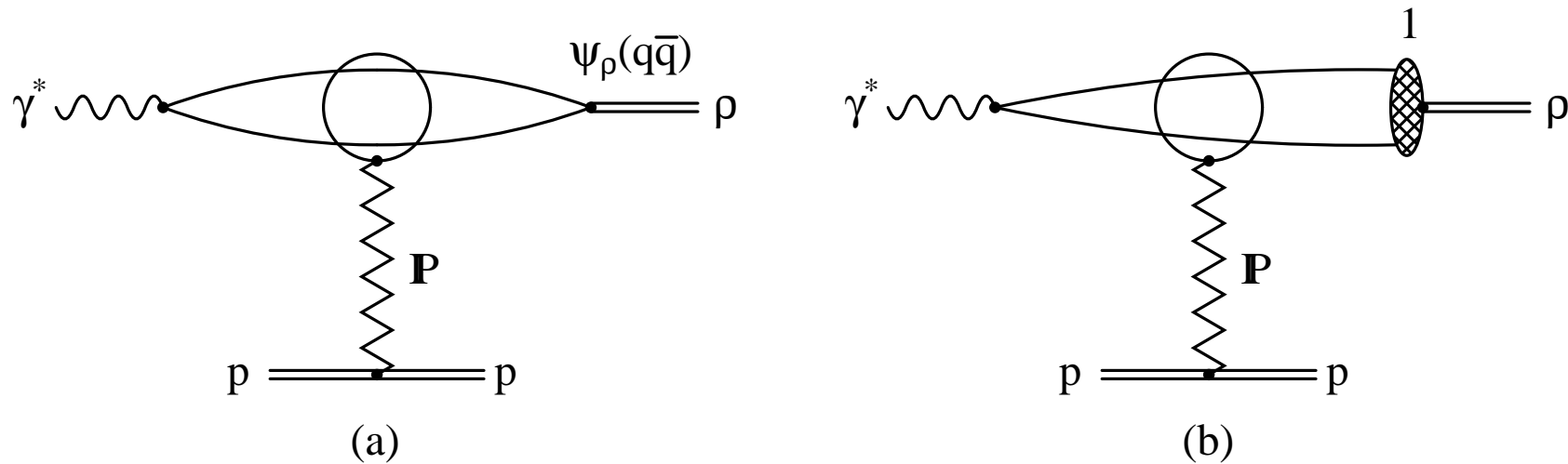
Plots thanks to Philipp Fleischmann (H1)

## Conclusions

- Diffractive VM production tests QCD and the gluon in the important semi-hard high energy regime
  - Theoretical QCD predictions indicate a very good sensitivity to the gluon and are able to describe most data
  - Current PDF fits do not constrain the gluon sufficiently at small  $x$  and small to intermediate scales. Use diffr. VM data! For this:
  - TH: better get QCD predictions under tighter control (generalized PDF, NLO impact factors, modelling of gluon in the IR)
  - EXP: get VM data in the largest possible range ( $Q^2, W, L/T, M_V, t$ )
  - ▶ Diffraction is getting understood quantitatively, and the exploitation of HERA data for the LHC is far from finished (EXP+TH)!
-

Some more details on the following pages

MRT avoid VM wave function: Use of **Parton Hadron Duality**:



**Assumption:**  $\gamma^* \rightarrow q\bar{q} \rightarrow \pi^+\pi^-$  cross section in the region  $M_{q\bar{q}} \sim M_\rho$  saturated by  $\rho$

(up to  $\sim 10\%$  for  $\omega$ ) when integrated over a *suitable* (universal?!) mass interval  $\Delta M$ :

$$\sigma(\gamma^* p \rightarrow \rho p) \simeq 0.9 \sum_{q=u,d} \int_{M_{min}^2}^{M_{max}^2} \frac{d\sigma(\gamma^* p \rightarrow (q\bar{q})p)}{dM^2} dM^2$$

+ **Projection** of  $q\bar{q}$  state on the correct VM Quantum Numbers  $J^P = 1^-$ .  
 ( $\rightsquigarrow$  Suppression of IR divergencies for contr. from transverse photon!)



## Contributions from the real part of the amplitude:

- The basic amplitudes  $\mathcal{A}^{L,T}$  are predominantly imaginary in the high energy (leading  $\log 1/x$ ) limit. Analytical expressions are for  $\text{Im } \mathcal{A}$  only.
- Account for the contributions from the real parts through (crossing symmetry + power behaviour  $\text{Im } T \sim s^\lambda$ ):

$$\text{Re } \mathcal{A} = \tan(\pi\lambda/2) \text{Im } \mathcal{A}.$$

- $\lambda(Q^2) = \frac{\partial \log \mathcal{A}}{\partial \log(1/x)}$  calculated numerically on *amplitude level*.
- Martin-Ryskin-T, PRD62,2000: *'The inclusion of the real part enhances the cross section of  $\rho$  production by 14 to 19% in the range where we compare to data,  $J/\psi$  production by 18 to 25%, and  $\Upsilon$  by about 30%, where the bigger effect always occurs at higher  $Q^2$ .'*

## Structure of the MRT code:

Contributions to  $\sigma(\gamma_{L,T}^* p \rightarrow V p)|_{t=0}$  from **Re, Im** for **L, T**, numerically  
(‘straightforward’, no iterative procedure for effective scales):

$$PHD: \int dM^2 \left[ \textit{Projection:} \int dk_T^2 \left( \textit{Skewed } \mathcal{A}'s \textit{ w. } K \textit{ fact.}; \textit{ Re:} \int dl_T^2 \right) \right]^2$$

## Pro and Con's of the MRT approach

- + MRT not just one more *'model'* but an approximation of QCD in a certain regime of parameter space.
- + Not just another fit of the data  $\rightsquigarrow$  predictive and can be improved.
- + Good description of basic observables:  $Q^2$  and  $W$  dependence of  $\sigma$ ,  $L/T$ .
- + Strong dependence on  $g(x, Q^2)$ .
- No prediction of  $t$  dependence (yet?)
- No  $q$  exchange needed for lower energies (could be added).
- PHD somewhat limiting: Details of VM wave-function in more exclusive measurements? Normalization not a good prediction.
- Full NLO still missing.