Vector meson electroproduction at small Bjorken-x and GPDs

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Outline:

- Handbag factorization and GPDs
- Structure of the amplitudes
- Modelling the GPDs
- The partonic subprocess
- Comparison with experiment
- Summary

based on work done in collaboration with

S.V. Goloskokov, hep-ph/0501242

Handbag factorization

Radyushkin (96); Collins et al (97):

proof of factorization

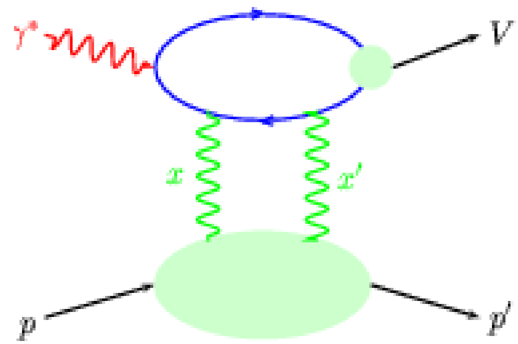
for $Q^2 \to \infty$ into

$\gamma^* g \to V g$ $(+\ \gamma^* q \to V q)$

and GPDs

(at small $x_{\rm Bj}$)

$x \neq x'$



dominant transition $\gamma^*_L \to V_L$ (others power suppressed)

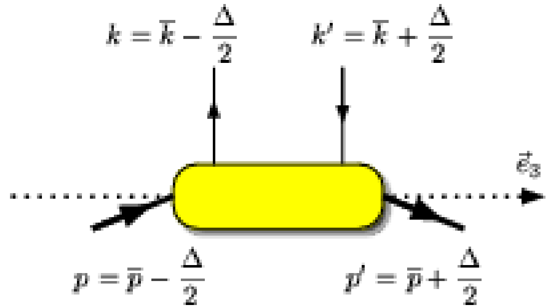
lead. $\ln(1/x_{\rm Bj})$ appr. (Brodsky et al): $x \simeq x' \simeq x_{\rm Bj}$; GPD $\to$ PDF

Generalized Parton Distributions

D. Müller et al (94), Ji(97), Radyushkin (97)

$$\xi = \frac{(p-p')^+}{(p+p')^+} \qquad \bar{x} = \frac{\overline{k}^+}{\overline{p}^+}$$

$$x = \frac{\bar{x}+\xi}{1+\xi} \qquad x' = \frac{\bar{x}-\xi}{1-\xi}$$



$$\int \frac{dz^-}{\pi} e^{i\bar{x}\,\overline{p}^+ z^-} \langle p'| G^{+\mu}(-\overline{z}/2)\, G^+_\mu(\overline{z}/2)\,|p\rangle =$$

$$\overline{u}(p')\gamma^+ u(p) H^g(\bar{x},\xi;t) + \overline{u}(p')i\sigma^{+\alpha}\frac{\Delta_\alpha}{2m}u(p)\; E^g(\bar{x},\xi;t)$$

(gauge $A^+ = 0$; $\overline{z} = [0, z^-, 0_\perp]$) $\qquad \widetilde{G}^+_\mu \longrightarrow \widetilde{H}^g, \widetilde{E}^g$

reduction formulas

$H^g(\bar{x},0;0) = \bar{x}g(\bar{x}) \qquad \widetilde{H}^g(\bar{x},0;0) = \bar{x}\Delta g(\bar{x})$

universality, polynomiality, evolution, positivity constraints, Ji's sum rule

The $\gamma^* p \to V p$ amplitudes

kinematical regime: large Q^2, W, small $x_{\rm Bj}(\lesssim 10^{-2})$, t;
kinematics fixes skewness: $\xi \simeq x_{\rm Bj}/2$

$$M_{\mu'+,\mu+} = \frac{e}{2}\mathcal{C}_V \int_0^1 \frac{d\bar{x}}{(\bar{x}+\xi)(\bar{x}-\xi+i\epsilon)} \times$$
$$\left\{\sum_\lambda H^V_{\mu'\lambda,\mu\lambda}\Big[H^g - \frac{\xi^2}{1-\xi^2}E^g\Big] + \sum_\lambda \lambda H^V_{\mu'\lambda,\mu\lambda}\Big[\widetilde{H}^g - \frac{\xi^2}{1-\xi^2}\widetilde{E}^g\Big]\right\}$$
$$M_{\mu'-,\mu+} = \cdots \frac{\sqrt{-t}}{2m} \quad \cdots E^g + \cdots \xi \widetilde{E}^g$$

Unpolarized protons: no flip-nonflip interference

$$|M_{\mu'-,\mu+}|^2 \propto t/m^2 \qquad \text{neglected}$$

$$M = M^H + M^{\widetilde{H}}: \qquad M^{H(\widetilde{H})}_{++,++} = +(-)M^{H(\widetilde{H})}_{-+,-+}$$
$$M^{H(\widetilde{H})}_{0+,++} = -(+)M^{H(\widetilde{H})}_{0+,-+}$$

as for (un)natural parity exchange

$$\Rightarrow Obs \sim |M^H|^2 + |M^{\widetilde{H}}|^2 \qquad \frac{\widetilde{H}}{H} \simeq \frac{|\Delta g|}{g} \ll 1$$

Electroprod. with unpolarized protons at small $x_{\rm Bj}$ and small t probes H^g

Modelling the GPD

GPD controlled by non-pert. QCD: lattice QCD (not yet)
extraction from experiment (not yet)
modelling

Factorizing in $\bar{x}, \xi$ and t probably incorrect (lattice, overlap)
restriction to $t \simeq 0$
Popular ansatz (Radyushkin) $n = 1, 2$

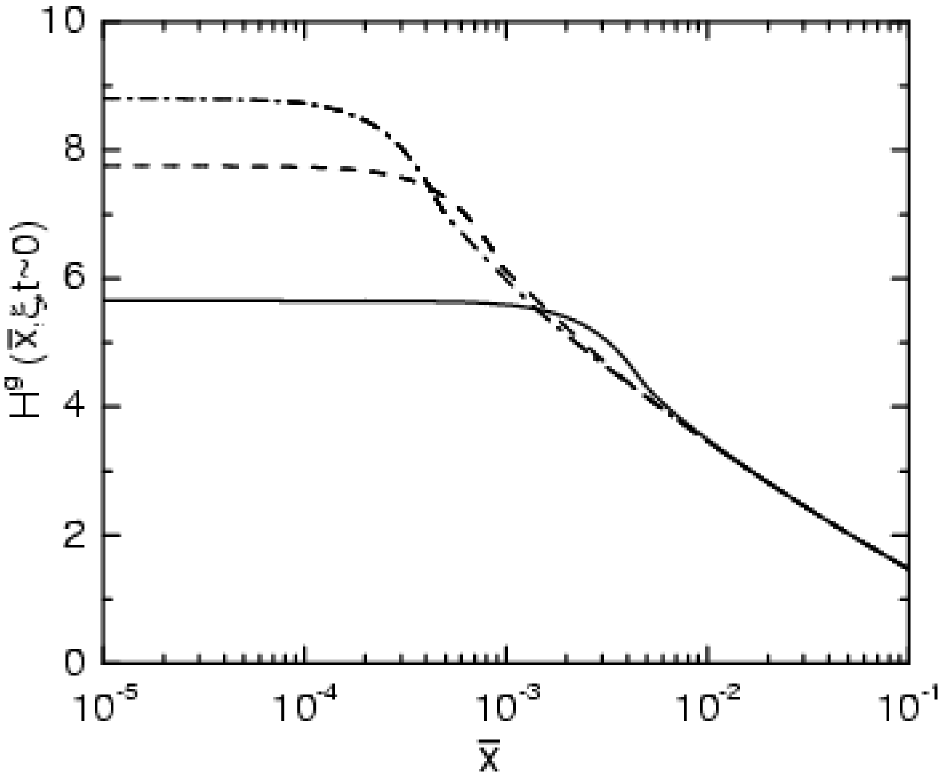
$$f(\beta, \alpha, t \simeq 0) = g(\beta) \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)} \frac{[(1-|\beta|)^2 - \alpha^2]^n}{(1-|\beta|)^{2n+1}}$$

double distribution; guarantees polynomiality of moments in ξ

$$H^g(\bar{x}, \xi) = \left[\Theta(0 \le \bar{x} \le \xi) \int_{\frac{\bar{x}-\xi}{1+\xi}}^{\frac{\bar{x}+\xi}{1+\xi}} + \Theta(\xi \le \bar{x} \le 1) \int_{\frac{\bar{x}-\xi}{1-\xi}}^{\frac{\bar{x}+\xi}{1+\xi}}\right] d\beta \frac{\beta}{\xi} f(\beta, \alpha = \frac{\bar{x}-\beta}{\xi})$$

$$+ \xi D^g(\bar{x}/\xi) \qquad [\text{support} - \xi \le \bar{x} \le \xi]$$

Input: NLO CTEQ5M

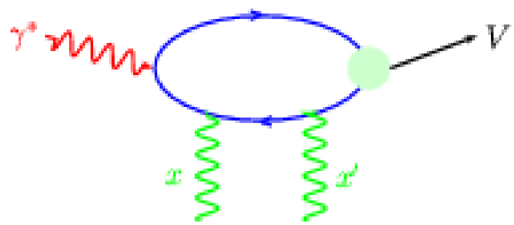


$n = 1$ ($n = 2$ similar)

solid(dashed, dash-dotted) line: $\xi = 5(1, 0.5) \cdot 10^{-3}$

The subprocess $\gamma^* g \to q\bar{q}g$, $q\bar{q} \to V$

modified perturbative approach (Sterman et al):
quark transverse momenta and gluonic radiative corrections (Sudakov) are taken into account;
supresses end-point regions ($\tau \to 0,1$) where q and $\bar{q}$ are separated by large transerse distances



similar to treatment in lead. $\ln(1/x_{\rm Bj})$ appr. (e.g. Frankfurt et al)

$$
\begin{aligned}
H_i^V &= \int d\tau \frac{d^2 k_\perp}{16\pi^2} \Psi_{Vi}(\tau, k_\perp) \\
&\times \frac{1}{\sqrt{2}} Tr \left[(\not{q}' + m_V)\not{\epsilon}_V T_0 - \frac{k_\perp^2 g_\perp^{\alpha\beta}}{2M_V} \{(\not{q}' + m_V)\not{\epsilon}_V, \gamma_\alpha\} \Delta T_\beta + \ldots \right]
\end{aligned}
$$

M_V soft parameter of order m_V
Gaussian wavefunction: $\Psi_{Vi} \sim f_{Vi} \exp[-a_{Vi}^2 k_\perp^2/(\tau(1-\tau)]$
parameters e.g. $f_{\rho L} = 0.216$ GeV $\quad a_{\rho_L} = 0.52\,\text{GeV}^{-1}$

Hierarchy:

$(\langle k_\perp^2\rangle^{1/2}/m_V \sim O(1))$

$$\begin{aligned}
\mathrm{L} \to \mathrm{L} \qquad & H^V \propto 1 \\
\mathrm{T} \to \mathrm{T} \qquad & \quad\ \propto \langle k_\perp^2\rangle^{1/2}/Q \\
\mathrm{T} \to \mathrm{L} \qquad & \quad\ \propto \sqrt{-t}/Q \\
\mathrm{L} \to \mathrm{T} \qquad & \quad\ \propto \sqrt{-t}\langle k_\perp^2\rangle^{1/2}/Q^2 \\
\mathrm{T} \to -\mathrm{T} \qquad & \quad\ \propto -t\langle k_\perp^2\rangle^{1/2}/Q^3
\end{aligned}$$

t dependence

Integrated cross sections (σ_L, σ_T)

exponential slopes of amplitudes $\sim e^{t\, B^V_{LL,TT}/2}$

i.e. we essentially need forward amplitudes (calculated)

and slopes (taken from experiment)

$B^V_{TT} \neq B^V_{LL}$? $\qquad |M_{TT}|^2 \propto \left(\frac{f^V_T}{M_V}\right) \frac{1}{B^V_{TT}}$

without precise t dependent data ($d\sigma/dt$, SDME) at disposal

only this product of parameters is probed

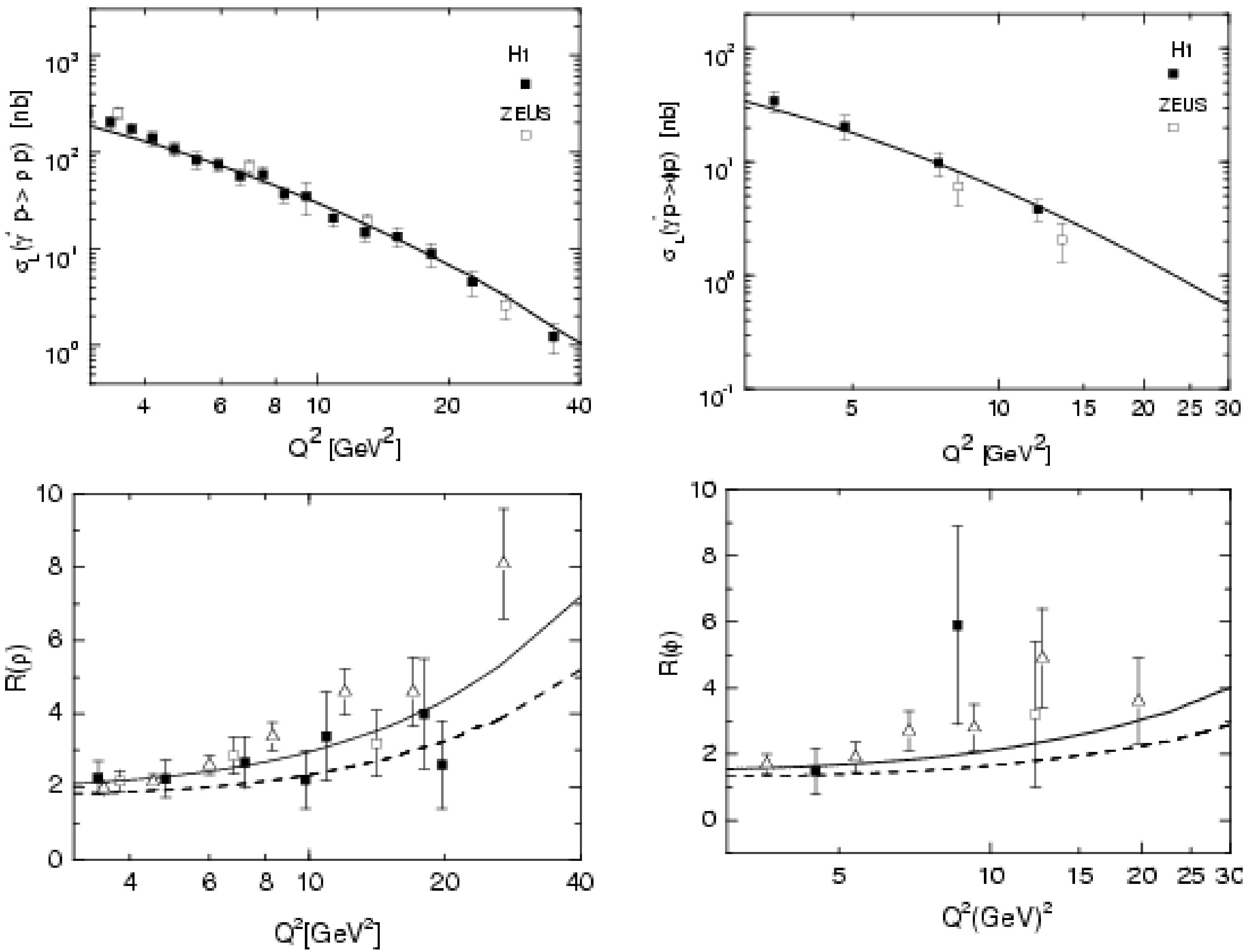
Examples:

$B^V_{TT} \simeq B^V_{LL}/2 \qquad M_V = m_V \qquad f^\rho_T = 250$ MeV

$B^V_{TT} \simeq B^V_{LL} \qquad M_V = m_V/2 \qquad f^\rho_T = 170$ MeV

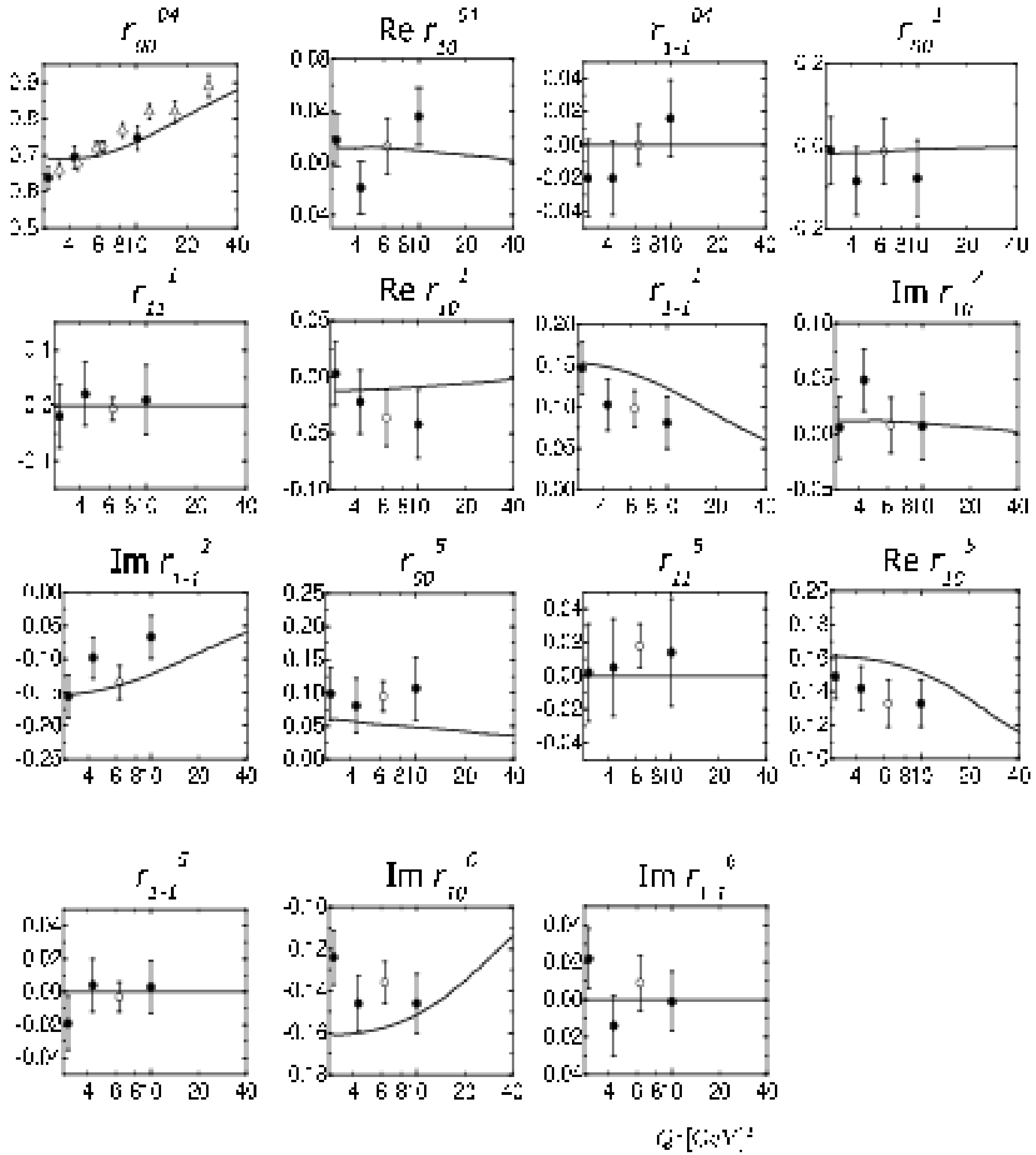
First example slightly favored by SDME

Improvements are required: concept clear but we need $H^g(\bar{x}, \xi, t)$



$W \simeq 75\,\mathrm{GeV}$; filled (open) symbols: H1 (ZEUS) data
solid (dashed) line: ratio of differential (integrated) cross sections, $\tilde{R}\,(R)$
$(t = -0.15\,\mathrm{GeV}^2;\ B^V_{TT} = B^V_{LL}/2)$

Spin density matrix elements of the ρ

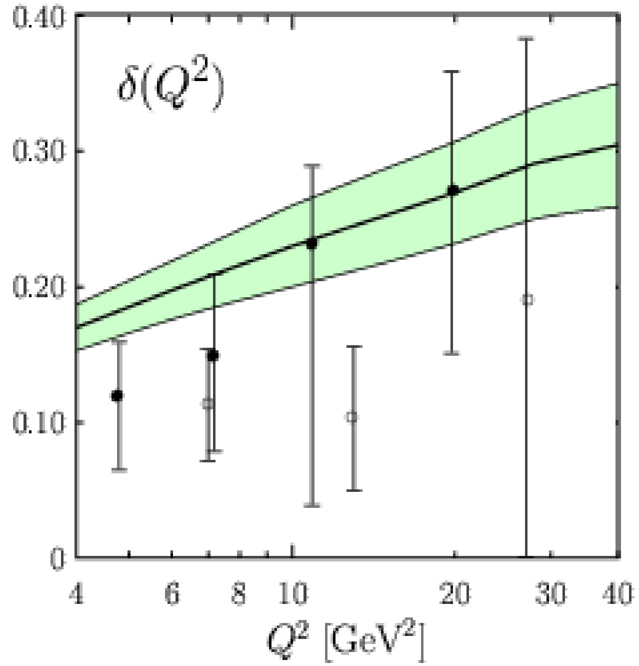


$W \simeq 75\,\mathrm{GeV}$ and $t \simeq -0.15\,\mathrm{GeV}^2$; filled (open) symbols: H1 (ZEUS) data

$$\sigma_L \sim \frac{|H^g(\xi,\xi)|^2}{B_{LL}^V Q^6} \sim W^{4\delta(Q^2)}$$

for fixed Q^2

from $\bar{x}g(\bar{x}) \sim \bar{x}^{-\delta}$ for $\bar{x} \to 0$



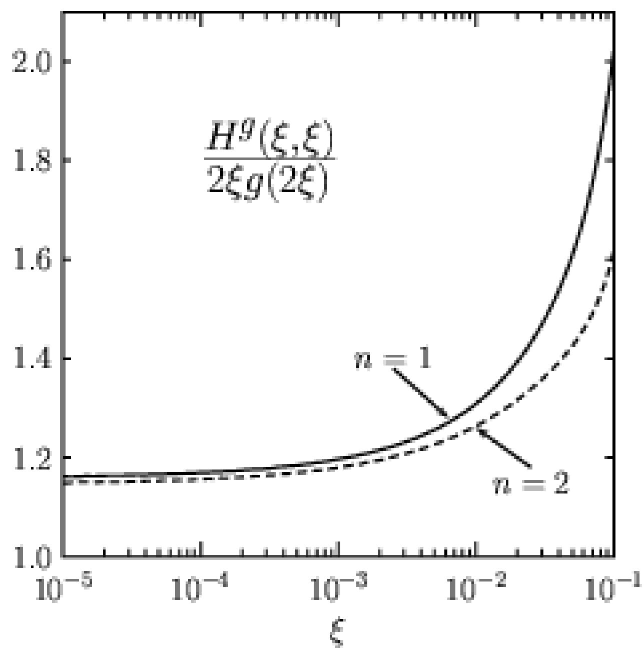
lead. $\ln(1/x_{\mathrm{Bj}})$ approx.:

$H^g(\xi,\xi) \to 2\xi g(2\xi)$

reasonable approx. at small ξ

ca. 18% enhancement

skewing effect (Martin et al, Belitsky et al)



Summary

- We analyzed electroproduction of light vector mesons at large $Q^2 (\gtrsim 4\,\mathrm{GeV}^2)$ and small $x_{\mathrm{Bj}} (\lesssim 10^{-2})$ within the handbag approach
- Only the gluonic GPD H^g contributes
- Subprocess is calculated within modified perturbative approach
- Fair agreement with integrated cross section and SDME is obtained with a few free parameters (essentially for the $T \to T$ transition)
- Amplitudes are calculated at $t = 0$, t dependence assumed to be exponential
- Accurate t dependent data ($d\sigma/dt$, SDME) required for improvement