Monte Carlo event generators

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Abstract. I review recent progress in the physics of parton shower Monte Carlos, emphasizing the ideas which allow the inclusion of higher-order matrix elements into the framework of event generators

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Reliable predictions of cross sections and final-state distributions for QCD processes are a crucial ingredient in high-energy collider experiments, not only as a test of QCD but also for new particle searches. All systematic approaches to this problem are based on fixed-order (FO) results in perturbation theory, and yield (usually at the next-to-leading order, NLO) the best available results for sufficiently inclusive observables. However, in many cases a more exclusive description of final states is needed. In such cases, in which one also combines perturbative calculations with a model for the conversion of partonic final states into hadrons, Monte Carlo (MC) simulations are generally adopted. MC's operate on partonic states with high multiplicity and low relative transverse momenta, which are obtained from a parton shower or dipole cascade approximation to QCD dynamics. This has to be confronted with FO results, which can describe the complementary region of small multiplicities, and large relative transverse momenta.

The lack of large transverse momentum emissions, and the fact that total rates are computed to leading order accuracy only, are serious problems in MC simulations, especially when the c.m. energies are in the TeV range. These problems can be solved by a suitable combination of MC and FO methods. Given the flexibility of MC's, it is actually desirable to embed as much as possible of FO information into the framework of MC simulations, since the other way around would just prove to be too complicated. In order to explain how this could be done, it is useful to briefly remind how an MC works: for a given process, which at the LO receives contribution from $2 \to n_0$ reactions, $(2+n_0)$-particle configurations are generated, according to exact tree-level matrix element (ME) computations. The quarks and gluons (partons henceforth) among these primary particles are then allowed to emit more quarks and gluons, which are obtained from a parton shower or dipole cascade approximation to QCD dynamics. To lessen the impact of this approximation on physical observables, one can devise two strategies. The first aims at having n_E extra hard partons in the final state; thus, in the example given above, the number of final-state hard particles would increase from n_0 to n_0+n_E. This approach is usually referred to as *matrix element corrections* (MEC), since the MC must use the $(2+n_0+n_E)$-particle ME's to generate the correct hard kinematics. The second strategy also aims at simulating the production of n_0+n_E hard particles, but improves the computation of rates as well, to $\mathrm{N}^{n_E}\mathrm{LO}$ accuracy. I'll generally denote the resulting MC as $\mathrm{N}^{n_E}\mathrm{LOwPS}$.

There are basically two major problems in the implementation of MEC. The first problem is that of achieving a fast computation of the ME's themselves for the largest possible $n_0 + n_E$, and an efficient phase-space generation. A variety of solutions exist nowadays for this problem, implemented in packages which I'll denote as ME generators; popular ones include AlpGEN [1], CompHEP [2], Grace [3], and MadEvent [4]. The second problem stems from the fact that multi-parton ME's are IR divergent. Clearly, in hard-particle configurations IR divergences don't appear; however, the definition of what hard means is, to a large extent, arbitrary. In practice, hardness is achieved by imposing some cuts on suitable partonic variables, such as p_T's and (η, φ)-distances. I collectively denote these cuts by δ_{sep}. One assumes that n hard partons will result (after the shower) into n jets; but, with a probability depending on δ_{sep}, a given n-jet event could also result from $n+m$ hard partons. This means that, when generating events at a fixed $n_0 + n_E$ number of primary particles, physical observables in general depend upon δ_{sep}; I refer to this as the δ_{sep}-bias problem. Any solution to the δ_{sep}-bias problem implies a procedure to combine consistently ME's with different $n_0 + n_E$'s. It should be stressed that, in presence of a δ_{sep} bias, the interface of an ME generator (which is responsible for producing the hard configurations, i.e. the initial conditions for the shower), and a parton shower code is *not*, strictly speaking, an event generator (EvG), since the events depend somehow on the value of δ_{sep}. In practice, the dependence is of the order of 20%, which is acceptable if one considers that, without MEC, multi-jet configurations predicted by standard MC's are completely unreliable. A solution to the δ_{sep}-bias problem has been presented, for e^+e^- collisions, in ref. [5] (CKKW henceforth), and subsequently extended (without formal proof) to hadronic collisions in ref. [6]; an alternative method for colour-dipole cascades has been presented in ref. [7]. Loosely speaking, CKKW achieve the following: if an n-jet observable is affected by the δ_{sep} bias in the following way

$$\sigma_n \sim \alpha_S^{n-2} \sum_k a_k \alpha_S^k \log^{2k} \delta_{sep}\,, \tag{1}$$

by applying the CKKW prescription one gets

$$\alpha_S^{n-2} (\delta_{sep}^a + \sum_k b_k \alpha_S^k \log^{2k-2} \delta_{sep})\,. \tag{2}$$

There is a considerable freedom in the implementation of the CKKW prescription in the case of hadronic collisions. This freedom is used to tune (some of) the EvG's parameters in order to reduce as much as possible the δ_{sep} dependence, which typically manifests itself in the form of discontinuities in the derivative of the physical spectra. A discussion on these issues, with practical examples of the implementation of CKKW in HERWIG and PYTHIA, can be found in ref. [8]. CKKW has also been implemented in SHERPA [9]; an alternative procedure, proposed by Mangano, is being implemented in AlpGEN.

I stress that the complete independence of δ_{sep} cannot be achieved; this would be possible only by including all diagrams (i.e., also the virtual ones) contributing to a given order in α_S. This fact appears to be pretty obvious: it is well known, and formally established by the BN and KLN theorems, that the infrared and collinear singularities of the real matrix elements are cancelled by the virtual contributions. One may in fact be surprised by the mild δ_{sep} dependence left in the practical implementation of CKKW (see for example ref. [8]); however, we should keep in mind that parton showers do contain part of the virtual corrections, thanks to the unitarity constraint which is embedded in the Sudakov form factors. However, to cancel exactly the δ_{sep} dependence there is no alternative way to that of inserting the exact virtual contributions to the hard process considered. In doing so, one is also able to include consistently in the computation the K factor. It is important to realize that this is *the only manner* to obtain this result in a theoretically consistent way. The procedure of reweighting the EvG's results to match those obtained with fixed-order codes for certain observables must be considered a crude approximation (since no fixed-order computation can keep into account all the complicated final-state correlations that are present when defining the cuts used in experimental analyses).

The desirable thing to do would be that of adding the virtual corrections of the same order as all of the real contributions to CKKW implementations. Unfortunately, this is unfeasible, for practical and principle reasons. The practical reason is that, at variance with real corrections, we don't know how to automatize efficiently the computations of loop diagrams in the Minkowskian kinematic region. The principle reason is that there's no known way of achieving the cancellation of infrared and collinear divergences in an universal and observable-independent manner beyond NLO. We have thus to restrict ourselves to the task of including NLO corrections in EvG's, i.e. $n_E = 1$ in the notation used above.

The fact that only one extra hard emission can be included in NLOwPS's is the reason why such codes must be presently seen as complementary to MEC. When one is interested in a small number of extra emissions, then NLOwPS's must be considered superior to MEC; on the other hand, for studying processes with many hard legs involved, such as SUSY signals or backgrounds, MEC implementations should be used. A realistic goal for the near future is that of incorporating the complete NLO corrections to all the processes with different n_E's in CKKW.

The striking feature of an NLOwPS is the computation of loop diagrams (which are necessary in order to compute total rates to NLO accuracy); this in general implies the presence of negative weights. This is a new feature in MC's, which however doesn't spoil their probabilistic nature. In fact, in NLOwPS the distributions of positive and negative weights are *separately* finite, at variance with what happens in NLO computations; thus, each of them can be unweighted and evolved separately, since no cancellation between large numbers is involved in this procedure. On the other hand, the contribution of loop diagrams implies that the δ_{sep}-bias problem which affects ME corrections is simply not present. This advantage comes at a price: KLN cancellation cannot be achieved any longer solely at the level of Sudakov form factors through unitarity, since virtual and real matrix elements are now explicitly present in the computation. Technically, this complicates enormously the problem wrt the case of MEC: KLN cancellation is inclusive by nature, and one wants to achieve it here in the context of a parton shower MC, whose final state is fully exclusive thanks to the use of a hadronization model. A solution to this problem has been presented for the first time in ref. [10]. It is based on the observation that, upon formally expanding an MC result in α_S, the first non-trivial order obtained in this way must match the behaviour of the fixed-order computations at the same order, and in the collinear limit. Thus, the $\mathscr{O}(\alpha_S)$ MC result can be used effectively to cancel locally the matrix element singularities. It can be shown [10] that, after the subtracted matrix elements are matched to the shower, the singularity cancellation achieved in this way is equivalent to the KLN one for inclusive observables, up to power-suppressed terms (which are not correctly included anyhow in results based on collinear factorization theorems).

The strategy outlined above has been implemented in MC@NLO [10, 11], which features a steadily-growing number of production processes in hadronic collisions, such as single vector and Higgs bosons, vector boson pairs, heavy quark pairs, lepton pairs, and Higgs boson in association with a W or Z [12]. Apart from MC@NLO there are at present only a couple of NLOwPS hadronic codes, Φ-veto [13] and GRACE_LLsub [14], which feature only single-Z production. On the other hand, there has been a substantial theoretical activity in the field in the past few years, which will certainly lead to more practical implementation of NLOwPS in the future. Nason [15] has proposed a method for constructing NLOwPS's that should result in a smaller number of negative weights wrt those obtained with MC@NLO. Collins and Zu [16] have defined a framework in which the shower can be improved beyond the LL accuracy; at the moment, the method cannot work in QCD, since it does not include a proper treatment of soft emissions. Soper and Nagy [17] attempt to introduce a formalism in which NLOwPS techniques are embedded in a CKKW framework, thus potentially improving the latter by adding an extra $\mathscr{O}(\alpha_S)$ accuracy for each emission. Finally, one should not forget that a lot of work is being done on different aspects of standard MC's: see ref. [18]).

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