Soft-gluon expansions through NNNLO

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Abstract. I present universal master formulas for soft-gluon corrections to hard-scattering cross sections through next-to-next-to-next-to-leading order (NNNLO). I also briefly discuss applications to some processes where these corrections enhance the cross section and decrease the scale dependence.

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SOFT-GLUON RESUMMATION

Cross sections in perturbative QCD can be calculated by employing factorization theorems as $\sigma = \sum_f \int [\prod_i dx_i\, \phi_{f/h_i}(x_i, \mu_F)]\, \hat{\sigma}(s,t,u,\mu_F,\mu_R)$, where $\hat{\sigma}$ is the perturbatively calculable hard-scattering cross section, and the parton distributions ϕ are determined from experiment. The renormalization and factorization scales are denoted by μ_R and μ_F respectively, and s,t,u are standard kinematical invariants formed from the momenta of the partons in the hard scattering.

Near threshold for the production of a specified system, such as a top quark pair or a Higgs boson, there is restricted phase space for real gluon emission. The incomplete cancellation of infrared divergences between real and virtual graphs results in the appearance of large logarithms. If we define $s_4 = s+t+u-\sum m^2$, with m the masses of the particles in the scattering, then $s_4 \to 0$ at threshold and these soft-gluon logarithmic corrections take the form of plus distributions, $\mathscr{D}_l(s_4) \equiv [\ln^l(s_4/M^2)/s_4]_+$, where M is a relevant hard scale, such as the mass of a heavy quark or the transverse momentum of a jet, and $l \le 2n-1$ for the n-th order corrections.

If we define moments of the cross section $\hat{\sigma}(N) = \int_0^\infty ds_4\, e^{-Ns_4/M^2}\, \hat{\sigma}(s_4)$ then the soft corrections are transformed as $[\ln^l(s_4/M^2)/s_4]_+ \to [(-1)^{l+1}/(l+1)] \ln^{l+1} N + \cdots$. We can formally resum these logarithms $\ln N$ to all orders in α_s by factorizing the soft gluons from the hard scattering [1, 2]. Although the formal resummation in moment space is well defined, when inverting back to momentum space we encounter ambiguities due to the infrared singularity which require a prescription. Unfortunately different prescriptions can give different numerical results as well as have dubious theoretical underpinnings (see discussion in Ref. [3]). However, fixed-order expansions can provide us with solid, prescription-independent, theoretical and numerical results [3, 4].

At next-to-leading order (NLO) in α_s, the cross section includes $\mathscr{D}_1(s_4)$ terms which are the leading logarithms (LL), and $\mathscr{D}_0(s_4)$ terms which are the next-to-leading logarithms (NLL). At next-to-next-to-leading order (NNLO), we have $\mathscr{D}_3(s_4)$ (LL), $\mathscr{D}_2(s_4)$ (NLL), $\mathscr{D}_1(s_4)$ (NNLL), and $\mathscr{D}_0(s_4)$ (NNNLL) terms. At next-to-next-to-next-to-leading order (NNNLO), we have $\mathscr{D}_5(s_4)$ (LL), $\mathscr{D}_4(s_4)$ (NLL), $\mathscr{D}_3(s_4)$ (NNLL),

$\mathscr{D}_2(s_4)$ (NNNLL), $\mathscr{D}_1(s_4)$ (NNNNLL), and $\mathscr{D}_0(s_4)$ (NNNNNLL) terms.

The threshold resummation formalism has been applied by now to many processes in hadron-hadron and lepton-hadron colliders, for both total and differential cross sections, in both single-particle-inclusive (1PI) and pair-invariant-mass (PIM) kinematics, for both simple and complex color flows, and in both $\overline{\rm MS}$ and DIS factorization schemes [4].

Specific processes for which soft-gluon corrections have been calculated at NNLO include top quark pair hadroproduction [3, 5], beauty and charm production [6], jet production [7], direct photon production [8], large-p_T W production [9], FCNC top production [10], and charged Higgs production [11]. Numerical results show that usually the soft corrections are a good approximation of the full NLO result. In all cases the higher-order corrections are sizable and produce a dramatic decrease of the scale dependence of the cross section.

The resummed cross section can be written for an arbitrary process as [4]

$$
\begin{aligned}
\hat{\sigma}^{res}(N) \;&=\; \exp\left[\sum_i E_i(N_i)\right] \, \exp\left[\sum_j E'_j(N_j)\right] \, \exp\left[2\, d_{\alpha_s} \int_{\mu_R}^{\sqrt{s}} \frac{d\mu'}{\mu'} \beta\left(\mu'\right)\right] \\
&\times \exp\left[\sum_i 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu'}{\mu'} \left(\frac{\alpha_s(\mu')}{\pi}\gamma_i^{(1)} + \gamma'_{i/i}\left(\mu'\right)\right)\right] \\
&\times {\rm Tr}\left\{H\left(\mu_R\right) \, \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu'}{\mu'} \Gamma_S^{\dagger}\left(\mu'\right)\right] \, S\left(s/\tilde{N}_j^2\right) \, \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu'}{\mu'} \, \Gamma_S\left(\mu'\right)\right]\right\}
\end{aligned} \tag{1}
$$

where $E_i(N_i)$ denotes contributions from the incoming partons and is given in the $\overline{\rm MS}$ scheme by

$$
E_i(N_i) = -\int_0^1 dz \frac{z^{N_i-1}-1}{1-z} \left\{\int_{(1-z)^2 s}^{\mu_F^2} \frac{d\mu'^2}{\mu'^2} A_i(\mu') + \nu_i\left[(1-z)^2 s\right]\right\} \tag{2}
$$

with $A_i = C_i[\alpha_s/\pi + (\alpha_s/\pi)^2 K/2] + \cdots$, $\nu_i = (\alpha_s/\pi)C_i + (\alpha_s/\pi)^2 \nu_i^{(2)} + \cdots$; $E_j(N_j)$ denotes contributions from massless final-state partons (if any) and is given by

$$
E'_j(N_j) = \int_0^1 dz \frac{z^{N_j-1}-1}{1-z} \left\{\int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} A_j\left(\lambda s\right) - B'_j[(1-z)s] - \nu_j\left[(1-z)^2 s\right]\right\} \tag{3}
$$

where $B'_j = (\alpha_s/\pi) B'^{(1)}_j + (\alpha_s/\pi)^2 B'^{(2)}_j + \cdots$; γ_i are parton anomalous dimensions; H are hard scattering matrices, independent of soft-gluon radiation; S are soft matrices which describe noncollinear soft-gluon emission; and Γ_S are soft anomalous dimension matrices which appear in the evolution of the S matrices. H, S, and Γ_S are matrices in the space of color exchanges; they become simple functions for processes with simple color structure, such as Drell-Yan production.

Expansions of the resummed cross section through NNLO were given in Ref. [4], where master formulas were derived and then used in calculations for specific processes [5-11]. Here we extend this expansion to NNNLO.

Expanding Eq. (1) to NLO, gives us the master formula for the NLO corrections

$$\begin{aligned}\hat{\sigma}^{(1)} &= \sigma^B \frac{\alpha_s(\mu_R^2)}{\pi} \{c_3\, \mathscr{D}_1(s_4) + c_2\, \mathscr{D}_0(s_4) + c_1\, \delta(s_4)\} \\ &\quad + \frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi} [A^c\, \mathscr{D}_0(s_4) + T_1^c\, \delta(s_4)] \end{aligned} \tag{4}$$

with $c_3 = \sum_i 2C_i - \sum_j C_j$, where for quarks $C_F = (N_c^2-1)/(2N_c)$ and for gluons $C_A = N_c$; $c_2 = c_2^\mu + T_2$, with $c_2^\mu = -\sum_i C_i \ln(\mu_F^2/M^2)$ and

$$T_2 = -\sum_i \left[C_i + 2C_i \ln\left(\frac{-t_i}{M^2}\right) + C_i \ln\left(\frac{M^2}{s}\right)\right] - \sum_j \left[B'^{(1)}_j + C_j + C_j \ln\left(\frac{M^2}{s}\right)\right] ; \tag{5}$$

and $c_1 = c_1^\mu + T_1$, with

$$c_1^\mu = \sum_i \left[C_i \ln\left(\frac{-t_i}{M^2}\right) - \gamma_i^{(1)}\right] \ln\left(\frac{\mu_F^2}{M^2}\right) + d_{\alpha_s} \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right), \tag{6}$$

where for quarks $B'^{(1)}_q = \gamma_q^{(1)} = 3C_F/4$ and for gluons $B'^{(1)}_g = \gamma_g^{(1)} = \beta_0/4$. The term A^c involves matrices and is given by $A^c = \mathrm{tr}\left(H^{(0)}\Gamma'^{(1)\dagger}_S S^{(0)} + H^{(0)} S^{(0)} \Gamma'^{(1)}_S\right)$. Finally T_1 and T_1^c can be read off a complete NLO calculation for a specific process.

Expanding the resummed cross section through NNLO and matching with the NLO result gives us the master formula for the NNLO corrections [4]

$$\begin{aligned}\hat{\sigma}^{(2)} &= \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \frac{1}{2} c_3^2\, \mathscr{D}_3(s_4) \\ &+ \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{\frac{3}{2} c_3\, c_2 - \frac{\beta_0}{4} c_3 + \sum_j C_j \frac{\beta_0}{8}\right\} \mathscr{D}_2(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \frac{3}{2} c_3\, A^c\, \mathscr{D}_2(s_4) \\ &+ \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln\left(\frac{\mu_R^2}{M^2}\right) + c_3 \frac{K}{2} - \sum_j \frac{\beta_0}{4} B'^{(1)}_j\right\} \mathscr{D}_1(s_4) \\ &+ \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \left\{\left(2\, c_2 - \frac{\beta_0}{2}\right) A^c + c_3\, T_1^c + F^c\right\} \mathscr{D}_1(s_4) + \mathscr{O}(\mathscr{D}_0(s_4)), \end{aligned} \tag{7}$$

where we show terms explicitly through NNLL. Here $F^c = \mathrm{tr}[H^{(0)} \left(\Gamma'^{(1)\dagger}_S\right)^2 S^{(0)} + H^{(0)} S^{(0)} (\Gamma'^{(1)}_S)^2 + 2H^{(0)} \Gamma'^{(1)\dagger}_S S^{(0)} \Gamma'^{(1)}_S]$.

Finally, expanding the resummed formula through NNNLO and matching with the NLO and NNLO results gives us the master formula for the NNNLO corrections

$$\hat{\sigma}^{(3)} = \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \frac{1}{8} c_3^3\, \mathscr{D}_5(s_4)$$

$$
+\sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{\frac{5}{8} c_3^2 c_2 - \frac{5}{2} c_3 X_3\right\} \mathscr{D}_4(s_4) + \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \frac{5}{8} c_3^2 A^c \mathscr{D}_4(s_4)
$$

$$
+\sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{c_3 c_2^2 + \frac{1}{2} c_3^2 c_1 - \zeta_2 c_3^3 + (\beta_0 - 4c_2) X_3 + 2c_3 X_2 - \sum_j C_j \frac{\beta_0^2}{48}\right\} \mathscr{D}_3(s_4)
$$

$$
+\frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \left\{\frac{1}{2} c_3^2 T_1^c + \left[2c_3 c_2 - \frac{\beta_0}{2} c_3 - 4X_3\right] A^c + c_3 F^c\right\} \mathscr{D}_3(s_4) + \mathscr{O}(\mathscr{D}_2(s_4)) \quad (8)
$$

where again we show terms explicitly through NNLL. Here $X_3 = (\beta_0/12)c_3 - \sum_j C_j \beta_0/24$ and $X_2 = -(\beta_0/4)T_2 + (\beta_0/8)c_3 \ln(\mu_R^2/M^2) + c_3 K/4 - \sum_j (\beta_0/8) B'^{(1)}_j$.

This calculation has recently been applied to charged Higgs production with a top quark via bottom gluon fusion at the LHC through NNLO [11], where for a charged Higgs mass of 500 GeV the NLO-NLL soft-gluon corrections provide a enhancement of 38% over the leading-order cross section and the NNLO-NLL corrections provide a enhancement of 11% over the NLO-NLL cross section. A new calculation of the NNNLO-NLL corrections shows that they provide an additional 7% enhancement over the NNLO-NLL cross section and further stabilize the scale dependence of the cross section. Similarly, for top quark pair production at the Tevatron the scale dependence is considerably decreased. More details will be given in a forthcoming paper.

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