Status Report of NNLO QCD Calculations

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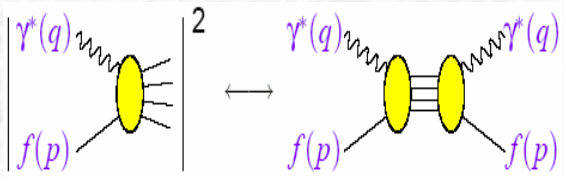
Structure Functions (1)

- Motivation:
  - Match experimental precision of final HERA data
  - Reduce PDF uncertainties for new physics searches at LHC
- Complete calculation of $F_{1,2,3}(x,Q^2)$ and $F_L=F_2-2xF_1$:
  - Anomalous dimensions: Σ @ 3-loop [hep-ph/0404111]
    → Coefficients of $1/\varepsilon$-poles in dim. reg.
  - Coefficient functions: c_L @ 3-loop [hep-ph/0411112]
    → Coefficients of ε^0 -terms in dim. reg.
- Very large effort:
  - 20 man-years: S. Moch, J. Vermaseren, A. Vogt (1997-2004)
  - Advances in mathematics ($H_{1..i}$) and computer algebra (FORM)
  - Applications to photon structure functions, e^+e^--annihilation

Structure Functions (2)

- Methods:
  - Optical theorem:

$$W_{\mu\nu} = e_{\mu\nu}\frac{1}{2x}F_L(x,Q^2) + d_{\mu\nu}\frac{1}{2x}F_2(x,Q^2) + i\varepsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{p\cdot q}F_3(x,Q^2)$$



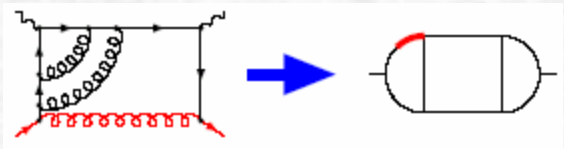
  - Mellin space:

$$F_i^{\gamma,N}(Q^2) = \int_0^1 dx\, x^{N-1}\mathcal{F}_i^\gamma(x,Q^2)\,, \qquad \mathcal{F}_i(x) = \frac{1}{x}F_i(x)$$

$$F_i^{\gamma,N}(Q^2) = \sum_{\alpha=\mathrm{ns,q,g},\gamma} C_{i,\alpha}^N\left(\frac{Q^2}{\mu^2}, a_s, a_{em}\right) A_{\Gamma,\alpha}^N(\mu^2)$$

$$\gamma = \begin{pmatrix} \gamma_{ns} & 0 & 0 & k_{ns} \\ 0 & \gamma_{qq} & \gamma_{qg} & k_q \\ 0 & \gamma_{gq} & \gamma_{gg} & k_g \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Topology mapping:
  - Composite $\rightarrow$ Basic building blocks
  - Non-planar $\rightarrow$ Benz $\rightarrow$ Ladder



- Integral relations:
  - Integration by parts
  - Scaling identities
  - Form factors relations

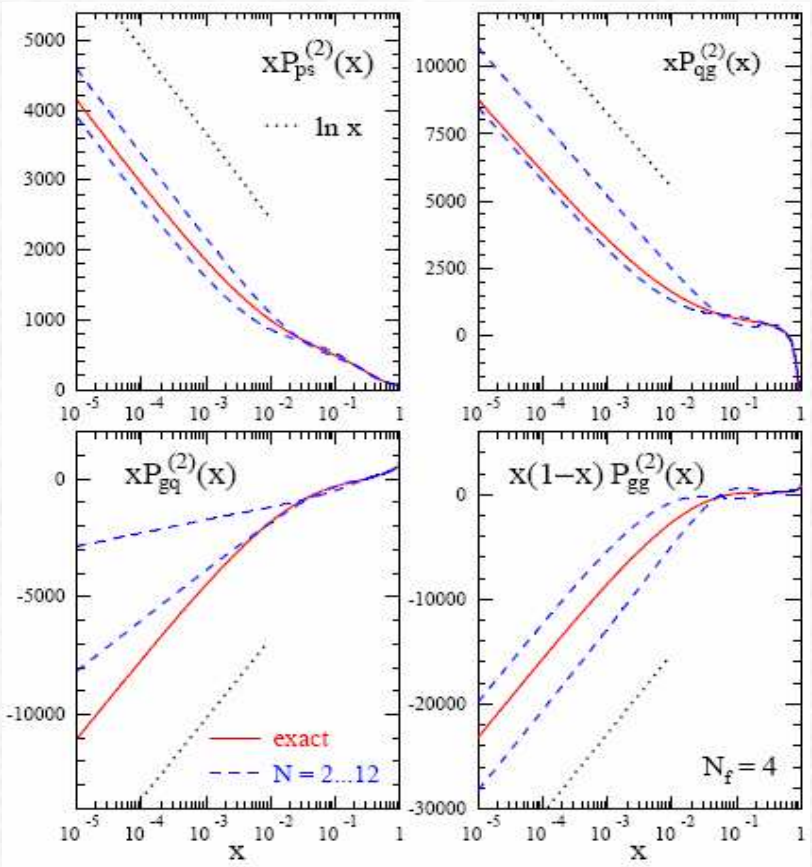
$$\int \prod_n d^D l_n \frac{\partial}{\partial l_i^\mu} l_j^\mu \times (\ldots) = 0$$

$$Q^\mu\frac{\partial}{\partial Q^\mu}\,, \quad P^\mu\frac{\partial}{\partial Q^\mu}\,, \quad P^\mu\frac{\partial}{\partial P^\mu}$$

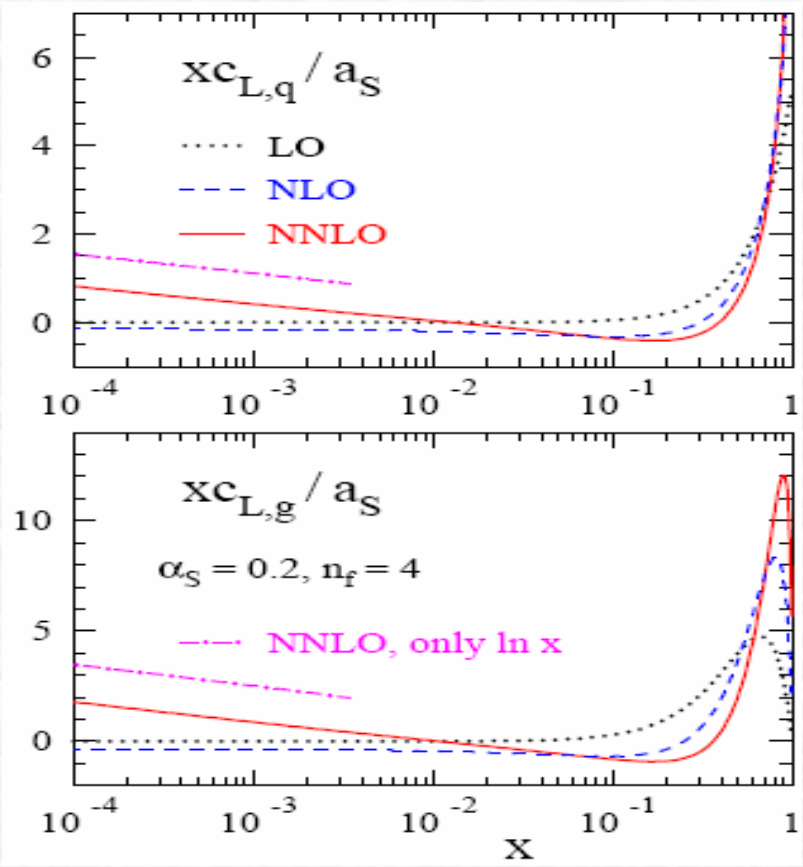
$$\int \prod_n d^D l_n\, l_i^\mu \times (\ldots) = Q^\mu I_Q + P^\mu I_P$$

Structure Functions (3)

- Splitting functions:

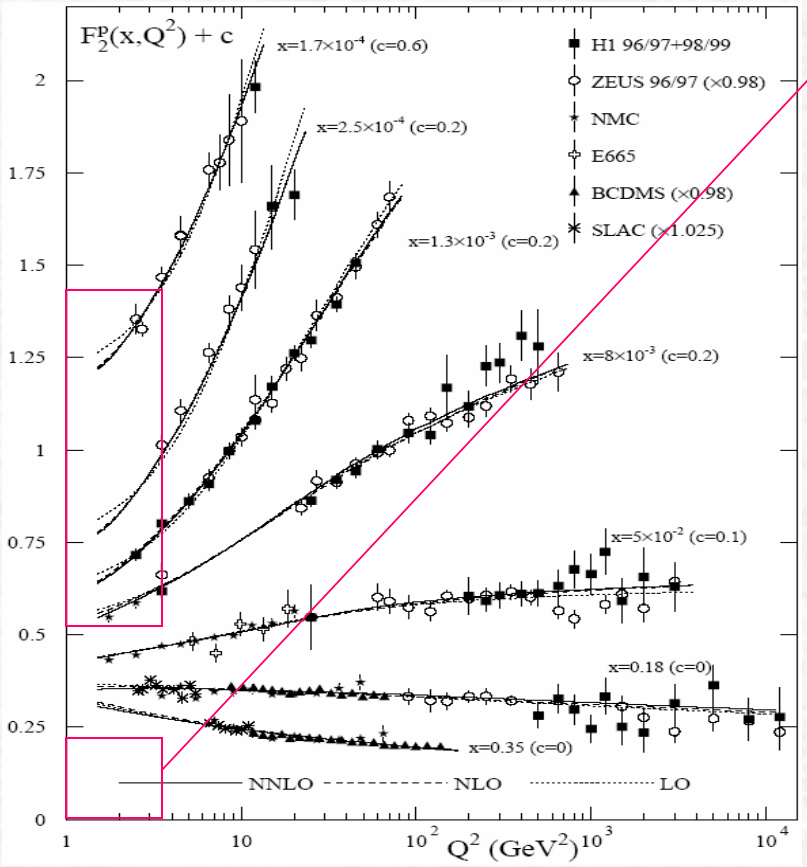


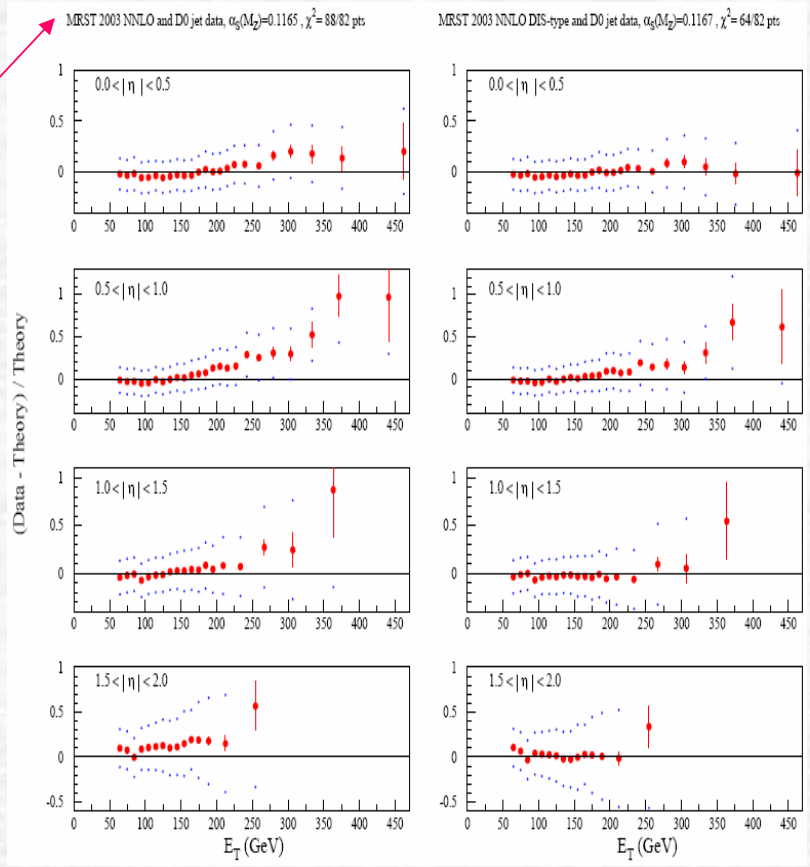
- Coefficient functions:



Structure Functions (4)

- MRST2002: Approx. NNLO
- MRST2004: Exact NNLO



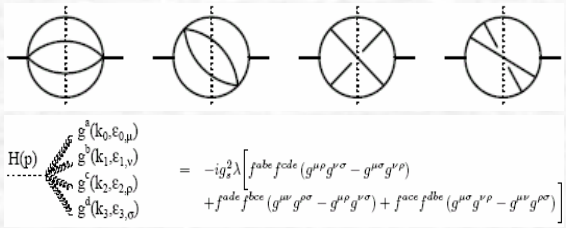


Jets (1)

- Motivation:
  - Determination of α_s and PDFs at LEP, HERA and Tevatron
  - Reduce background uncertainties for jet signatures at LHC
- So far $2 \to 2$ processes only:
  - $e^+e^- \to 3$ jets $\Leftrightarrow$ $ep \to 2$ jets [analytic continuation]
  - $p\,p \to 2$ jets: qq helicity amplitudes [JHEP 0409 (2004) 039]
- Ongoing effort (1997-):
  - A. Gehrmann, T. Gehrmann, N. Glover, E. Remiddi, …
  - Z. Bern, L. Dixon, D. Kosower, S.Moch, P.Uwer, S. Weinzierl, …
  - Advances in mathematics: Differential eqs., nested sums
  - Applications to event shapes, multijets @ 1-loop, photons

Jets (2)

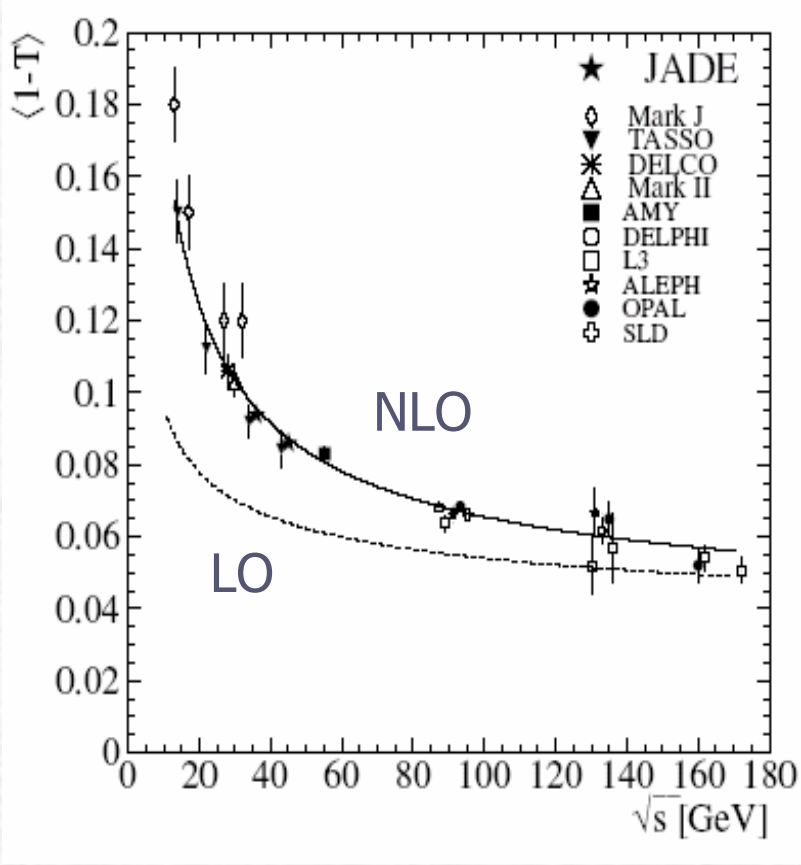
- Ingredients:
  - $1 \to 3$ @ 2-loop: Inteference with $1 \to 3$ @ 0-loop √
  - $1 \to 4$ @ 1-loop: Single soft and/or collinear regions √
  - $1 \to 5$ @ 0-loop: Double soft … triple collinear regions √
- Methods:
  - Optical theorem: 3-loop propagators
  - Subtraction terms: Antenna functions



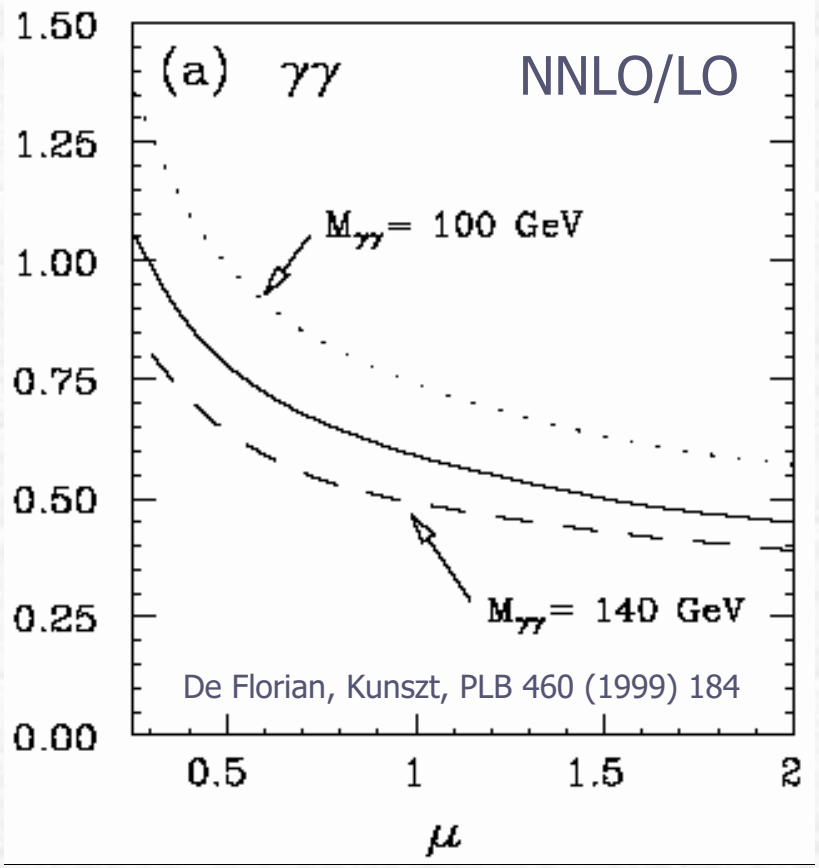
- First (preliminary) numerical result:
  - Average thrust: $\langle 1-T\rangle = \int (1-T)\frac{1}{\sigma_0}\frac{d\sigma}{dT} = C_F\left[\left(\frac{\alpha_s}{2\pi}\right)A + \left(\frac{\alpha_s}{2\pi}\right)^2 B + \left(\frac{\alpha_s}{2\pi}\right)^3 C + \ldots\right]$
  - A = 1.57, B = 32.3, C = (-20.4±4) C_F^2 + …
  - A. Gehrmann, T. Gehrmann, N. Glover, LL 2004

Jets (3)

- Thrust: Higher twist?

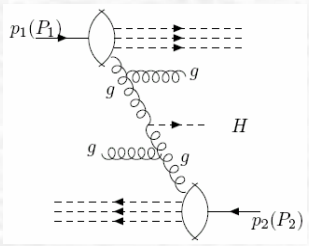


- Diphotons: Higgs from gg?

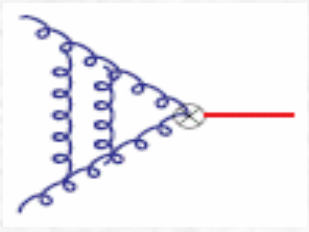


Higgs (1)

- Inclusive cross section: gg,(bb) → h(,H,A)+X
  - R. Harlander, B. Kilgore (2002)
  - C. Anastasiou, K. Melnikov (2002)
  - V. Ravindran, J. Smith, W. van Neerven (2003)



- Differential cross section:
  - C. Anastasiou, K. Melnikov, F. Petriello (2004)



- The ggh-vertex:
  - Effective Lagrangian:
  - Coefficient function:
  - Renormalization constant:

$$\mathcal{L} = \frac{1}{4v} C_1 Z_1 G^a_{\mu\nu} G^{a\mu\nu} H$$

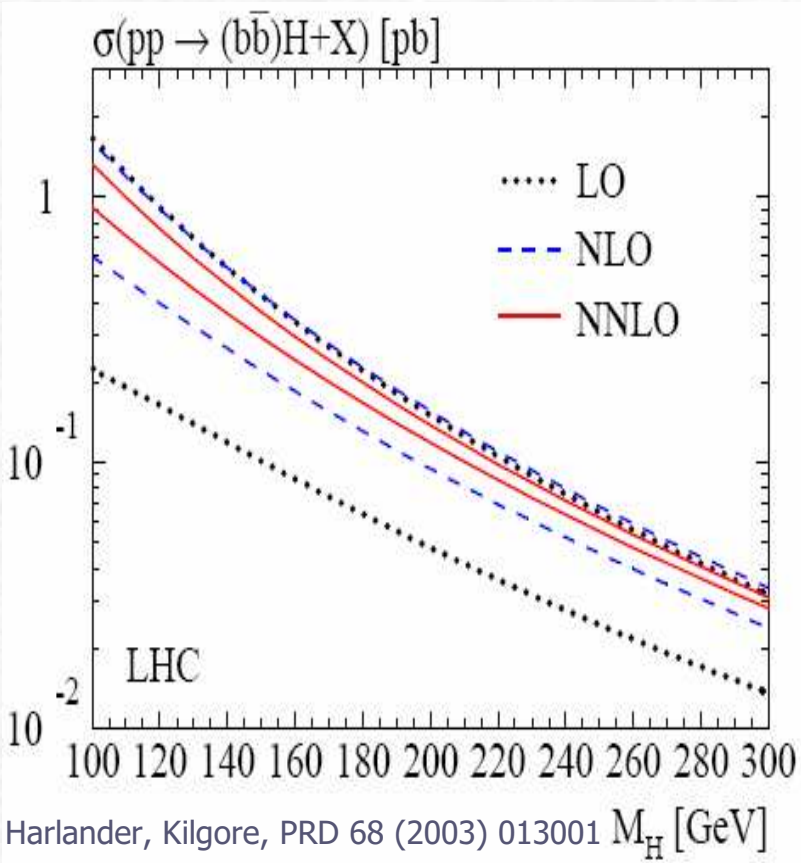
$$C_1 = -\frac{1}{3\pi}\left\{1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{2777}{288} + \frac{19}{16}L_t + n_f\left(-\frac{67}{96} + \frac{1}{3}L_t\right)\right] + \mathcal{O}\left(\alpha_s^3\right)\right\}$$

$$Z_1 = 1 - \frac{\alpha_s}{\pi}\frac{\beta_0}{\epsilon} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{\epsilon}\right] + \mathcal{O}\left(\alpha_s^3\right)$$

- Ingredients:
  - gg→h @ 2-loop;gg,qq→hg @ 1-loop;gg,qq→hgg,qq @ 0-loop

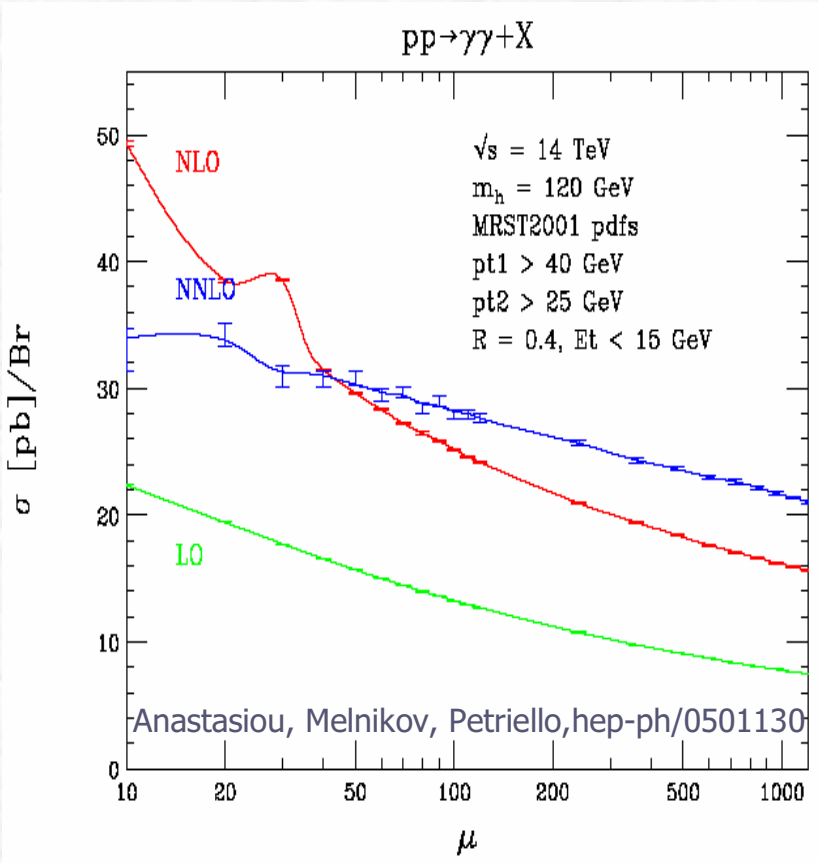
Higgs (2)

- Forward-going b-jets:



Harlander, Kilgore, PRD 68 (2003) 013001

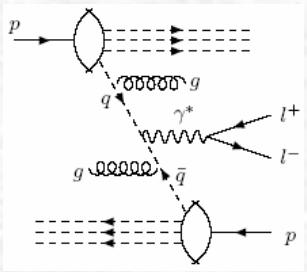
- Diphotons + no hard jets:

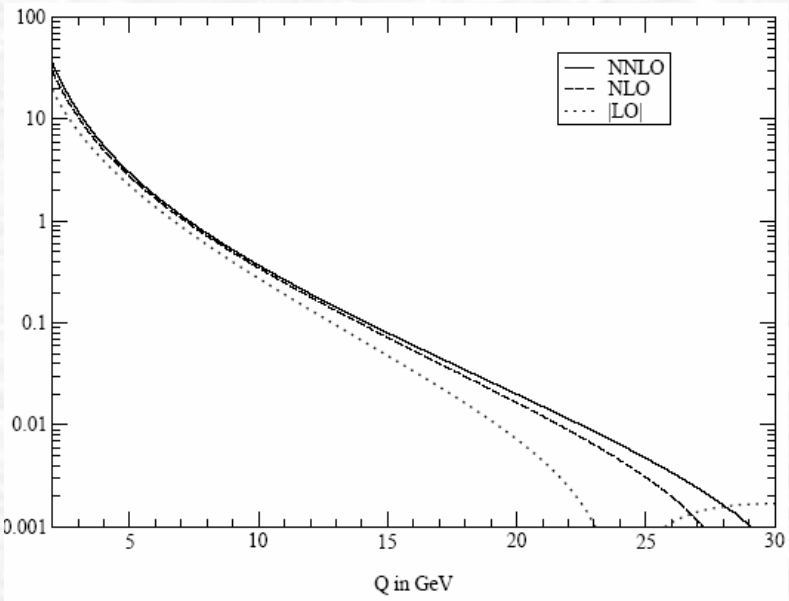


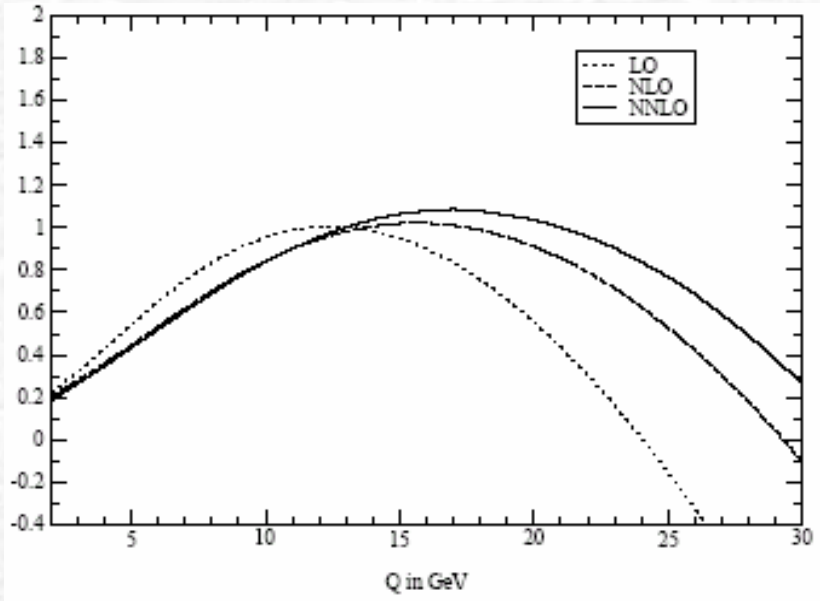
Anastasiou, Melnikov, Petriello,hep-ph/0501130

Polarized Drell-Yan Process

- NNLO coefficient function:
  - V. Ravindran, J. Smith, W. van Neerven (2004)
- γ_5 : HVBM-scheme, evanescent counterterms
- Cross section: Asymmetry:

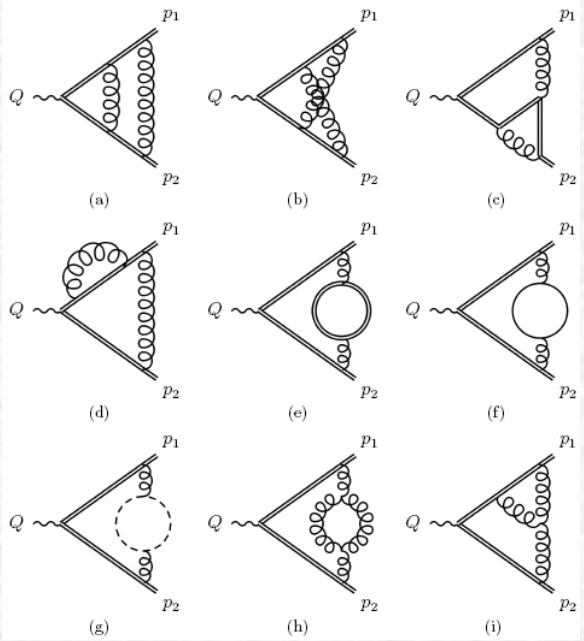






Heavy Quarks

- $e^+e^- \rightarrow QQ$: Vertex corrections @ 2-loop
  - Vector: W. Bernreuther et al., NPB 706 (2005) 245
  - Axial-vector: W. Bernreuther et al., NPB 712 (2005) 229
- Method:
  - Two scale problem: Q, m_Q
  - Laporta reduction, IBP, Lorentz invar.
  - 17 master integrals: Polylogarithms
  - Renormalization: Z_m^{OS}, Z_2^{OS}; Z_g^{MS}, Z_3^{MS}
  - Confirm threshold expansion results
- Still missing: Real emission
  - 1-loop single-subtraction terms
  - 0-loop double-subtraction terms



Twistor Methods

- New approach to multiparticle amplitudes:
  - Relation of pert. gauge theory to string instanton expansion:
    $\rightarrow$ Twistor space: Represent 4-momenta as bi-spinors
  - Motivated by N=4 SYM, but works for all massless theories:
    $\rightarrow$ MHV amplitudes [scalar vertices] + scalar propagators
    $\rightarrow$ F. Cachazo, P. Svrcek, E. Witten, JHEP 0409 (2004) 006
- Tree-level: Established
  - Pure gauge theory: Next-to-MHV, recursion relations
  - Processes with fermions: MHV with quarks and gluinos
- Loop-level: Cut constructibility/new CSW-diagrams
  - Works for N = 4 MHV @ 1-loop for up to 7-gluons
  - Extension to N<4, next-to-MHV and 2-loop unclear

Summary

- Inclusive processes:
  - NNLO calculation completed for $F_{1,2,3}$ $\rightarrow$ first NNLO fits
  - NNLO calculation completed for Higgs and DY production
- Less inclusive processes:
  - e^+e^- $\rightarrow$ 3 jets: $1 \rightarrow 4$ subtraction terms partially completed
    - $\rightarrow$ First (preliminary and partial) result for average thrust
  - e^+e^- $\rightarrow$ QQ : $1 \rightarrow 2$ vector and axial-vector vertex @ 2-loop
    - $\rightarrow$ Subtraction terms still missing
  - p p $\rightarrow$ 2 jets: $2 \rightarrow 2$ quark helicity amplitudes completed
    - $\rightarrow$ Subtraction terms still missing
- Multi-particle processes: Twistor methods
  - Tree-level: Works for non-SUSY, non-MHV, also fermions
  - Loop-level: Works for N=4 SYM @ 1-loop, extension unclear