Polarized Structure Functions from Lattice QCD

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Abstract. In this talk I present recent lattice results on polarized structure functions obtained by the QCDSF Collaboration.

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INTRODUCTION

Structure functions probe how hadrons are made up from quarks and gluons. The basis for theoretical investigation is the operator product expansion (OPE), which connects moments of structure functions with hadronic matrix elements of local operators. A complete theoretical understanding of the underlying dynamics of quarks and gluons thus requires the calculation of an appropriate set of matrix elements in QCD. This is a nonperturbative problem, and lattice QCD holds the tools to solve it [1].

Polarized structure functions are of particular interest, ever since it was discovered that only a small fraction of the spin of the nucleon is carried by the spin of the quarks. They contain a wealth of information on the distribution of spin and transversity in the fast moving nucleon, and their derivation provides a challenge, both experimentally and theoretically.

In this talk I will concentrate on the axial and tensor charge of the nucleon, on the nucleon's second spin dependent structure function g_2, as well as on the orbital angular momentum of the quarks.

The lattice simulations are done with $N_f = 2$ flavors of light dynamical quarks. To reduce cut-off effects, we use non-perturbatively $O(a)$ improved Wilson fermions. We work on lattices as large as $24^3\,48$ and lattice spacings as small as 0.07 fm. The operators are renormalized non-perturbatively as well throughout this talk.

AXIAL AND TENSOR CHARGE

The nucleon's tensor charge g_T measures the net number of transversely polarized valence quarks in the transversely polarized nucleon, while the axial charge g_A measures the number of longitudinally polarized valence quarks in the longitudinally polarized nucleon. One could argue that the two charges should be the same by rotational invariance. This would be the case if the nucleon was made of free quarks. However, in the

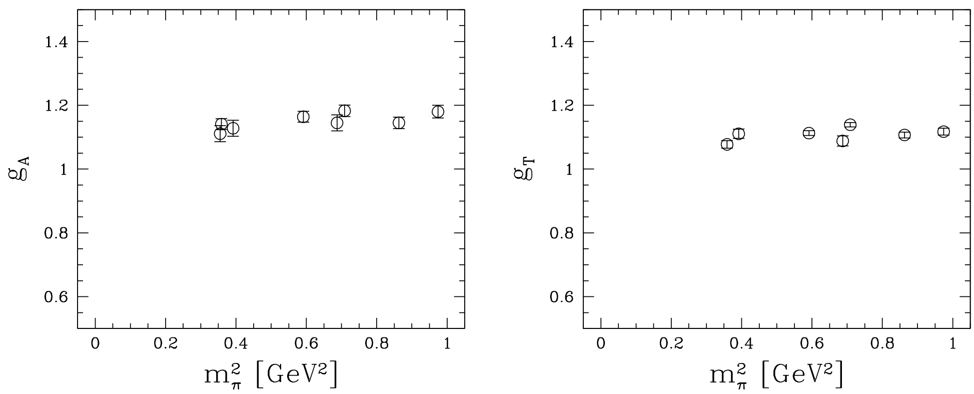

FIGURE 1. The axial and tensor charge of the nucleon.

infinite momentum frame rotational invariance is highly nontrivial and the rotation operators involve interactions. Thus, the difference of axial and tensor charges tells us about the interactions of quarks in the fast moving nucleon. In Fig. 1 I plot the axial and tensor charge of the nucleon as a function of the pion mass squared. The tensor charge refers to the $\overline{MS}$ scheme at 4 GeV^2. We find little difference between g_A and g_T.

$G_2(X,Q^2)$ AND HIGHER TWIST

The nucleon's second spin dependent structure function $g_2(x,Q^2)$ is of considerable phenomenological interest because at leading order in Q^2 it receives contributions from both twist-2 and twist-3 operators. Here we shall be interested in the second moment of g_2 only, and in particular in its twist-3 contribution d_2:

$$d_2 = \int_0^1 dx x^2 g_2(x,Q^2) + \frac{2}{3}\int_0^1 dx x^2 g_1(x,Q^2). \tag{1}$$

In Fig. 2 I plot d_2 as a function of the pion mass squared for proton and neutron target. The lattice data involve different lattice spacings. For an analysis lattice spacing by spacing and an attempt of a continuum extrapolation see [2]. In the chiral limit d_2 turns out to be consistent with zero, both for proton and nucleon. For the twist-3 contribution to the first moment we find $d_1^q = (2m_q/m_N)\delta q$ (m_q being the quark mass), which vanishes in the chiral limit as well. This suggest that [3]

$$g_2(x,Q^2) = \int_x^1 \frac{dy}{y} g_1(y,Q^2) - g_1(x,Q^2). \tag{2}$$

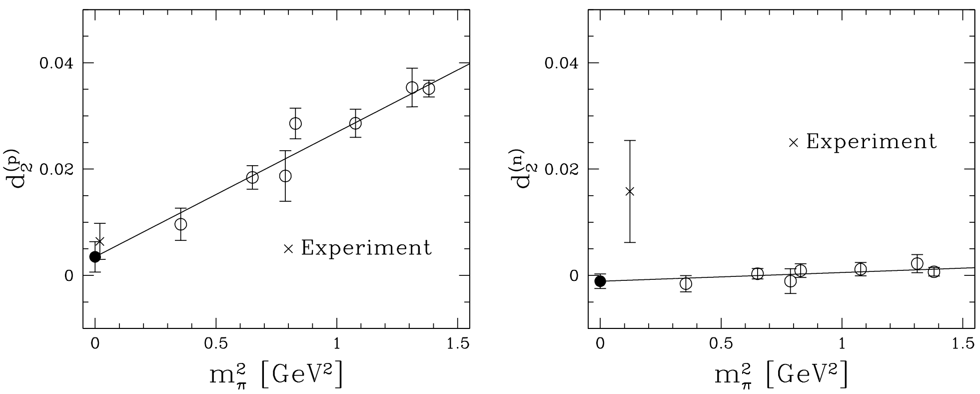

FIGURE 2. The twist-3 contribution to the second moment of g_2 in the $\overline{MS}$ scheme at 5 GeV^2.

ORBITAL ANGULAR MOMENTUM

The spin of the nucleon decomposes into the following contributions:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g \,, \tag{3}$$

where $\Delta\Sigma$ (ΔG) is the quark (gluon) spin contribution and L_q (L_g) the contribution of the orbital angular momentum of the quarks (gluon). The angular momentum $J^q = L^q + \Delta q/2$, $\sum_q \Delta q = \Delta\Sigma$, can be computed from the nucleon matrix of the energy-momentum tensor:

$$\begin{aligned}
\frac{\mathrm{i}}{2}\langle p'|\bar{q}\gamma_{\{\mu}\overset{\leftrightarrow}{D}_{\nu\}}q|p\rangle &= A_2^q(\Delta^2)\,\bar{u}(p')\gamma_{\{\mu}\bar{p}_{\nu\}}u(p) - B_2^q(\Delta^2)\,\frac{\mathrm{i}}{2m_N}\bar{u}(p')\Delta^\alpha\sigma_{\alpha\{\mu}\bar{p}_{\nu\}}u(p) \\
&+ C_2^q(\Delta^2)\,\frac{1}{m_N}\bar{u}(p')u(p)\Delta_{\{\mu}\Delta_{\nu\}}\,, \qquad (4) \\
J^q &= \frac{1}{2}\left(A_2^q(0) + B_2^q(0)\right)\,, \qquad (5)
\end{aligned}$$

where $\bar{p} = (p+p')/2$ and $\Delta = p' - p$. In Fig. 3 I show the generalized form factors (GFFs) A_2, B_2 and C_2 together with a dipole fit, and in Fig. 4 I show the GFFs extrapolated to $\Delta^2 = 0$, from which we can read off the total angular momentum J. All numbers given refer to valence quarks. If we subtract the contribution of Δq, which is known from an independent calculation [4], we obtain

$$L^{u+d} = 0.03(7) \quad L^{u-d} = -0.45(6)\,. \tag{6}$$

While the total contribution of u and d quarks appears to be consistent with zero, this is not the case for the individual contributions.

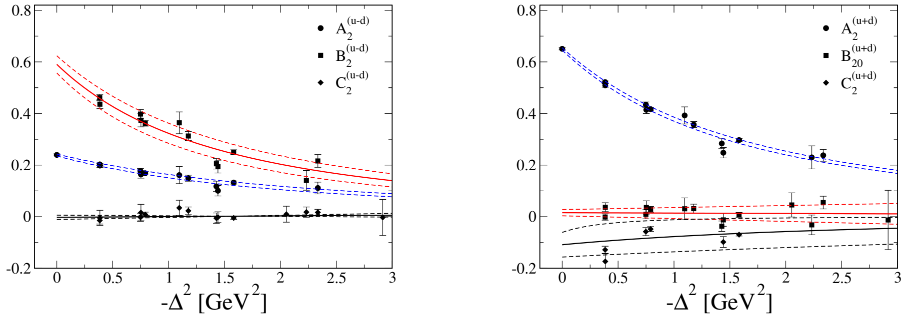

FIGURE 3. The generalized form factors A_2, B_2 and C_2 on the $24^3 48$ lattice at $\beta = 5.4$, $\kappa = 0.135$.

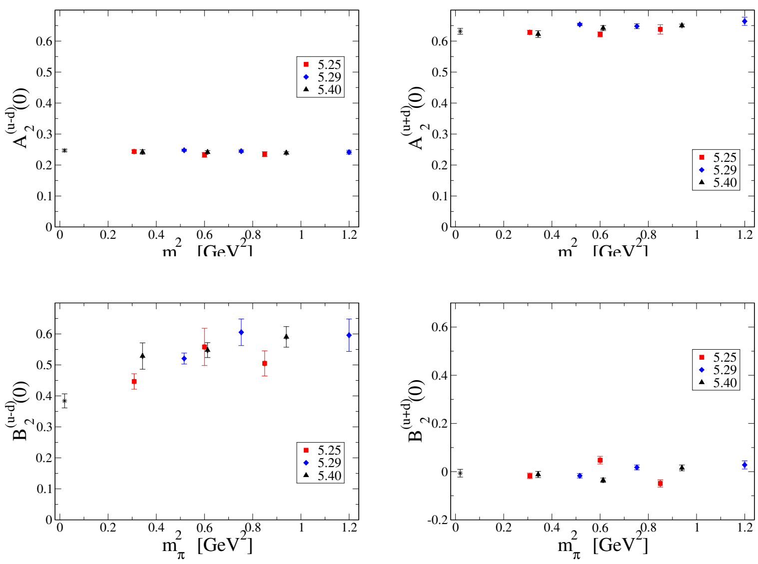

FIGURE 4. The generized form factors A_2 and B_2 extrapolated to $\Delta^2 = 0$.

SUMMARY

Due to space limitations I could show only a selection of results. For more details I refer the interested reader to recent talks and publications of the QCDSF collaboration.

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