Gaps between Jets: Matching two Approaches

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Abstract. We calculate the parton level cross section for the production of two jets that are far apart in rapidity, subject to a limitation on the total transverse momentum Q_0 in the interjet region. We specifically address the question of how to combine the approach which sums all leading logarithms in Q/Q_0 (where Q is the jet transverse momentum) with the BFKL approach, in which leading logarithms of the scattering energy are summed. Using an "all orders" matching, we obtain results for the cross section which correctly reproduce the two approaches in the appropriate limits.

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INTRODUCTION

Final states with high-p_T jets separated by large rapidity gaps at hadron colliders offer the possibility to better understand QCD in the high energy limit and also to understand QCD radiation in "gap" events. There are two major approaches to the production of two gap-separated jets. In the BFKL [1] approach, parton-parton elastic scattering with a QCD colour singlet exchange is regarded as providing the leading contribution to the cross-section. The leading-Y terms (Y is the rapidity interval between the jets) are summed i.e. terms $\sim \alpha_s^n Y^n$ [2, 3]. The observable calculated in this approach does not consider any radiation into the interjet region. Experiments though, impose an upper bound on this radiation by necessity. In the second approach soft radiation with transverse energy below Q_0 is allowed in the interjet region. This gives rise to logarithms of Q/Q_0 where Q is the transverse momentum of the jets. The global leading logarithms of Q/Q_0 (LLQ_0) have been summed for various jet definitions [4, 5] i.e terms $\sim \alpha_s^n Y^m L^n$ $(m \leq n)$ where $L = \ln Q^2/Q_0^2$. Non-global effects have been considered in [5, 6]. In order to get a better understanding of the gaps-between-jets processes at colliders it is desirable to combine the two approaches. This is the main issue in this contribution, for details see [7]

SUMMING LOGARITHMS IN Q_0

As the first step we recalculate the cross section for two-jet production in the high-energy (i.e. high rapidity separation) limit, with limited total scalar transverse momentum in the interjet region. We require this transverse momentum to be below Q_0 and consider the region $Q_0^2 \ll Q^2 \ll \hat{s} = \mathrm{e}^Y Q^2$. Since we are not sensitive to collinear emission, we work at the parton level and calculate the all-orders gap cross section $\sigma \equiv \frac{\mathrm{d}\sigma(\hat{s},Q_0,Y)}{\mathrm{d}Q^2}$ for the process $qq' \to qq'$. $\sigma^{(n)}$ denotes the cross section at $\mathscr{O}(\alpha_s^n)$. Our approximation implies the eikonal (soft gluon) approximation. To generate the leading logs in Q_0, we make the approximation of strongly-ordered transverse momenta for real and virtual gluons.

As the basis for the calculation to all orders we employ the following theorem. Let us denote by $\mathscr{A}_1^{(n)}(Q_0)\mathbf{C}_1$ ($\mathscr{A}_8^{(n)}(Q_0)\mathbf{C}_8$) the singlet (octet) component of the $\mathscr{O}(\alpha_s^n)$ $qq' \to qq'$ amplitude in the approximation defined above, with the phase space for the gluons constrained to the gap region in rapidity and with transverse momentum above Q_0 ($\mathbf{C}_{1,8}$ are the colour factors). With $\mathscr{B}(Q_0)$ denoting the production amplitude (including colour factor) for more than 2 particles, the theorem reads

$$\sigma^{(k)} = |\mathscr{A}_1(0)|^2\mathbf{C}_1^2 + |\mathscr{A}_8(0)|^2\mathbf{C}_8^2 + |\mathscr{B}(Q_0)|^2 = |\mathscr{A}_1(Q_0)|^2\mathbf{C}_1^2 + |\mathscr{A}_8(Q_0)|^2\mathbf{C}_8^2 \tag{1}$$

where the squares are to be read symbolically representing the sums over $\mathscr{A}^{(n)*}\mathscr{A}^{(m)}$ (and $\mathscr{B}(Q_0)$, respectively). This is clearly a major simplification, since it means that we never have to calculate any real emission or triple-gluon-vertex diagrams. This theorem provides the basis for the matching with BFKL. We calculate $\mathscr{A}(Q_0)_{1,8}$ and hence σ to all orders. Besides the double-leading-logarithmic (*DLL*) terms we include those terms sub-leading in Y that arise from the imaginary parts of the loop integrals.

MATCHING WITH BFKL

To combine the gap cross section with the BFKL approach order-by-order we need to prevent double counting and make sure the divergences arising from the BFKL approach (at each order in α_s) cancel in the jet cross section. To this end we calculate the leading-Y approximation of the singlet component $\mathscr{A}_1^{(n)}(Q_0)$:

$$\mathscr{A}_{1,S}^{(n)}(Q_0) \equiv \mathscr{A}_1^{(n)}(Q_0)\Big|_{LY}. \tag{2}$$

$\mathscr{A}_{1,S}^{(n)}(0)$ is divergent at each order and it is this contribution to σ that is also included in the BFKL result.

Fixed order matching. We denote by $\mathscr{A}_{BFKL}^{(n)}\mathbf{C}_1$ the $\mathscr{O}(\alpha_s^n)$ elastic quark scattering amplitude with colour singlet exchange in the leading-Y approximation. We want σ to include $\mathscr{A}_{BFKL}^{(n)}$. However, $\mathscr{A}_1^{(n)}(0)$ also includes terms sub-leading in Y which we have to keep; they are given by $(\mathscr{A}_1(0) - \mathscr{A}_{1,S}(0))^{(n)}$. We therefore define the following fixed order gap cross section (again omitting the sum over indices in the first line).

$$\sigma_{gap}^{(k)} \equiv |\mathscr{A}_{BFKL} + \mathscr{A}_1(0) - \mathscr{A}_{1,S}(0)|^2\mathbf{C}_1^2 + |\mathscr{A}_8(0)|^2\mathbf{C}_8^2 + |\mathscr{B}(Q_0)|^2 \tag{3}$$

$$= \sigma^{(k)} + \sum_{m+n=k}\left[2\mathrm{Im}\mathscr{A}_1^{(m)}(0)\cdot(-i\delta^{(n)}) + \delta^{(m)}\delta^{(n)*}\right]\mathbf{C}_1^2 \tag{4}$$

$$\text{with} \quad \delta^{(n)} = \mathscr{A}_{BFKL}^{(n)} - \mathscr{A}_{1,S}^{(n)}(0) \tag{5}$$

where, in the last line we have invoked the theorem (1). This cross section combines the two approaches without double counting. However, not surprisingly, the strong ordering approximation cannot cancel the divergence in the BFKL amplitude at any order. The second term in (4) and hence $\sigma_{gap}^{(k)}$ is divergent for $k \geq 6$. Via (4) we can therefore combine the all-orders cross section σ with the BFKL result up to $\mathscr{O}(\alpha_s^5)$.

The theorem (1) holds beyond the high energy approximation, the matching with BFKL can therefore be extended to full (global) LLQ_0 accuracy in a straightforward way [7].

All orders matching. Although the order-by-order combination of the LLQ_0 and the BFKL result can only work for the first few orders it is possible to construct an all-orders cross section that does smoothly interpolate the LLQ_0 and BFKL results, agreeing with each in its region of validity and avoiding any double-counting. Central to this are the following two observations. First, the amplitude $\mathscr{A}_{1,S}^{(n)}(Q_0)$ summed to all orders reads:

$$\mathscr{A}_{1,S}(Q_0) = -i\,\frac{N_c^2-1}{2N_c^3}\,\frac{\pi}{Y}\,\mathscr{A}_8^{(1)} \cdot \left[1 - \exp\left(-\frac{N_c\alpha_s}{2\pi}\,YL\right)\right]. \tag{6}$$

The exponential vanishes as $Q_0 \to 0$. In contrast to the fixed order result, $\mathscr{A}_{1,S}(0)$ is therefore finite. Secondly, we find the following relation between the (finite) all-orders results for the BFKL $2 \to 2$ cross section σ_{BFKL} [3] and the gap cross section σ:

$$\sigma_{BFKL}|_{Y\to 0} = \sigma|_{Y\to\infty} = \sigma_S \equiv |\mathscr{A}_{1,S}(0)|^2\mathbf{C}_1^2 = \sigma^{(2)}\,\frac{N_c^2-1}{N_c^4}\,\frac{\pi^2}{Y^2} \tag{7}$$

which implies $\mathscr{A}_{BFKL}|_{Y\to 0} = \mathscr{A}_{1,S}(0)$. Using these two remarkable results we construct three different matched cross sections (δ is given by (5) summed to all orders).

Simple matching: $\sigma_{gap} = \sigma + N_c^2|\delta|^2$
Cross section matching: $\sigma_{gap} = \sigma + \sigma_{BFKL} - \sigma_S$
Amplitude matching: $\sigma_{gap} = \frac{1}{4}(N_c^2-1)|\mathscr{A}_8(Q_0)|^2 + N_c^2|\mathscr{A}_1(Q_0)+\delta|^2$

In the first scheme we have replaced all expressions in (4) with the (finite) all-orders results and exploited the fact that $\mathscr{A}_1(0)$ is zero. In all three cases we subtract from the sum of the LLQ_0 and $BFKL$ amplitudes (cross sections) the double-counted term $\mathscr{A}_{1,S}(0)$ (σ_s). In all schemes $\sigma_{gap} \to \sigma$ for $Y \to 0$ since $\delta \to 0$ ($\sigma_{BFKL} - \sigma_S \to 0$), see (7). As $Y \to \infty$ we have $\sigma, \sigma_S \to 0$ and $\mathscr{A}_{1,8}(Q_0), \mathscr{A}_{1,S}(0) \to 0$ (i.e. $\delta \to \mathscr{A}_{BFKL}$) and hence $\sigma_{gap} \to \sigma_{BFKL}$. Each scheme therefore achieves our goal of having a smooth matching of the two all-orders cross sections, in that for small and large Y it agrees with the LLQ_0 and BFKL cross sections respectively avoiding any double-counting.

As a measure of the uncertainty inherent in the matching procedure fig. 1 shows numerical results of all three schemes. Indeed, they all match the two cross sections in the small and large Y limits and the differences are not large in between.

CONCLUSION

Working in the high energy limit we have calculated the (partonic) cross section for the production of two jets distant in rapidity and with limited transverse energy flow into the

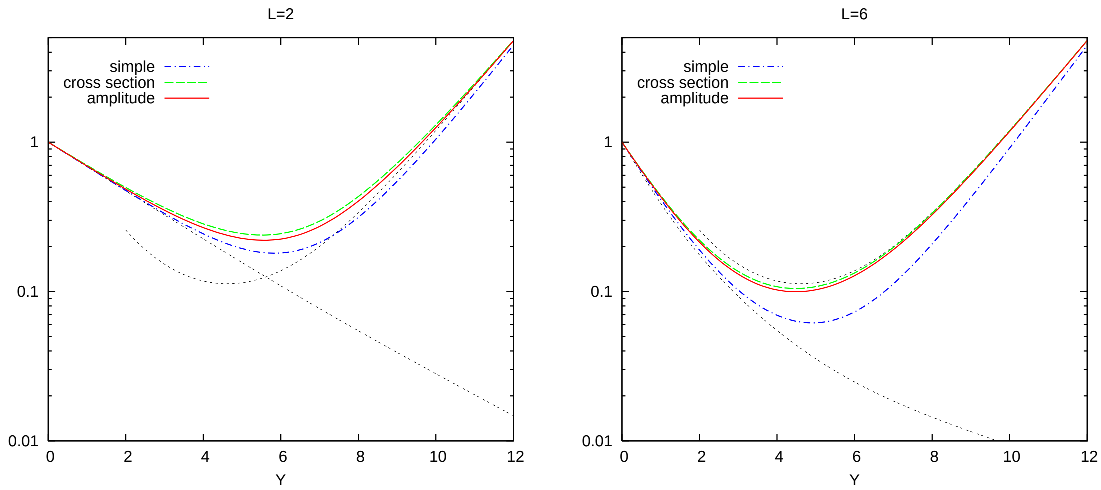

FIGURE 1. The gap cross section in the three matching schemes for $L = 2$ and 6 ($\alpha_s = 0.2$) compared to σ_{BFKL} (dots) and σ (double-dots)

region between the jets. Besides the *DLL* terms, we have summed terms sub-leading in Y stemming from the imaginary parts of the loop integrals. This allowed us to consistently combine the terms of the LLQ_0 series and the BFKL series to $\mathscr{O}(\alpha_s^5)$ accuracy without double counting. In the LLQ_0A, the inclusion of higher orders of the BFKL cross section in this way is not possible since it implies a divergent cross section.

We have also studied several "all order" matching schemes that effectively interpolate between the LLQ_0 and BFKL results. Although they all yield similar results, the differences between them cannot be resolved without further work, specifically understanding the role of real-emission contributions in the high energy limit. We have made a first step towards the unification of the two main approaches to the "jet–gap–jet" process.

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