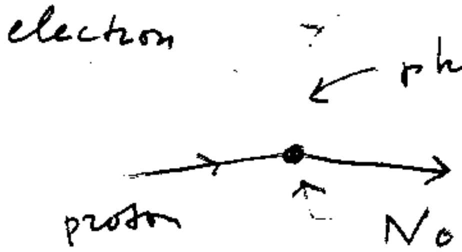
Boris Efremov 5/10/05

Electron - Proton Collisions

1. Low energy:

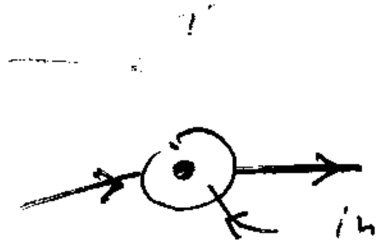


$\lambda \gtrsim r_p$

No structure of the proton

2. Larger energies:

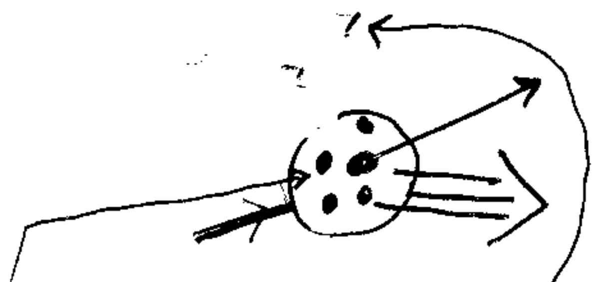
$\lambda < r_p$



interacts with constituents, probing the structure

3. High energies:

$\lambda \ll r_p$



The proton is exploded into fragments ≡ Deep Inelastic Scattering

Investigation of the proton structure

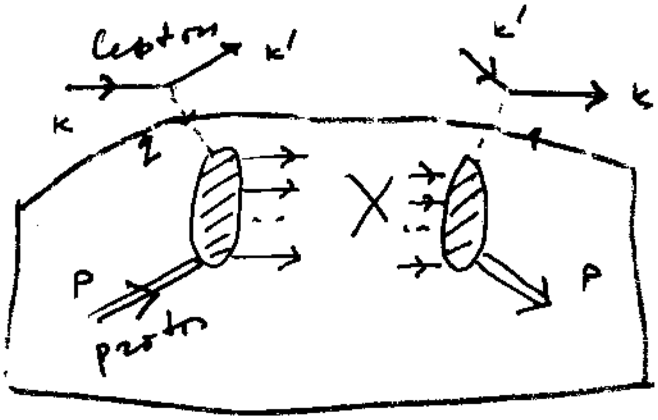
$\lambda \sim \frac{1}{E_{photon}}$

The only registered particle

... structure function g_1 at $x \ll 1$

Ermolaev, Greco, Troyan

Deep Inelastic Scattering:



$= W_{\mu\nu}(p,q)$ - hadronic tensor

Spin-dependent part, $W_{\mu\nu}^{spin} =$

$$i \varepsilon_{\mu\nu\lambda\rho} \frac{q_\lambda m}{pq} \left\{ S_\rho \underline{g_1} + \left(S_\rho - \frac{Sq}{pq} p_\rho\right) \underline{g_2} \right\} \approx$$

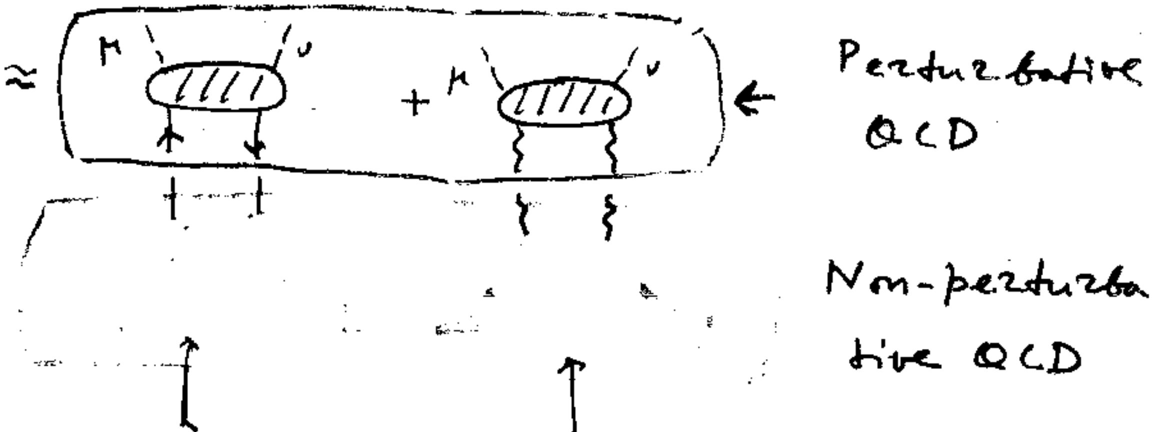
$$\approx i \varepsilon_{\mu\nu\lambda\rho} \frac{q_\lambda m}{pq} \left\{ S_\rho^{\parallel} \underline{g_1} + S_\rho^{\perp} \underline{(g_1 + g_2)} \right\}$$

$g_{1,2} = g_{1,2}(x, Q^2)$, with $x = Q^2/2pq$,

$Q^2 \equiv -q^2 > 0$, $0 < x < 1$

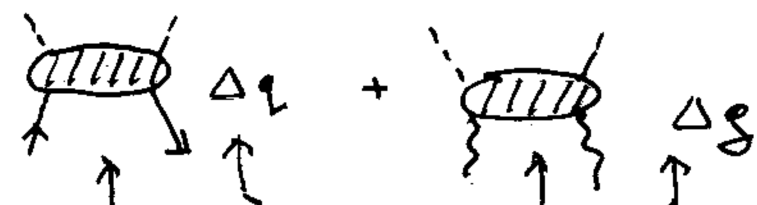
When $2Pq$, $Q^2 \gg$ mass scale, factorization:

$W_{\mu\nu} \approx$



Probabilities to find a quark/gluon in the hadron

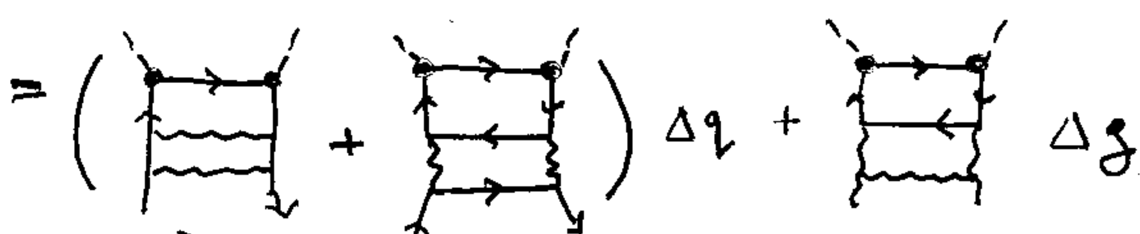
$\approx$



initial parton densities

QCD evolution of Δq, Δg

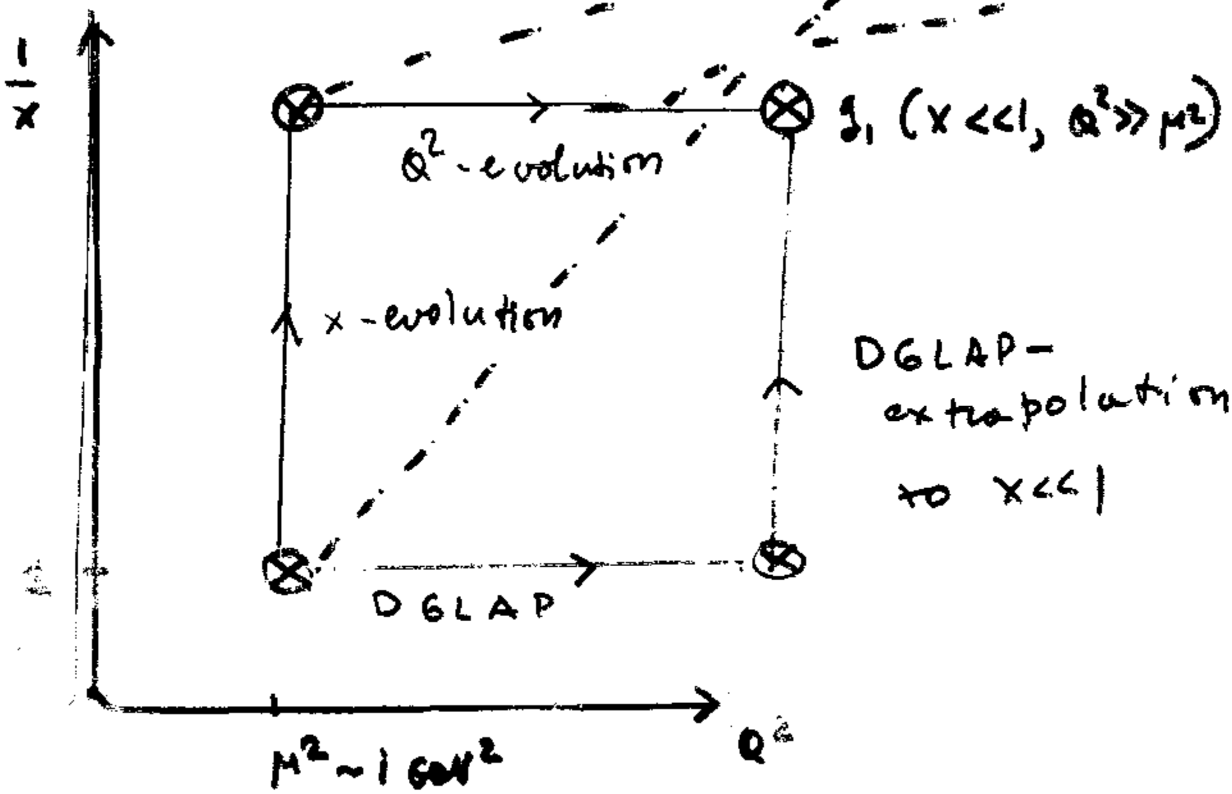
$=$



non-singlet

singlet

$$g_1 = \frac{e_q}{2} \int_{-i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right) C(\omega)\, \delta q(\omega)\, e \ \ldots \ \mu^2$$



Standard Theoretical Approach-DGLAP

$$C(\omega) = 1 + \alpha_s C^{(1)}(\omega) + \alpha_s^2 C^{(2)}(\omega) + \cdots$$

$$H(\omega) = \alpha_s \gamma^{(0)}(\omega) + \alpha_s^2 \gamma^{(1)}(\omega) + \cdots$$

Leading Order

NLO

NNext Leading Order

$\alpha_s \quad P_{ik}$ — [illegible]

$\alpha_s^2 \quad P^{(1)}_{ik}$ — Floratos, Ross, Sachrajda, Gonzalez-Arroyo, Lopez, Yndurain, Floratos, Kounnas, Lacaze, Curci, Furmanski, Petronzio, Zijlstra, Mertig, Van Neerven, Glück, Reya, Vogelsang

$\alpha_s \, C^{(1)}_{k}$ — Bardeen, Buras, Muta, Duke, Altarelli, Kodaira, Anselmino, Efremov, Leader

$\alpha_s^2 \quad C^{(2)}_{k}$ — Zijlstra, Van Neerven

$\alpha_s^3 \quad C^{(3)}_{k}$ — Larin, Tkachov, Vermaseren

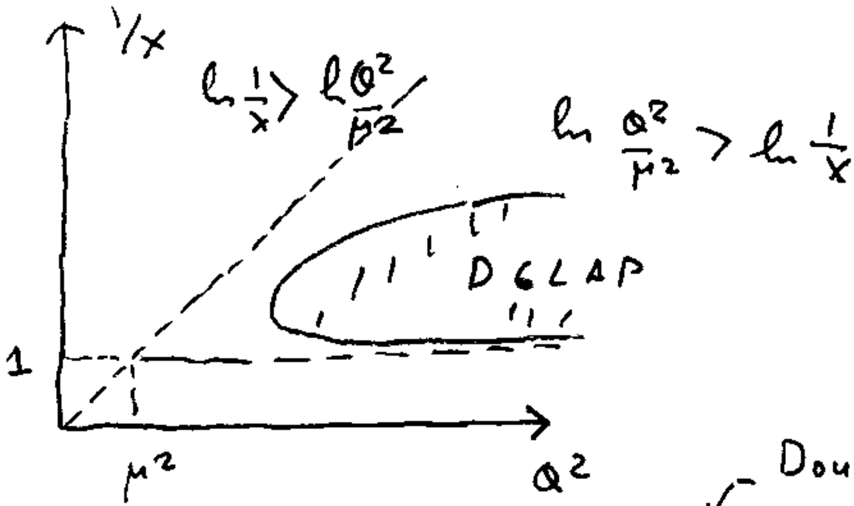
[illegible]

Fits: $\Delta g, \Delta q$

$$N_\eta \, x^{\alpha} (1 + \gamma x^{\delta})$$

$$N_\eta \left[\left(\ln \frac{1}{x}\right)^{\alpha} + \gamma \left(\ln \frac{1}{x}\right)^{\beta} \right]$$

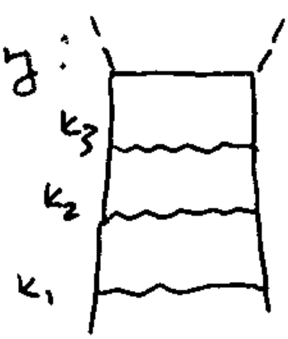
Parameters



Double-logarithmic terms

Contributions $\sim \left(\alpha_s \ln^2 \frac{1}{x}\right)^k$, $k > 2$ are neglected in DGLAP $\Rightarrow$ extrapolating DGLAP into $x \ll 1$ region looks important contributions

Neglecting $\left(\alpha_s \ln^2 \frac{1}{x}\right)^k$ is due to the ordering:



$$\mu^2 < \underbrace{k_{1\perp}^2 < k_{2\perp}^2 < k_{3\perp}^2}_{\text{good when } x \lesssim 1} < Q^2$$

When $x \ll 1$, $\mu^2 < k_{1\perp}^2, k_{2\perp}^2, k_{3\perp}^2 < s$

and no ordering

Being extrapolated into the region $x \ll 1$,

DGLAP predicts

$$g_1 \underset{x \ll 1}{\sim} e^{c\sqrt{\ln\frac{1}{x}\ln\frac{\ln(Q^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}}}$$

When double-logarithmic contributions are accounted for, the small-x asymptotics is different:

$$g_1 \sim \left(\frac{1}{x}\right)^{\Delta}\left(\frac{Q^2}{\mu^2}\right)^{\Delta/2}$$

(Δ ← intercept)

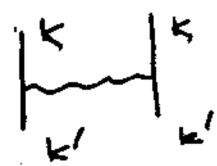
Bartels – Ermolaev – Manaenkov – Ryskin

$$\Delta_{NS} = \sqrt{\frac{2\alpha_s C_F}{\pi}} \qquad C_F = \frac{N^2-1}{2N} = \frac{4}{3}$$

$$\Delta_S = 3.5\sqrt{\frac{\alpha_s N}{2\pi}} \qquad N = 3$$

Here α_s is fixed at an unknown scale – double-logarithmic accuracy

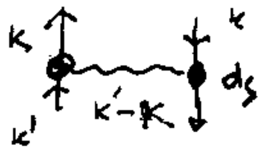
DGLAP: α_s is running:



$\alpha_s = \alpha_s(k_\perp^2)$. Theoretical basis of such a parametrization – Amati – Bassetto – Ciafaloni – Marchesini – Veneziano

However $\alpha_s = \alpha_s(k_\perp^2)$ is true only when $x \lesssim 1$.

When $x \ll 1$, $\alpha_s = \alpha_s\left((k-k')^2\right) \simeq$



$$\simeq \alpha_s\left(\frac{k_\perp^2 + k_\perp'^2}{\beta}\right)$$

β ← fraction of $k^\parallel$

When $x \sim 1$ $\begin{cases} \beta \sim 1 \Rightarrow \text{back to DGLAP} \\ k_\perp^2 > k_\perp'^2 \end{cases}$

Therefore, before extrapolating to $x \ll 1$ DGLAP should be improved:

1. Contributions $\sim \left(\alpha_s \ln^2 \frac{1}{x}\right)^n$; $\left(\alpha_s \ln \frac{1}{x}\right)^n$ should be accounted for to all orders in α_s
2. Argument $k_\perp^2$ of α_s should be changed
3. It should be a two-dimensional evolution equation respecting both Q^2- and x- evolutions

Factorization, $W_{\mu\nu} \approx$  is true when

$k_{i\perp} > \mu$
← a mass scale.

$\alpha_s = \frac{1}{b \ln \frac{k^2}{\Lambda^2_{QCD}}}$

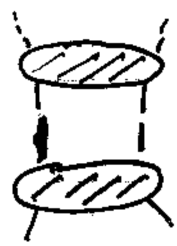
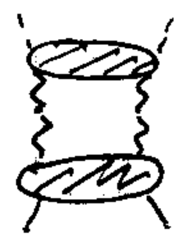
$\mu \gg \Lambda_{QCD}$, otherwise it is arbitrary.

$k_{i\perp}\ \mu \Rightarrow \mu$ is the Infra Red cut-off $\Rightarrow$ Infra Red Evolution Equations (IREE) instead of DGLAP.

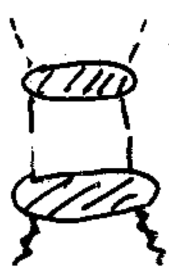
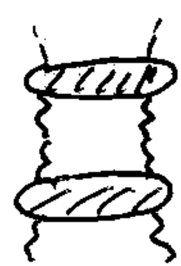
$g_1 =$  Δq +  Δg

g_q g_g.

IREE for g_q, g_g:

g_q  =  +  g_q +  g_g

Born H_{qq} H_{gq}

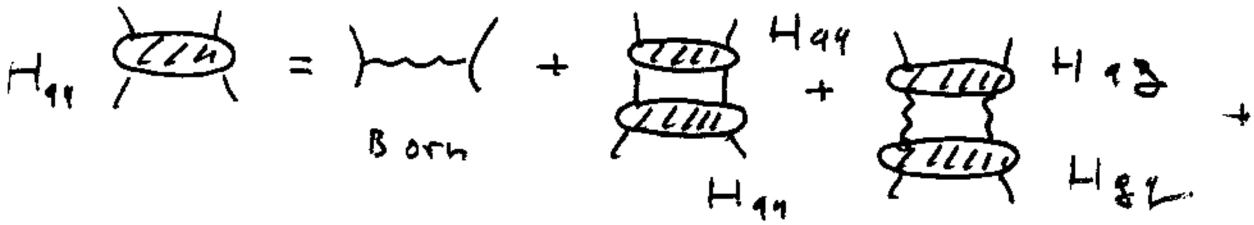
g_g  =  g_q +  g_g

H_{qg} H_{gg}

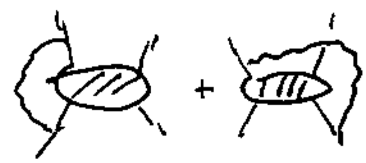
H_{qq}, H_{gq}, H_{qg}, H_{gg} – new anomalous dimensions

They account for DL of x to all orders.

IREE for H_{ik} looks similar. Indeed:



Difference to DGLAP:



LO DGLAP

⇓

$\gamma_{qq} \sim \alpha_s$

NLO DGLAP

⇓

$\gamma_{qq} \sim \alpha_s^2$

Solutions: after the Mellin transform

$$g_1^{NS} = \frac{e_q^2}{2}\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega} \frac{\omega\,\Delta q(\omega)}{\omega - H^{NS}}\left(\frac{Q^2}{\mu^2}\right)^{H^{NS}}$$

with NON-SINGLET $H_{NS} = \frac{1}{2}\left(\omega - \sqrt{\omega^2 - 4b_{qq}}\right)$,

$$b_{qq} = a_{qq} + V_{qq},$$

$$a_{qq} = \frac{A(\omega)\,C_F}{2\pi}, \quad V_{qq} = \frac{m_{qq}}{\pi^2} D(\omega), \quad m_{qq} = \frac{C_F}{2N}$$

$$A(\omega) = \frac{1}{b}\left[\frac{\eta}{\eta^2 + \pi^2} - \int_0^\infty \frac{d\rho\, e^{-\omega\rho}}{(\rho+\eta)^2 + \pi^2}\right]$$

$$D(\omega) = \frac{1}{2b^2}\int_0^\infty d\rho\, e^{-\omega\rho} \ln\left(\frac{\rho+\eta}{\eta}\right)\left[\frac{\rho+\eta}{(\rho+\eta)^2+\pi^2} + \frac{1}{\rho+\eta}\right]$$

$$b = \frac{33 - 2n_f}{12\pi}, \quad \eta = \ln\left(\mu^2/\Lambda^2_{QCD}\right)$$

Singlet g_1 is given by similar but more complicated expressions

When $x \to 0$, $g_1^{NS} \sim \frac{e_q^2}{2} \left(\frac{1}{x}\right)^{\Delta^{NS}} \left(\frac{Q^2}{\mu^2}\right)^{\Delta^{NS}/2} \Delta q(\omega)$

$$g_1^S \sim \frac{\langle e_q^2 \rangle}{2} \left[Z_1 \Delta q + Z_2 \Delta g \right] \left(\frac{1}{x}\right)^{\Delta_S} \left(\frac{Q^2}{\mu^2}\right)^{\Delta_S/2}$$

intercepts: non-singlet intercept

$\Delta^{NS} = 0.4$

singlet intercept

$\Delta^{S} = 0.86$

Factors $Z_{1,2}$: $Z_1 = -1.2$, $Z_2 = -0.08$

It is known that $\Delta q > 0 \Rightarrow g_1^{NS} > 0$

$g_1^S > 0$ if $15 \Delta q + \Delta g < 0$, otherwise it is negative.

$\Delta_{NS} = 0.4$ is confirmed by fitting experimental data

[illegible]
[illegible]
[illegible]
[illegible]
[illegible] Peshekhonov
Kwiecinski [illegible]
Ziaja

We predict Asymptotic Scaling when $x \to 0$, $Q^2 \to \infty$

DGLAP asymptotics

$$g_1 \sim e^{\sqrt{\ln\frac{1}{x}\ln\left(\frac{\ln Q^2/\Lambda^2}{\ln \mu^2/\Lambda^2}\right)}}$$

Our asymptotics

$$g_1 \sim \left(\frac{Q^2}{x^2\mu^2}\right)^{\Delta/2},$$

with $\Delta_{NS} = 0.42$,

$\Delta_S = 0.86$

Our prediction for the singlet g_1 intercept

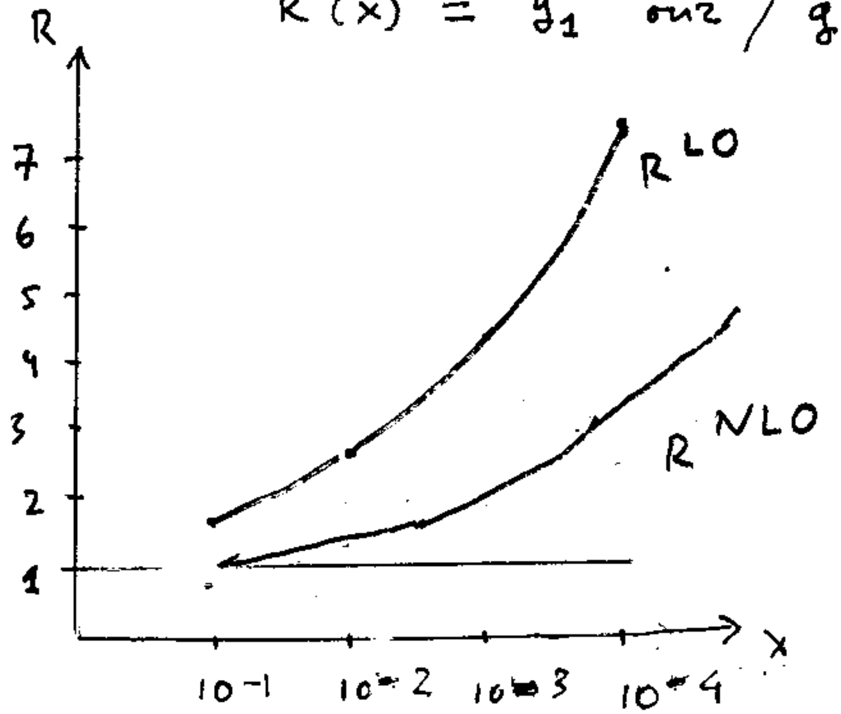
$$\Delta_S = 0.86$$

agrees perfectly with the fits to the HERMES data

$$\Delta_S = 0.88 \pm 0.14$$

Finite values of X

$$R(x) = g_1^{NS}{}_{\text{our}} \,/\, g_1^{NS}{}_{\text{DGLAP}} \quad \text{when } \Delta q = 1$$



$Q^2 = 20\ \text{GeV}^2$

DGLAP works reliably at the region

$$G = \{ x \gtrsim 0.05 \}$$

Standard DGLAP fits for $\delta q(x)$:

$$\delta q_{NS} = x^{\alpha}(1 + \gamma x^{\delta})(1-x)^{\beta}$$

$\beta = 2.7$, $\gamma = 3.4$, $\delta = 0.75$

$\underline{\alpha = -0.57}$, δq_{NS} singular when $x \to 0$

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega} C(\omega)\,\delta q(\omega)\, e^{H(\omega)\ln\frac{Q^2}{\mu^2}}$$

DGLAP	We
$C(\omega) \approx 1 - \frac{a}{\omega^2}$	$C(\omega) = \frac{\omega}{\omega + \sqrt{\underline{\omega^2 - B(\omega)}}}$
$\delta q(\omega) \simeq \frac{1}{\omega - \underline{0.57}}$	Branch point $\omega = 0.43$
from fitting exp data	as result of total resum

Hence, singular part of δq^{DGLAP} mimics our total resummation

DGLAP | OUR APPROACH

DGLAP

Exact two-loop contributions to C, H:

DL + SL + const

⇓

good at $x \lesssim 1$ and not accurate at $x \ll 1$

OUR APPROACH

Total resummation of DL + SL, no constants

⇓

good at $x \ll 1$ and not accurate at $x \lesssim 1$

⇓ ⇓

Synthesis of our approach and DGLAP

Expand our C, H into series:

$$H = \underbrace{\frac{A(\omega)C_F}{2\pi}\left(\frac{1}{\omega}+\frac{1}{2}\right) + \left(\frac{A C_F}{2\pi}\right)^2\left(\frac{1}{\omega}+\frac{1}{2}\right)^2}_{\equiv \tilde{H}} + O(A^3)$$

$$C = \underbrace{1 + \frac{A C_F}{2\pi}\left(\frac{1}{\omega^2}+\frac{1}{2\omega}\right)}_{\equiv \tilde{C}} + O(A^2).$$

$$H^{New} = H - \tilde{H} + H_{DGLAP}$$

$$C^{New} = C - \tilde{C} + C_{DGLAP}$$

} synthetic C, H.

Conclusion

1. Total resummation of double-logs of x and accounting for running α_s at the same time leads to expressions for g_1 singlet and non-singlet with coefficient functions and anomalous dimensions accounting for all singular ($\sim \alpha_s^n/\omega^{2n+1}$) terms.

2. Small-x asymptotics of these expressions is - contrary to DGLAP predictions - Regge-like

3. This result is confirmed by several independent groups that made fitting of experimental data.

4. The impact of the higher-loop corrections is becoming sizable at $x \lesssim 5 \cdot 10^{-2}$

5. DGLAP is successful at small x because the fits imitate the impact of higher loop contributions