Status Report of NNLO QCD Calculations

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Abstract. We review recent progress in next-to-next-to-leading order (NNLO) perturbative QCD calculations with special emphasis on results ready for phenomenological applications. Important examples are new results on structure functions and jet or Higgs boson production. In addition, we describe new calculational techniques based on twistors and their potential for efficient calculations of multiparticle amplitudes.

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INTRODUCTION

Despite the fact that the theory of strong interactions, quantum chromodynamics (QCD), is today a well-established part of the Standard Model of particle physics, it continues to be an extremely active field of experimental and theoretical research. This is as true for non-perturbative (and often lattice) determinations of meson and baryon spectra, decay constants, or form factors for electric dipole moments or *B*-meson decays, as it is for perturbative calculations. Today, the experimental precision of high-energy collider data always requires calculations beyond the leading order (LO) in the strong coupling constant, $\alpha_s(\mu)$, and often even beyond the next-to-leading order (NLO) for reliable comparisons and determinations of unknown Standard Model parameters.

After a relatively slow start due to large technical difficulties in the mid-1990s, QCD calculations at next-to-next-to-leading order (NNLO) have recently attained their goals for several phenomenological applications. The aim of this Report is therefore to review recent progress in this field with special emphasis on phenomenologically relevant results, including structure functions, jet, and Higgs boson production. Very recently, twistor methods have received particular attention, as they may offer a route to efficient multiparticle calculations, even beyond tree-level. They will therefore be briefly discussed, before we present our conclusions.

STRUCTURE FUNCTIONS

Since its start-up in 1992, the DESY *ep* collider HERA has provided a wealth of data on the structure functions of the proton, and thus its parton densities (PDFs), in a large region of Bjorken-x and Q^2 [1]. The PDFs represent at the same time an important

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ingredient in the search for new physics, that is currently underway at the Fermilab Tevatron and is an important research goal at the CERN LHC [2]. In particular, QCD uncertainties in PDF determinations, *e.g.* from renormalization and factorization scale and scheme [3, 4] variations, have to be under control before disagreement between theory and experiment can be interpreted as evidence for new physics.

It is thus fortunate that, after completion of the three-loop singlet splitting functions (obtained from the $1/\varepsilon$-poles in dimensional regularization) and longitudinal coefficient function (obtained from the finite terms) [5], a full NNLO calculation of the structure functions $F_{1,2}(x,Q^2)$ (and thus also of $F_L = F_2 - 2xF_1$), is now available. These results have been obtained using a large variety of techniques, such as application of the optical theorem, transformation to Mellin space, mapping of diagrams with composite topologies to basic building blocks, and integration by parts. They also relied on recent advances in mathematics (harmonic sums) and computer algebra (QGRAF, FORM). Fortunately, the results can also be applied to other physical observables such as photon structure functions and total cross sections in e^+e^--annihilation.

While the numerical effects of the NNLO QCD corrections are well visible in splitting functions and coefficient functions *separately* and show clear differences in normalization, although not in shape, from earlier leading-ln(x) estimates (see Fig. 1), the total

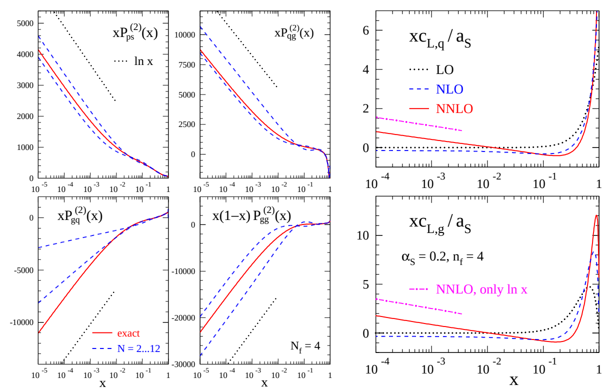

FIGURE 1. Splitting functions (left) and longitudinal coefficient functions (right) at NNLO [5].

effect in the *convoluted* structure functions is only important for very small or large x and small Q^2. For this reason, the NNLO corrections have not yet been included in the global CTEQ analysis, as the authors find sufficient stability of their fit at NLO [6], whereas the MRST collaboration have now implemented the exact NNLO splitting functions, but find minor effects w.r.t. a DIS-scheme motivated variation of the input gluon density [4] or an earlier analysis using approximate NNLO splitting functions [7].

JET PRODUCTION IN ELECTRON-POSITRON ANNIHILATION

Calculations of observables less inclusive than deep inelastic scattering (DIS) structure functions and total e^+e^- cross sections do not allow for a straight-forward application of the optical theorem, as they require multiparticle cuts in the three-loop photon propagator. For this reason, NNLO jet calculations in e^+e^--annihilation have been lagging behind, although recently considerable progress has also been achieved in this field. The interference terms of $1 \to 3$ tree-level and two-loop amplitudes are now known, as are the single soft and/or collinear regions of $1 \to 4$ squared one-loop amplitudes. What remains to be calculated are the maximally double-soft and triple-collinear regions of $1 \to 5$ squared tree-level amplitudes. Here, antenna functions have successfully been used for quark-gluon and gluon-gluon final states [8].

A first preliminary numerical result has been obtained for the NNLO contribution to the C_F^2 color class of the average thrust [9]

$$\langle 1-T \rangle \;=\; \int (1-T) \frac{1}{\sigma_0} \frac{d\sigma}{dT} \;=\; C_F \left[\left(\frac{\alpha_s}{2\pi} \right) A + \left(\frac{\alpha_s}{2\pi} \right)^2 B + \left(\frac{\alpha_s}{2\pi} \right)^3 C + \ldots \right],$$

where $A = 1.57$ and $B = 32.3$ were known and now $C(C_F^2) = -20.4 \pm 4$. The full result will be important for reliable estimates of higher-twist effects in e^+e^- event shapes. Furthermore, analytic continuation allows to apply the results obtained for three-jet production in e^+e^--annihilation also to dijet production in DIS.

HIGGS BOSON PRODUCTION AT HADRON COLLIDERS

The inclusive NNLO cross section for light/heavy scalar and pseudo-scalar Higgs production in gg scattering at hadron colliders has been known for several years, but the relevance of these calculations has been limited by the need for experimental cuts on associated photons, jets or b-quarks. Fully differential cross sections, such as a recent NNLO calculation for Higgs plus dijet production [10], allow for transverse energy vetos on the additional jets and thus for a suppression of the QCD background in the diphoton (or other) Higgs decay channels (see Fig. 2).

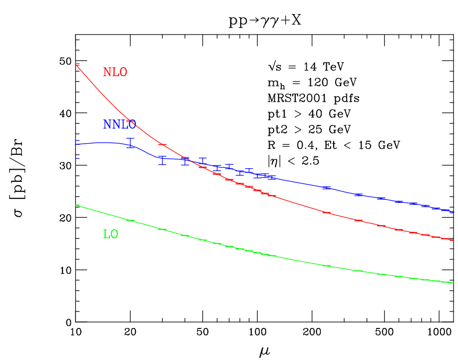

FIGURE 2. Higgs production at the LHC with diphoton decay and no additional hard jets [10].

JET HADROPRODUCTION AND TWISTOR METHODS

For hadron-hadron scattering, the two-loop $2 \to 2$ parton amplitudes have been known for some time, and the result for polarized $qq \to qq$ scattering has recently been confirmed using supersymmetric methods [11]. Since the soft/collinear singular regions of one-loop $2 \to 3$ amplitudes are also known, recent work has focused on multiple soft/collinear emissions in tree-level multiparton amplitudes and on extending the dipole subtraction formalism to NNLO, where two alternatives to previously known methods have been proposed [12, 13]. Although the subtraction terms are in principle known, the singular regions often overlap and need to be disentangled to avoid double-counting. Furthermore, phase space factorization and integration of the singular matrix elements still needs to be resolved. For this reason, further progress towards a full NNLO calculation for jet hadroproduction has been slow, and numerical integration methods using sector decomposition [14] may have to be developped further.

Multiparticle amplitudes may be calculated efficiently by relating perturbative gauge theory to the D-instanton expansion for a topological string in twistor space. While originally motivated by $N = 4$ super Yang-Mills theory, this approach has been shown to work at tree-level for generic QCD amplitudes. $N = 4$ supersymmetric and cut-constructible one-loop diagrams have also been calculated, and extensions to non-supersymmetric theories and two-loop amplitudes are underway [15].

CONCLUSION

Although this brief Report is necessarily far from complete, it should be clear that NNLO QCD calculations represent a theoretically challenging, phenomenologically important, and fast-moving field of research allowing for interesting advances in the fields of mathematics, computer science and last, but not least, elementary particle physics.

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