Soft-gluon expansions through NNNLO

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- Soft-gluon corrections
- Resummed cross sections
- NNLO corrections
- NNNLO corrections
- Application to tH^- production

Factorization in perturbative QCD

$$\sigma = \sum_f \int \left[\prod_i dx_i \, \phi_{f/h_i}(x_i, \mu_F^2)\right] \hat{\sigma}(s, t, u, \mu_F, \mu_R)$$

Hard-scattering cross section - perturbatively calculable

Near threshold for production of the system restricted phase space for real gluon emission

Incomplete cancellation of infrared divergences between real and virtual graphs $\rightarrow$ large logarithms

Soft and collinear corrections $\longrightarrow$ plus distributions

Define $s_4 = s + t + u - \sum m^2 \rightarrow 0$ at threshold

$$\mathcal{D}_l(s_4) \equiv \left[\frac{\ln^l(s_4/M^2)}{s_4}\right]_+$$

with $l \leq 2n - 1$ for the n-th order corrections

Define moments of the cross section

$\hat{\sigma}(N) = \int_0^\infty ds_4 \, e^{-N s_4/M^2} \, \hat{\sigma}(s_4)$

Soft corrections

$$\mathcal{D}_l(s_4) \equiv \left[\frac{\ln^l(s_4/M^2)}{s_4}\right]_+ \rightarrow \frac{(-1)^{l+1}}{l+1} \ln^{l+1} N + \cdots$$

We can formally resum these logarithms to all orders in α_s: factorize soft gluons from the hard scattering

Invert back to momentum space

At NLO, $\underset{\text{LL}}{\mathcal{D}_1(s_4)}$ and $\underset{\text{NLL}}{\mathcal{D}_0(s_4)}$ terms

At NNLO, $\underset{\text{LL}}{\mathcal{D}_3(s_4)}$, $\underset{\text{NLL}}{\mathcal{D}_2(s_4)}$, $\underset{\text{NNLL}}{\mathcal{D}_1(s_4)}$, and $\underset{\text{NNNLL}}{\mathcal{D}_0(s_4)}$ terms

At NNNLO, $\mathcal{D}_5(s_4)$, $\mathcal{D}_4(s_4)$, $\mathcal{D}_3(s_4)$, $\mathcal{D}_2(s_4)$, $\mathcal{D}_1(s_4)$, and $\mathcal{D}_0(s_4)$ terms

Threshold resummation formalism applied to:

- Hadron-hadron and lepton-hadron colliders
- Total and differential cross sections
- 1PI and PIM kinematics
- Simple and complex color flows
- $\overline{\text{MS}}$ and DIS factorization schemes

Specific processes:

- Top quark pair hadroproduction
- Beauty and charm production
- Single-jet and dijet production
- Direct photon production
- Large-p_T W and Z production
- FCNC top production
- Charged Higgs production

Numerical results:
Soft corrections a good approx. of full NLO result
Higher-order corrections are sizable
Dramatic decrease of scale dependence

Soft-gluon resummation

A unified approach

$$
\begin{aligned}
\hat{\sigma}^{res}(N) \quad = \quad & \exp\left[\sum_i E_i(N_i)\right] \exp\left[\sum_j E'_j(N_j)\right] \\
& \times \exp\left[\sum_i 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu'}{\mu'}\left(\frac{\alpha_s(\mu'^2)}{\pi}\gamma_i^{(1)} + \gamma'_{i/i}\left(\alpha_s(\mu'^2)\right)\right)\right] \\
& \times \exp\left[2\, d_{\alpha_s}\int_{\mu_R}^{\sqrt{s}} \frac{d\mu'}{\mu'}\beta\left(\alpha_s(\mu'^2)\right)\right] \\
\times \mathrm{Tr}\Bigg\{ & H\left(\alpha_s(\mu_R^2)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu'}{\mu'}\Gamma'^{\dagger}_S\left(\alpha_s(\mu'^2)\right)\right] \\
& \times \tilde{S}\left(\alpha_s(s/\tilde{N}_j^2)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}_j} \frac{d\mu'}{\mu'}\,\Gamma'_S\left(\alpha_s(\mu'^2)\right)\right]\Bigg\}
\end{aligned}
$$

with ($\overline{\mathrm{MS}}$ scheme)

$$
E_i(N_i) = -\int_0^1 dz \frac{z^{N_i-1}-1}{1-z}\left\{\int_{(1-z)^2 s}^{\mu_F^2} \frac{d\mu'^2}{\mu'^2} A_i\left(\alpha_s(\mu'^2)\right) + \nu_i\left[\alpha_s((1-z)^2 s)\right]\right\}
$$

with $A_i(\alpha_s) = C_i\,[\alpha_s/\pi + (\alpha_s/\pi)^2 K/2] + A_i^{(3)} + \cdots$

$\nu_i = (\alpha_s/\pi)C_i + (\alpha_s/\pi)^2\nu_i^{(2)} + \cdots$

and (for any massless final-state partons at lowest order)

$$
\begin{aligned}
E'_j(N_j) \quad = \quad & \int_0^1 dz \frac{z^{N_j-1}-1}{1-z}\left\{\int_{(1-z)^2}^{1-z} \frac{d\lambda}{\lambda} A_j\left(\alpha_s\left(\lambda s\right)\right)\right. \\
& \left. - B'_j\left[\alpha_s((1-z)s)\right] - \nu_j\left[\alpha_s((1-z)^2 s)\right]\right\}
\end{aligned}
$$

where $B'_j = (\alpha_s/\pi)B'^{(1)}_j + (\alpha_s/\pi)^2 B'^{(2)}_j + \cdots$

with $B'^{(1)}_q = 3C_F/4$ and $B'^{(1)}_g = \beta_0/4$

γ_i are parton anomalous dimensions; H are hard scattering matrices;

S are soft matrices (noncollinear soft-gluon emission);

Γ_S are soft anomalous dimension matrices

NLO master formula

$$\begin{aligned}\hat{\sigma}^{(1)} &= \sigma^B \frac{\alpha_s(\mu_R^2)}{\pi}\{c_3\,\mathcal{D}_1(s_4)+c_2\,\mathcal{D}_0(s_4)+c_1\,\delta(s_4)\}\\ &+\frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi}\left[A^c\,\mathcal{D}_0(s_4)+T_1^c\,\delta(s_4)\right]\end{aligned}$$

with

$$c_3=\sum_i 2\,C_i-\sum_j C_j$$

For quarks $C_F=(N_c^2-1)/(2N_c)$ For gluons $C_A=N_c$

$$c_2=c_2^\mu+T_2$$

with

$$c_2^\mu=-\sum_i C_i \ln\left(\frac{\mu_F^2}{M^2}\right)$$

$$\begin{aligned}T_2 &= -\sum_i\left[C_i+2\,C_i\,\ln\left(\frac{-t_i}{M^2}\right)+C_i\,\ln\left(\frac{M^2}{s}\right)\right]\\ &-\sum_j\left[B_j'^{(1)}+C_j+C_j\,\ln\left(\frac{M^2}{s}\right)\right]\end{aligned}$$

For quarks $B_q'^{(1)}=\gamma_q^{(1)}=3C_F/4$ For gluons $B_g'^{(1)}=\gamma_g^{(1)}=\beta_0/4$

Also $c_1=c_1^\mu+T_1$, with

$$c_1^\mu=\sum_i\left[C_i\,\ln\left(\frac{-t_i}{M^2}\right)-\gamma_i^{(1)}\right]\ln\left(\frac{\mu_F^2}{s}\right)+d_{\alpha_s}\frac{\beta_0}{4}\ln\left(\frac{\mu_R^2}{s}\right)$$

Matrix terms

$$A^c=\mathrm{tr}\left(H^{(0)}\Gamma_S'^{(1)\,\dagger}S^{(0)}+H^{(0)}S^{(0)}\Gamma_S'^{(1)}\right)$$

$$T_1^c=\mathrm{tr}\left(H^{(1)}S^{(0)}+H^{(0)}S^{(1)}\right)+A^c\ln\left(\frac{M^2}{s}\right)$$

NNLO master formula

$$\hat{\sigma}^{(2)} = \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \frac{1}{2} c_3^2 \, \mathcal{D}_3(s_4)$$

$$+ \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \frac{3}{2} c_3 \, c_2 - \frac{\beta_0}{4} c_3 + \sum_j C_j \frac{\beta_0}{8} \right\} \mathcal{D}_2(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \frac{3}{2} c_3 \, A^c \, \mathcal{D}_2(s_4)$$

$$+ \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} C_{D_1}^{(2)} \, \mathcal{D}_1(s_4) + \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \left\{ \left(2\, c_2 - \frac{\beta_0}{2} \right) A^c + c_3 \, T_1^c + F^c \right\} \mathcal{D}_1(s_4)$$

$$+ \sigma^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} C_{D_0}^{(2)} \, \mathcal{D}_0(s_4)$$

$$+ \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R^2)}{\pi^2} \left\{ \left[c_1 - \zeta_2 \, c_3 + \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right) + \frac{\beta_0}{4} \ln\left(\frac{M^2}{s}\right) \right] A^c + \left(c_2 - \frac{\beta_0}{2} \right) T_1^c \right.$$

$$\left. + F^c \ln\left(\frac{M^2}{s}\right) + G^c \right\} \mathcal{D}_0(s_4)$$

Here

$$C_{D_1}^{(2)} = c_3 \, c_1 + c_2^2 - \zeta_2 \, c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln\left(\frac{\mu_R^2}{M^2}\right) + c_3 \frac{K}{2} - \sum_j \frac{\beta_0}{4} B'^{(1)}_j$$

$$C_{D_0}^{(2)} = c_2 \, c_1 - \zeta_2 \, c_3 \, c_2 + \zeta_3 \, c_3^2 - \frac{\beta_0}{2} T_1 + \frac{\beta_0}{4} c_2 \ln\left(\frac{\mu_R^2}{M^2}\right) + \cdots$$

$$F^c = \mathrm{tr}\left[H^{(0)} \left(\Gamma'^{(1)\,\dagger}_S \right)^2 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma'^{(1)}_S \right)^2 + 2 H^{(0)} \Gamma'^{(1)\,\dagger}_S S^{(0)} \Gamma'^{(1)}_S \right]$$

$$G^c = \mathrm{tr}\left[H^{(1)} \Gamma'^{(1)\,\dagger}_S S^{(0)} + H^{(1)} S^{(0)} \Gamma'^{(1)}_S + H^{(0)} \Gamma'^{(1)\,\dagger}_S S^{(1)} + H^{(0)} S^{(1)} \Gamma'^{(1)}_S \right.$$

$$\left. + H^{(0)} \Gamma'^{(2)\,\dagger}_S S^{(0)} + H^{(0)} S^{(0)} \Gamma'^{(2)}_S \right]$$

NNNLO master formula

$$\hat{\sigma}^{(3)} = \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \frac{1}{8} c_3^3 \, \mathcal{D}_5(s_4)$$

$$+ \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{ \frac{5}{8} c_3^2 \, c_2 - \frac{5}{2} c_3 \, X_3 \right\} \mathcal{D}_4(s_4) + \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \frac{5}{8} c_3^2 \, A^c \, \mathcal{D}_4(s_4)$$

$$+ \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{ c_3 \, c_2^2 + \frac{c_3^2}{2} c_1 - \zeta_2 \, c_3^3 + (\beta_0 - 4c_2) \, X_3 + 2c_3 \, X_2 - \sum_j C_j \frac{\beta_0^2}{48} \right\} \mathcal{D}_3(s_4)$$

$$+ \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \left\{ \frac{1}{2} c_3^2 \, T_1^c + \left[2 \, c_3 \, c_2 - \frac{\beta_0}{2} c_3 - 4 \, X_3 \right] A^c + c_3 \, F^c \right\} \mathcal{D}_3(s_4)$$

$$+ \sigma^B \frac{\alpha_s^3(\mu_R^2)}{\pi^3} \left\{ \frac{3}{2} c_3 \, c_2 \, c_1 + \frac{1}{2} c_2^3 - 3 \, \zeta_2 \, c_3^2 \, c_2 + \frac{5}{2} \zeta_3 \, c_3^3 + \left(-3 \, c_1 + \frac{27}{2} \zeta_2 \, c_3 \right) X_3 \right.$$

$$+ (3 \, c_2 - \beta_0) \, X_2 - \frac{3}{2} c_3 \, X_1 - \sum_i C_i \frac{\beta_1}{8} + \sum_j \frac{\beta_0^2}{16} B_j'^{(1)} + \sum_j \frac{3}{32} C_j \, \beta_1$$

$$\left. + \sum_j C_j \frac{\beta_0}{16} \left[\beta_0 \ln \left(\frac{\mu_R^2}{M^2} \right) + 2 \, K \right] \right\} \mathcal{D}_2(s_4)$$

$$+ \frac{\alpha_s^{d_{\alpha_s}+3}(\mu_R^2)}{\pi^3} \left\{ \left(\frac{3}{2} c_3 \, c_2 - 3 \, X_3 \right) T_1^c + \frac{3}{2} \left[c_2 + c_3 \ln \left(\frac{M^2}{s} \right) \right] F^c \right.$$

$$+ \left[\frac{3}{2} c_2^2 + \frac{3}{2} c_3 \, c_1 - 3 \, \zeta_2 \, c_3^2 + 3 \, X_2 + \frac{\beta_0^2}{4} - \frac{3}{4} \beta_0 \left(c_2 - \frac{c_3}{2} \ln \left(\frac{\mu_R^2}{M^2} \right) \right) \right.$$

$$\left. \left. - \frac{3\beta_0}{8} c_3 \ln \left(\frac{M^2}{s} \right) \right] A^c + \frac{3}{2} c_3 \, G^c + \frac{1}{2} K_3^c \right\} \mathcal{D}_2(s_4) \quad + \cdots$$

Here $X_3 = (\beta_0/12)c_3 - \sum_j C_j \beta_0/24$

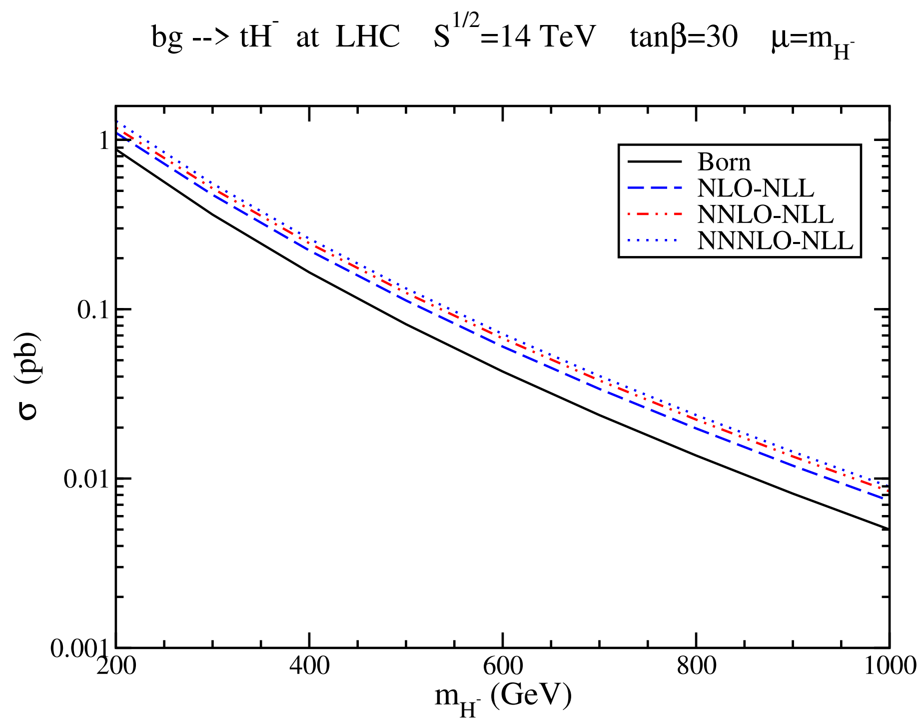
$X_2 = -(\beta_0/4)T_2 + (\beta_0/8)c_3 \ln(\mu_R^2/M^2) + c_3 K/4 - \sum_j (\beta_0/8) B_j'^{(1)}$

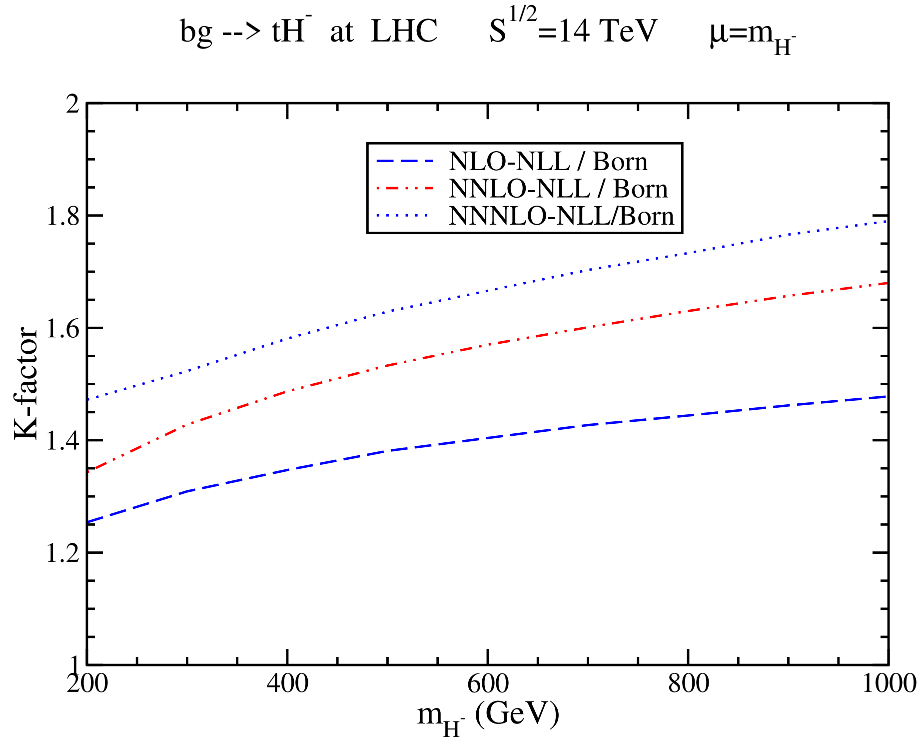
$X_1 = c_2 \, c_1 - \zeta_2 \, c_3 \, c_2 + \zeta_3 \, c_3^2 + (\beta_0/4) \, \zeta_2 \, c_3 - \sum_j C_j (\beta_0/8)\zeta_2 - C_{D_0}^{(2)}$

$$K_3^c = \mathrm{tr} \left[H^{(0)} \left(\Gamma_S'^{(1)\dagger} \right)^3 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S'^{(1)} \right)^3 \right.$$

$$\left. + 3 \, H^{(0)} \left(\Gamma_S'^{(1)\dagger} \right)^2 S^{(0)} \Gamma_S'^{(1)} + 3 \, H^{(0)} \Gamma_S'^{(1)\dagger} S^{(0)} \left(\Gamma_S'^{(1)} \right)^2 \right]$$

NNNLO soft-gluon corrections for charged Higgs production





Summary

- Soft-gluon resummation
- Soft-gluon threshold corrections are sizable
- NLO, NNLO, and NNNLO soft-gluon expansions
- Important for greater theoretical accuracy