Polarized Structure Functions from Lattice QCD

G. Schierholz

Deutsches Elektronen-Synchrotron DESY

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Special mention:

M. Göckeler, P. Hägler, R. Horsley, D. Pleiter, P. Rakow and J. Zanotti

Outline

Preliminaries

Axial and Tensor Charge $g_A\ ,\quad g_T$

The Structure Function $g_2(x, Q^2)$ and Higher Twist $d_1\ ,\quad d_2\ ,\quad \cdots$

(Orbital) Angular Momentum

Generalized Parton Distributions Spin & Transversity

Conclusions

Unpublished

Preliminaries

OPE

$$\langle p, s|\, \mathcal{O}^{q}_{\{\mu_1\cdots\mu_n\}}\, |p, s\rangle = v^q_n\, \bar{u}(p, s)\, \big(\gamma_{\mu_1} p_{\mu_2} \cdot\cdots\cdot p_{\mu_n}\big)\, u(p, s)$$

$$\langle p, s|\mathcal{O}^{5q}_{\{\mu\,\mu_1\cdots\mu_n\}}|p, s\rangle = a^q_n\, \bar{u}(p, s)\, \big(\gamma_{\{\mu}\gamma_5 p_{\mu_1}\cdots p_{\mu_n\}}\big)\, u(p, s)$$

$$\langle p, s|\mathcal{O}^{5q}_{[\mu\{\mu_1]\cdots\mu_n\}}|p, s\rangle = d^q_n\, \bar{u}(p, s)\, \big(\gamma_{[\mu}\gamma_5 p_{\{\mu_1]}\cdots p_{\mu_n\}}\big)\, u(p, s)$$

$$\langle p, s|\mathcal{O}^{Tq}_{\mu\nu\{\mu_1\cdot\cdot\mu_n\}}|p, s\rangle = t^q_n\, \bar{u}(p, s)\, \big(\sigma_{\mu\nu} p_{\{\mu_1}\cdot\cdots\cdot p_{\mu_n\}}\big)\, u(p, s)$$

$$\mathcal{O}^{q}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_{\mu_1} \overleftrightarrow{D}_{\mu_1}\cdots \overleftrightarrow{D}_{\mu_n} q$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\gamma_{\sigma}\gamma_5 \overleftrightarrow{D}_{\mu_1}\cdots \overleftrightarrow{D}_{\mu_n} q$$

$$\mathcal{O}^{Tq}_{\mu\nu\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\sigma_{\mu\nu} \overleftrightarrow{D}_{\mu_1}\cdots \overleftrightarrow{D}_{\mu_n} q$$

In particular

$$v_n^q(\mu) = \int_0^1 dx\, x^{n-1} q(x,\mu^2) = \langle x^{n-1}\rangle^q$$

$$a_n^q(\mu) = \int_0^1 dx\, x^n \Delta q(x,\mu^2) = \Delta^n q \qquad a_0^q = \Delta q, \qquad \boxed{g_A = \Delta u - \Delta d}$$

$$t_n^q(\mu) = \int_0^1 dx\, x^n \delta q(x,\mu^2) = \delta^n q \qquad t_0^q = \delta q, \qquad \boxed{g_T = \delta u - \delta d}$$

$$2\int_0^1 dx\, x^n g_1(x,Q^2) = e_{1,n}(Q^2/\mu^2, g(\mu^2))\, a_n(\mu)$$

$$2\int_0^1 dx\, x^n g_2(x,Q^2) = \frac{n}{n+1}\left[e_{2,n}(Q^2/\mu^2, g(\mu^2))\, d_n(\mu) - e_{1,n}(Q^2/\mu^2, g(\mu^2))\, a_n(\mu)\right]$$

$\uparrow$

Twist-3

No parton model interpretation

Off forward

$$\langle p_1, s|\, \mathcal{O}^q_{\{\mu_1\cdots\mu_n\}}\, |p_2, s\rangle = \bar{u}(p_1, s)\left[A^q_n(\Delta^2)\gamma_{\{\mu_1} + \frac{i\Delta^\alpha}{2m_N} B^q_n(\Delta^2)\sigma_{\alpha\{\mu_1}\right]\bar{p}_{\mu_2}\cdots\bar{p}_{\mu_n\}}\, u(p_2, s) + \cdots$$

$$\langle p_1, s|\mathcal{O}^{5q}_{\{\mu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\left[\tilde{A}^q_{n+1}(\Delta^2)\gamma_{\{\mu}\gamma_5\bar{p}_{\mu_1}\cdots\bar{p}_{\mu_n\}}\right]u(p_2, s) + \cdots$$

$$\langle p_1, s|\mathcal{O}^{Tq}_{\{\mu\nu\mu_1\cdot\cdot\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\left[A^{Tq}_{n+1}(\Delta^2)\,\sigma_{\mu\nu}\,\bar{p}_{\{\mu_1}\cdots\bar{p}_{\mu_n\}}\right]u(p_2, s) + \cdots$$

$$A^q_n(\Delta^2) = \int_0^1 dx\, x^{n-1} H^q(x, \Delta^2) \qquad H^q(x, 0) = q(x)$$

$$\tilde{A}^q_n(\Delta^2) = \int_0^1 dx\, x^{n-1} \tilde{H}^q(x, \Delta^2) \qquad \tilde{H}^q(x, 0) = \Delta q(x)$$

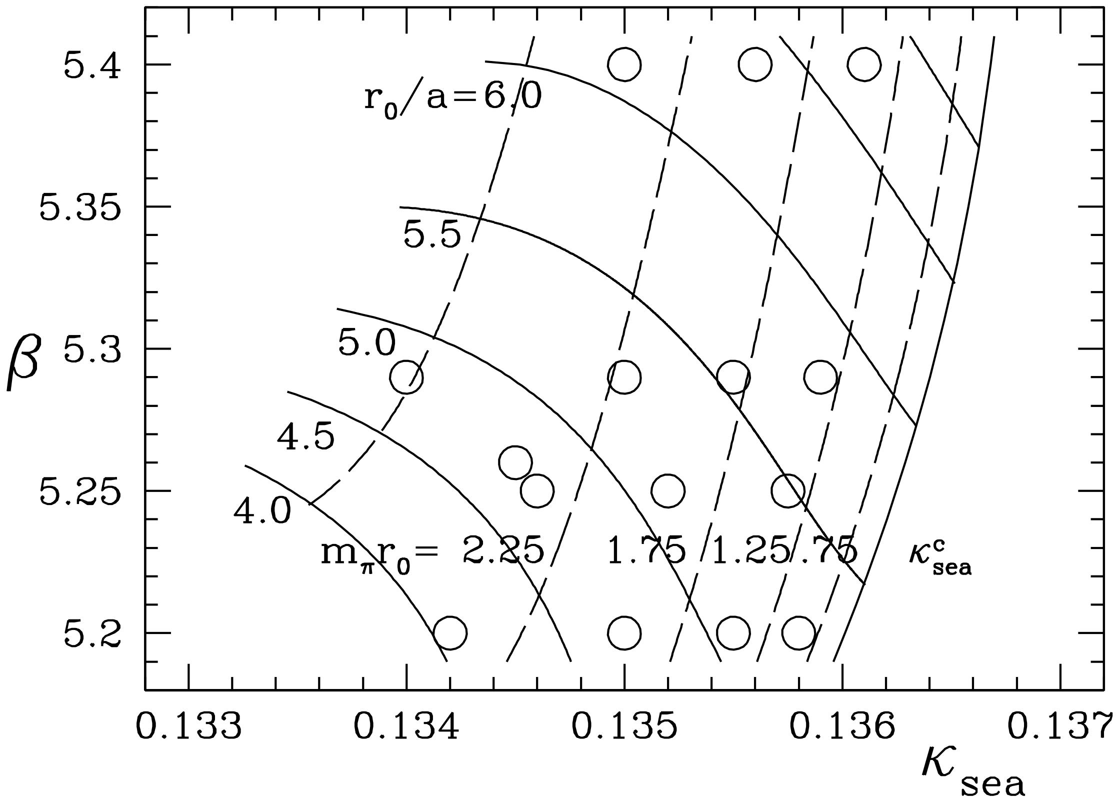
$$H^{Tq}_n(\Delta^2) = \int_0^1 dx\, x^{n-1} H^{Tq}(x, \Delta^2) \qquad H^{Tq}(x, 0) = \delta q(x)$$

↑ GFFs

↑ GPDs

$$\boxed{A^q_2(0) + B^q_2(0) = J^q}$$

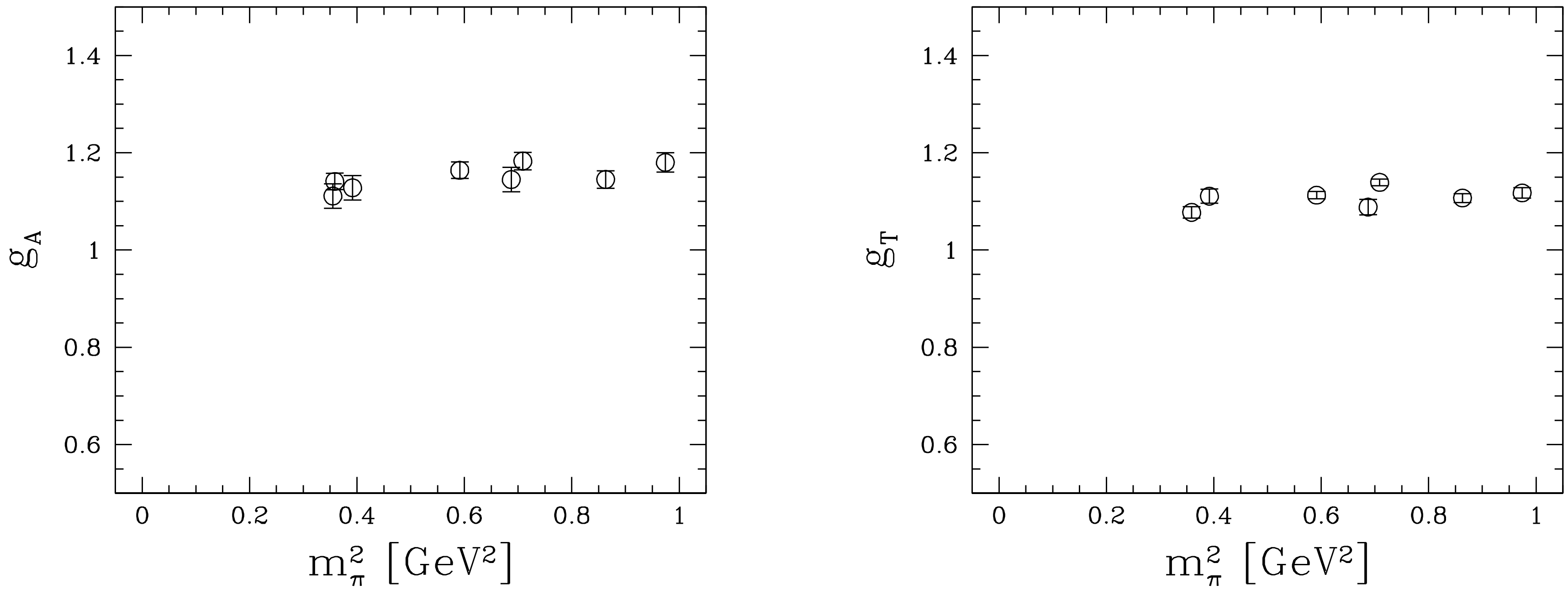
Dynamical Wilson Fermions $N_f = 2$ Nonperturbatively $O(a)$ improved



$0.07 \text{ fm} \lesssim a \lesssim 0.12 \text{ fm}$, $1 \text{ fm} \lesssim L \lesssim 2.2 \text{ fm}$

Axial and Tensor Charge

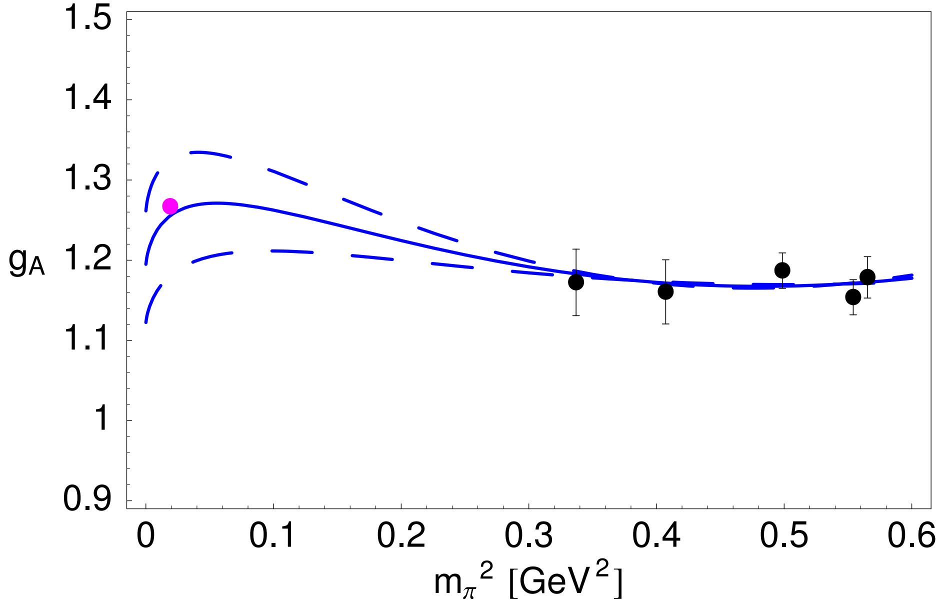
$\overline{MS}$, 4 GeV^2



$g_T \approx g_A$

Soffer bound $2g_T \leq 1 + g_A$ saturated

Chiral extrapolation



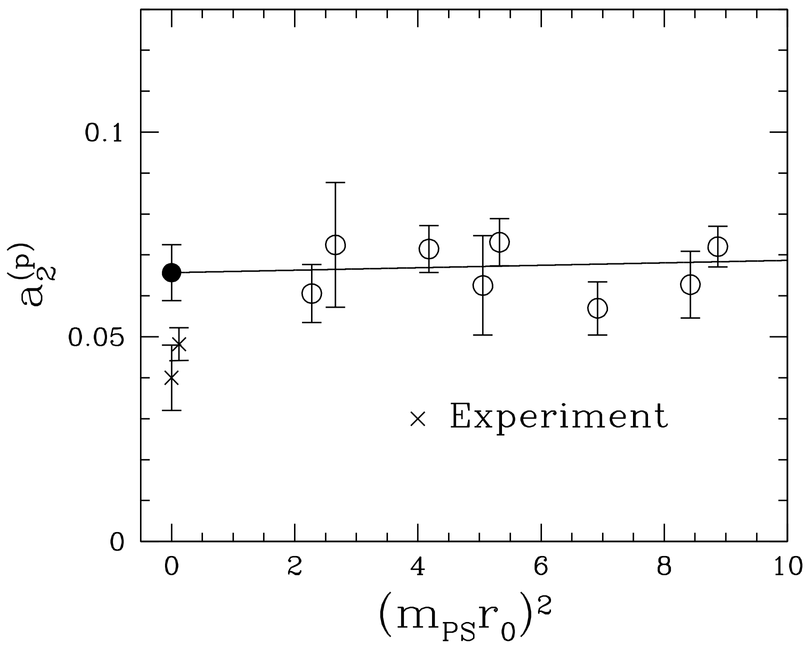
Hemmert, Procura & Weise

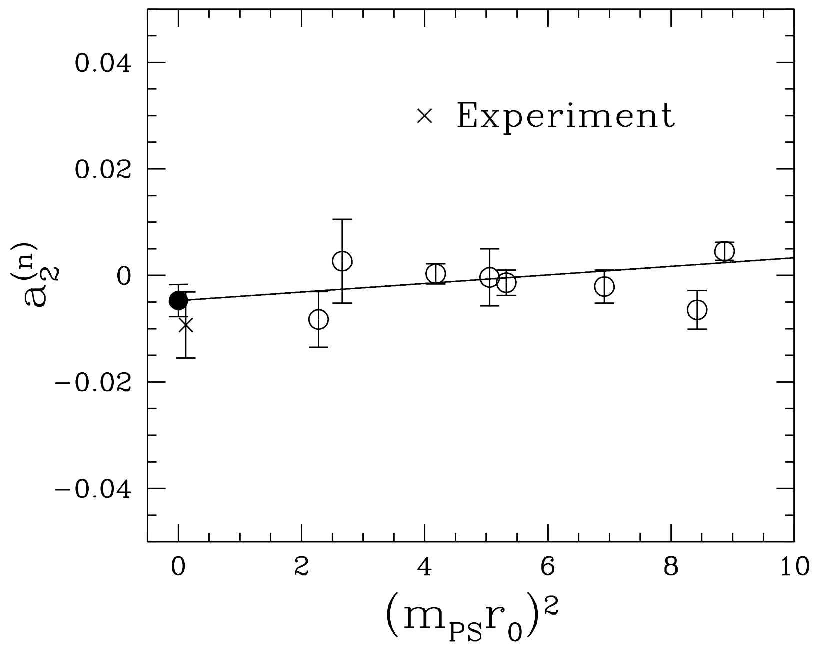
The Structure Function $g_2(x, Q^2)$ and Higher Twist

Second moment

$$\int_0^1 dx\, x^2\, g_1(x, Q^2) = \frac{1}{2}\left(1 + O(g^2)\right)\, a_2(Q^2)$$

Benchmark



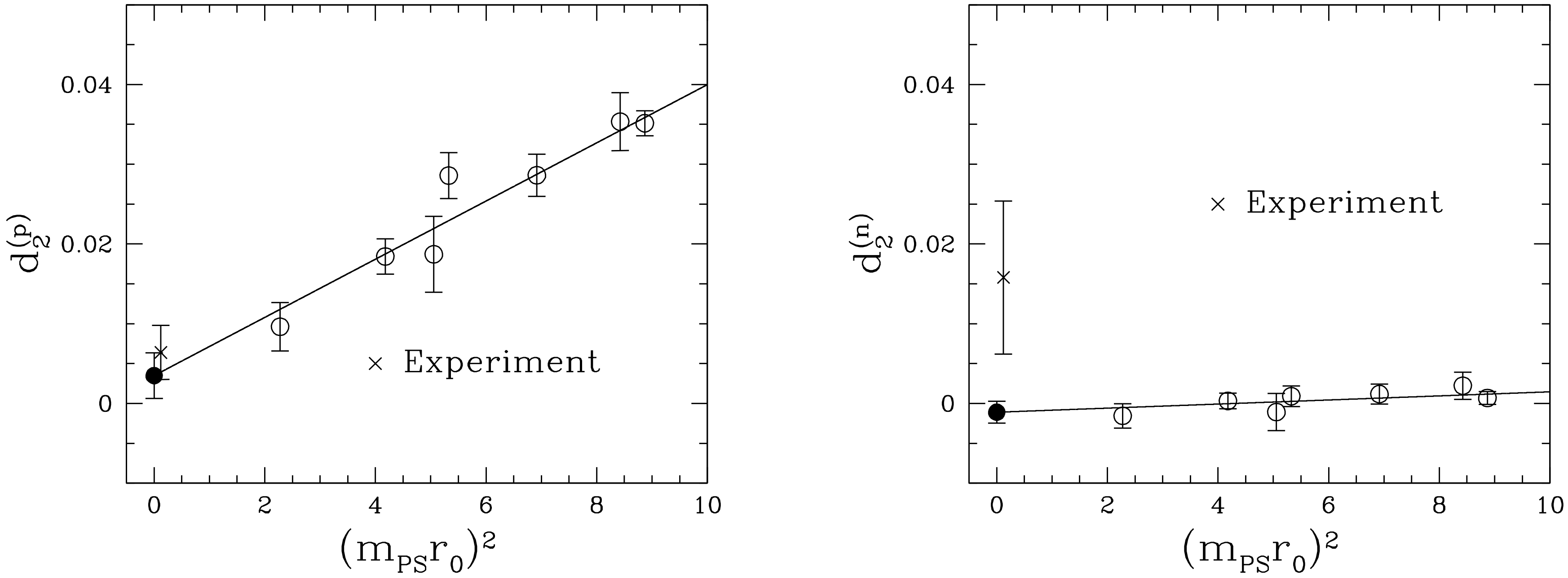


$\overline{MS}$, $Q^2 = 5$ GeV^2

$$\int_0^1 dx\, x^n\, g_2(x,Q^2) + \frac{n}{n+1}\int_0^1 dx\, x^n\, g_1(x,Q^2) = \frac{n}{2(n+1)} d_n \quad \approx \ 0$$

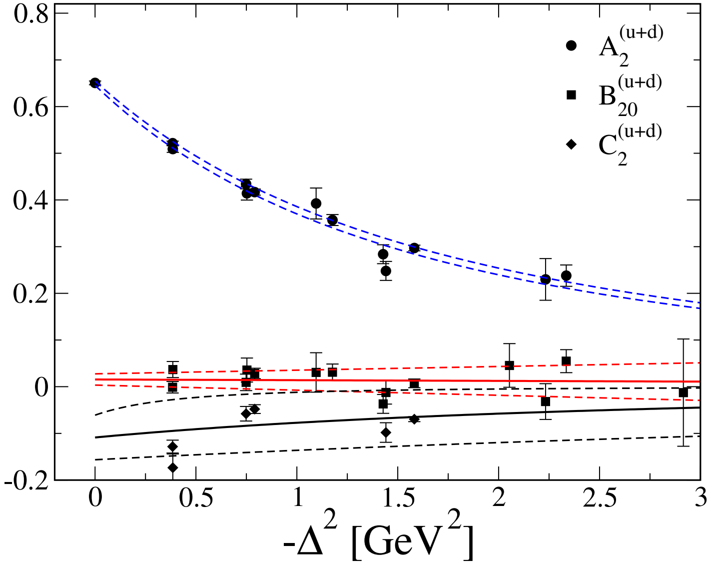
$n = 1$: $\quad d_1^q = \frac{2m_q}{m_N}\,\delta q \longrightarrow 0 \qquad \Uparrow$

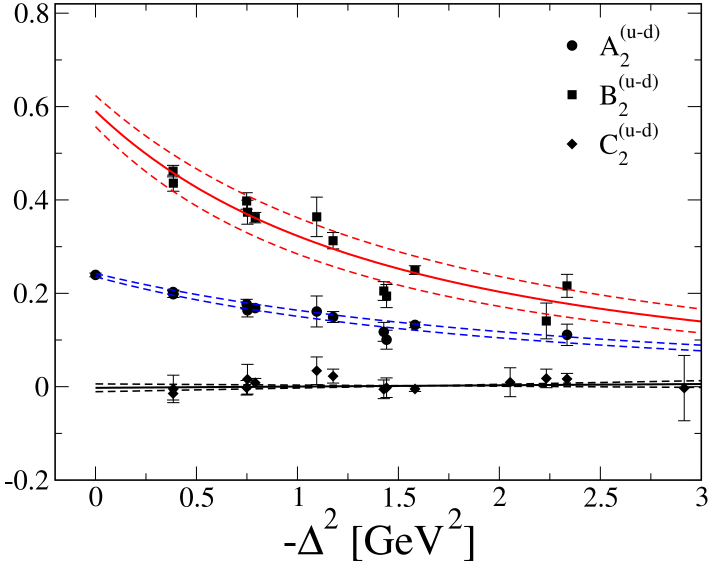
$n = 2$:



This suggests: $\quad g_2(x,Q^2) = \int_x^1 \frac{dy}{y}\, g_1(y,Q^2) - g_1(x,Q^2)$ Wandzura-Wilczek

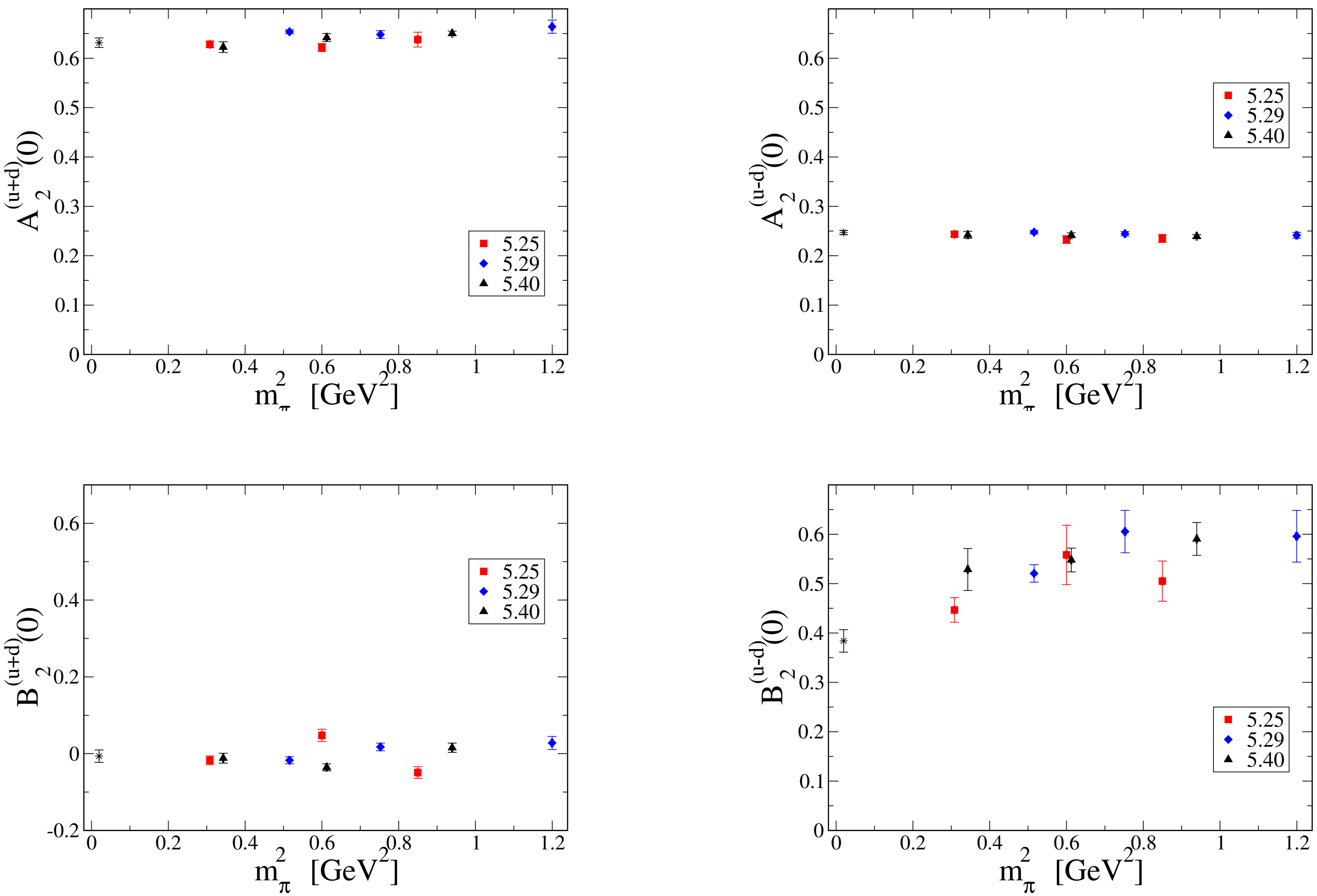
(Orbital) Angular Momentum





Dipole fit

$$A_2(\Delta^2) = \frac{A_2(0)}{(1 - \Delta^2/M_2^2)^2} \qquad B_2(\Delta^2) = \frac{B_2(0)}{(1 - \Delta^2/\hat{M}_2^2)^2}$$



$$J^{u+d} \approx \frac{1}{2} \langle x \rangle^{u+d}$$

$$J^{u-d} \approx \frac{5}{4} \langle x \rangle^{u-d}$$

Orbital angular momentum

$$J^q = L^q + S^q, \quad S^q = \Delta q$$

$$\boxed{L^{u+d} = 0.03(7)}$$

$\uparrow$

Valence quarks only

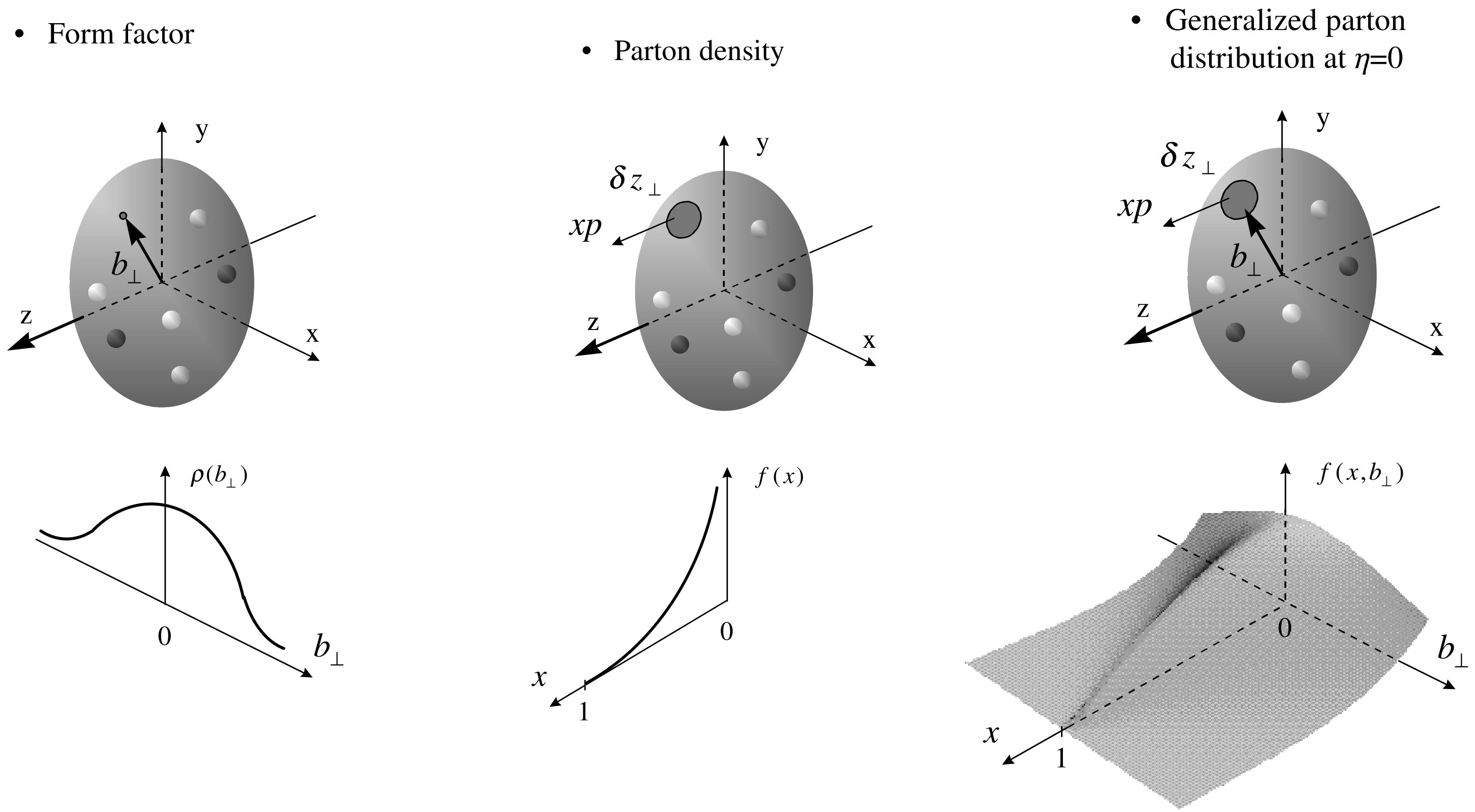
$$\boxed{L^{u-d} = -0.45(6)}$$

$\cdots$ but strong cancellations

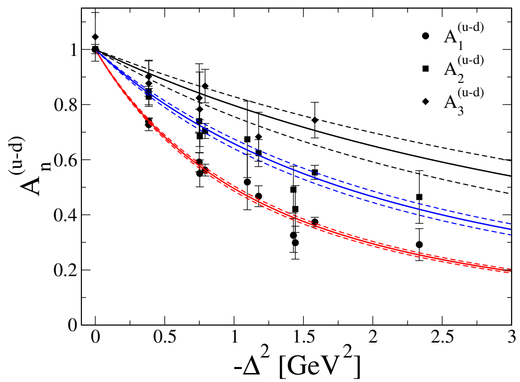
Generalized Parton Distributions

$$H(x, b^2) = \int \frac{d^2\Delta}{(2\pi)^2} e^{ib\Delta} H(x, \Delta^2)$$

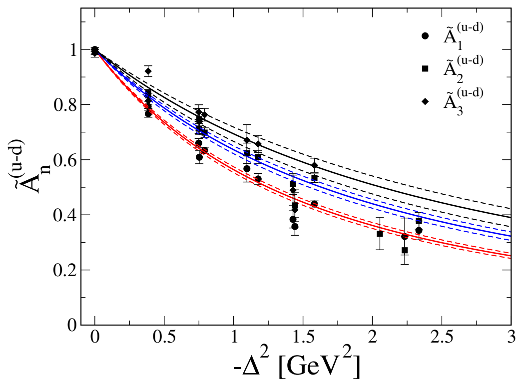
$\Rightarrow$ Probability interpretation



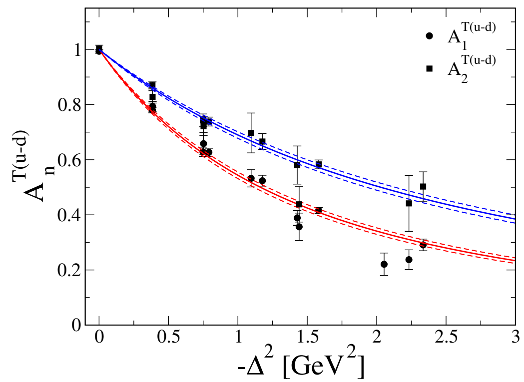
Spatial resolution: $\delta z_\perp \sim 1/Q$



Axial

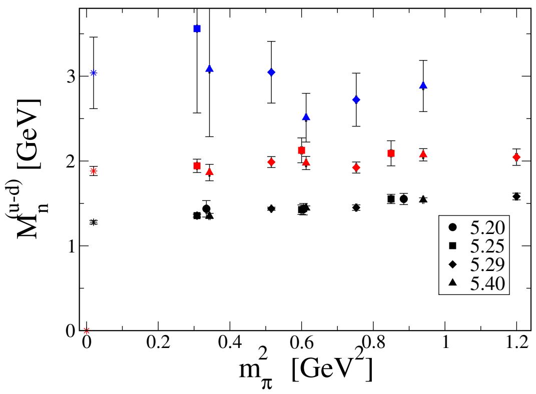


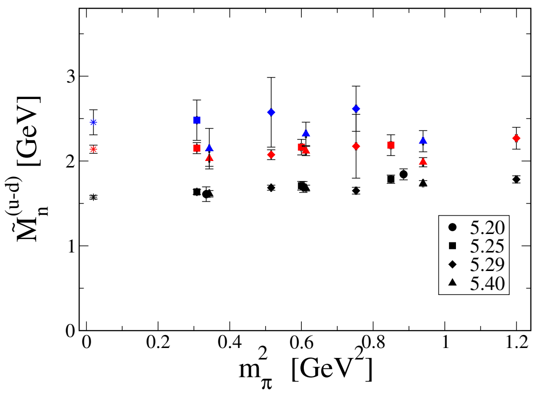
Tensor

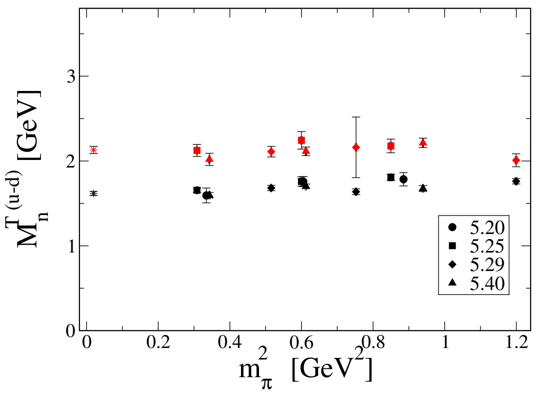


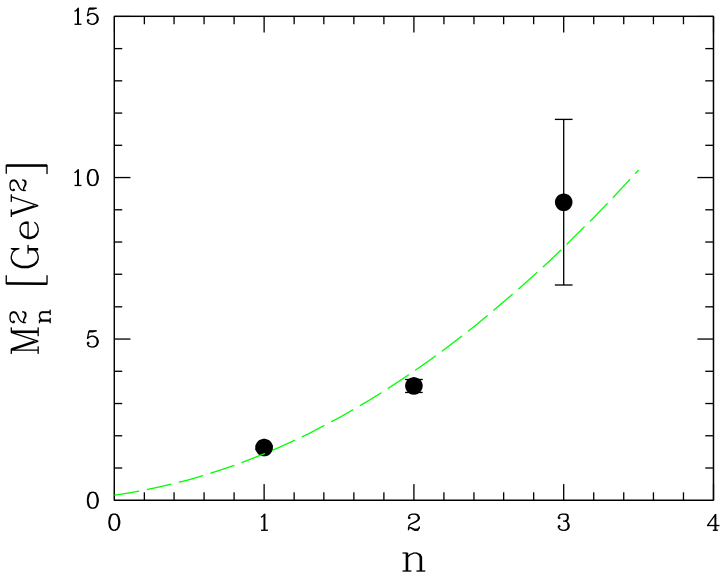
Dipole ansatz: $A_n(\Delta^2) = \dfrac{A_n(0)}{(1 - \Delta^2/M_n^2)^2}$

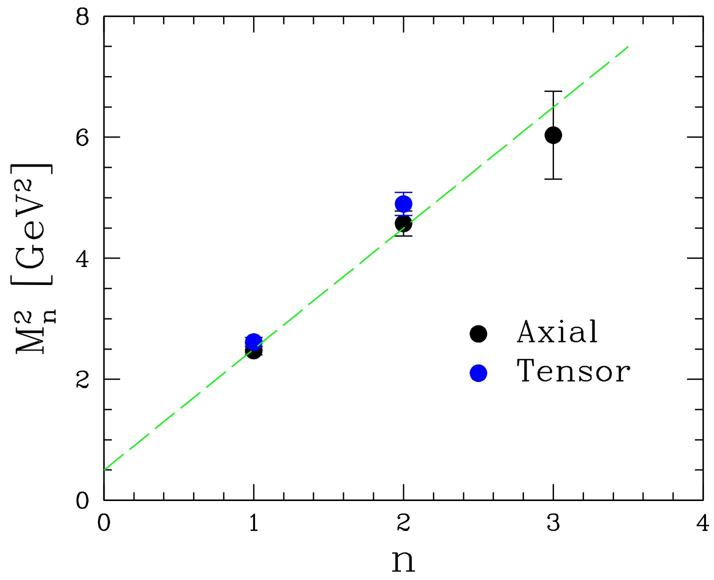
Chiral extrapolation











$$n = \alpha_0 + \alpha_1 \sqrt{M_n^2}$$

square root

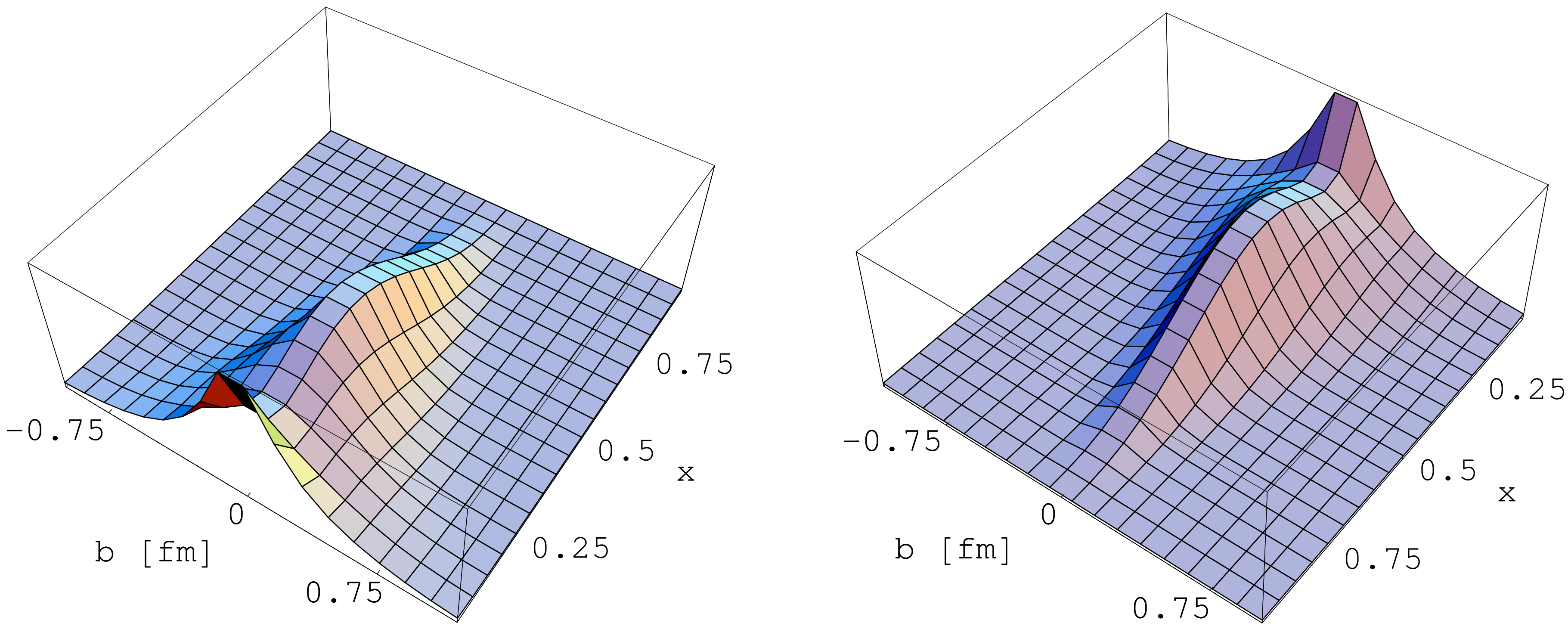
$$n = \alpha_0 + \alpha' \, M_n^2$$

linear trajectory

$$\tilde{H}^q(x,b^2) = \int_x^y \frac{dy}{y} C^q\left(\frac{x}{y}, b^2\right) \Delta q(x)$$

$$\int_0^1 dx\, x^n C^q(x,\Delta^2) = \frac{\tilde{A}^q_{n+1}(\Delta^2)}{\tilde{A}^q_{n+1}(0)}$$

Valence



$Q^2 = 4\ \text{GeV}^2$

Conclusions

- Spin and transversity distributions look very similar

  In particular: $g_A \approx g_T$

- $d_1 \approx d_2 \approx \cdot\cdot ? \cdot\cdot \approx 0$

  Higher twist contributions turn out to be surprisingly small

- $L^{u+d} \approx 0$

- 3-D Modelling of the nucleon in terms of quarks can be done on the lattice. First results on the charge, spin and transversity distribution look promising

  Waiting for experimental results

- To better constrain the generalized form factors need results for higher moments and on larger lattices

- To safely extrapolate to the chiral limit need to do simulations at $m_\pi \lesssim 300$ MeV

  Probably this can only be achieved with overlap fermions