The Effect of Positivity Constraints on Polarized Parton Densities

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Abstract. The impact of positivity constraints on the polarized parton densities has been studied. Special attention has been paid to the role of positivity constraints in determining the polarized strange quark and gluon densities, which are not well determined from the present data on inclusive polarized DIS.

Keywords: QCD, polarized parton densities, positivity constraints
PACS: 13.60.Hb, 12.38-t, 13.88.+e, 14.20.Dh

Spurred on by the famous European Muon Collaboration (EMC) experiment [1] at CERN in 1987, there has been a huge growth of interest in the partonic spin structure of the nucleon, *i.e.*, how the nucleon spin is built up out from the intrinsic spin and orbital angular momentum of its constituents, quarks and gluons. Our present knowledge about the spin structure of the nucleon comes from polarized inclusive and semi-inclusive DIS experiments at SLAC, CERN, DESY and JLab, polarized proton-proton collisions at RHIC and polarized photoproduction experiments. The determination of the longitudinal polarized parton densities in QCD is one of the important aspects of this knowledge. Many analyses [2, 3] of the world data on inclusive polarized DIS have been performed in order to extract them. It was shown that if the convention of a flavor symmetric sea is used[1] the polarized valence quarks are well determined, while the polarized strange sea and polarized gluon densities are weakly constrained[2].

In this talk we will discuss the effect of positivity constraints on the polarized parton densities and will demonstrate their importance in determining the strange and gluon densities, especially at high x.

The polarized parton densities have to satisfy the positivity condition, which in LO QCD implies:

$$|\Delta f_i(x,Q^2)| \leq f_i(x,Q^2), \quad |\Delta \bar{f}_i(x,Q^2)| \leq \bar{f}_i(x,Q^2). \tag{1}$$

The constraints (1) are the consequence of a probabilistic interpretation of the parton densities in the naive parton model, which is still valid in LO QCD. Beyond LO the parton densities are not physical quantities and the positivity constraints on the polarized parton densities are more complicated. They follow from the positivity condition for the

[1] In the absence of polarized charged current neutrino experiments a flavor decomposition is not possible.
[2] About the situation in semi-inclusive DIS see the talk by R. Sassot at this Workshop [4].

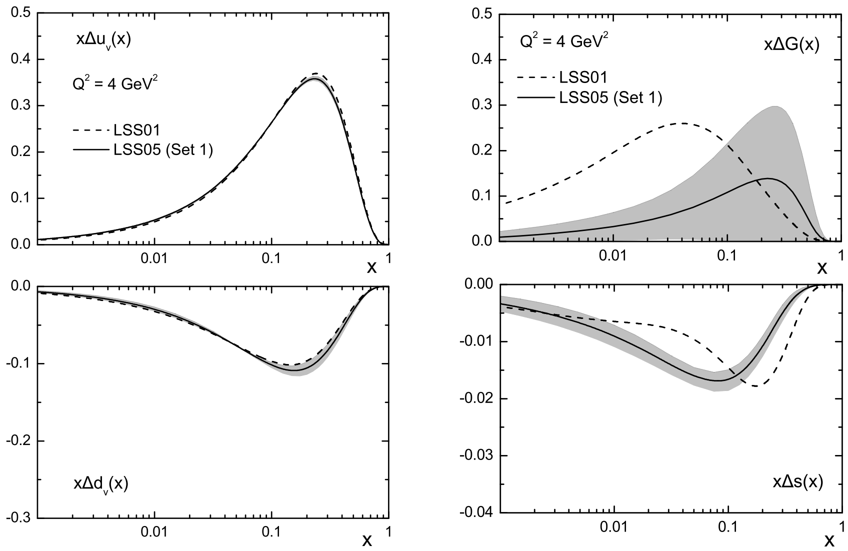

FIGURE 1. Comparison between our two sets of NLO($\overline{\text{MS}}$) polarized parton densities, LSS'01 and LSS'05(Set 1), at $Q^2 = 4\ GeV^2$.

polarized lepton-hadron cross-sections $\Delta\sigma_i$ in terms of the unpolarized ones ($|\Delta\sigma_i| \leq \sigma_i$) and include also the Wilson coefficient functions. It was shown [5], however, that for all practical purposes it is enough, at the present stage, to consider LO positivity bounds for LO as well as for for NLO parton densities, since NLO corrections are only relevant at the level of accuracy of a few percent. Note that, if the positivity constraints (1) are imposed at some Q_0^2, they are satisfied at any $Q^2 > Q_0^2$ [6]. So, in order to control easily the positivity conditions (1) it is enough to impose them for the minimum value of $Q^2 = Q_0^2$ in the data set used in the QCD analysis.

Let us consider how the use of different positivity constraints influences the results on the polarized parton densities. In Fig. 1 we compare LSS'05(Set 1) NLO($\overline{\text{MS}}$) polarized parton densities [3] with LSS'01 parton densities [7] presented on the HEPDATA web site. Both sets are determined from the data by the same method but using different positivity constraints. While the LSS'05 polarized PD are compatible with the positivity bounds (1) imposed by the MRST'02 unpolarized parton densities [8], those of the LSS'01 set are limited by the Barone et al. unpolarized parton densities [9]. As seen from Fig. 1, the valence quark densities Δu_v and Δd_v of the two sets are close to each other, while the polarized strange sea quark and gluon densities are significantly different. This comparison is a good illustration of the fact that the present inclusive polarized DIS data allow a much better determination of the valence quark densities (if SU(3) symmetry of the flavour decomposition of the sea is assumed) than the polarized strange quarks $\Delta s(x,Q^2)$ and the polarized gluons $\Delta G(x,Q^2)$. This is especially true for the high x region, where the values of $\Delta s(x,Q^2)$ and $\Delta G(x,Q^2)$ are very small and the precision of the data is not enough to extract them correctly. That is why different unpolarized sea quark and gluon densities (see Fig. 2) used on the RHS of the positivity constraints (1) are important and crucial in determining $\Delta s(x,Q^2)$ and $\Delta G(x,Q^2)$ in

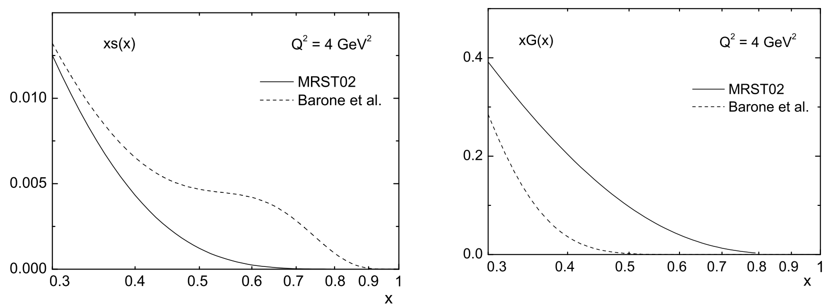

FIGURE 2. Comparison between the NLO($\overline{\mathrm{MS}}$) unpolarized strange quark sea and gluon densities determined by MRST'02 [8] and Barone at al. [9].

this region. The more restrictive $s(x,Q^2)_{\mathrm{MRST'02}}$ at high x leads to a smaller value of $|\Delta s(x,Q^2)|_{\mathrm{LSS'05}}$ in this region, while the smaller $G(x,Q^2)_{\mathrm{Bar.et.al}}$ provides a stronger constraint on $\Delta G(x,Q^2)_{\mathrm{LSS'01}}$ (see Fig. 1).

To illustrate this fact once more, we compare the LSS'05 (Set 1) PPD at $Q^2 = 4\ GeV^2$ with those [2] obtained by GRSV, Blumlein, Bottcher and the Asymmetry Analysis Collaboration (AAC) using almost the same set of data. Note that all these groups have used the GRV unpolarized parton densities [10] for constraining their polarized parton densities at large x. In this x region the unpolarized GRV and MRST'02 gluons are practically the same, while the magnitude of the unpolarized GRV strange sea quarks is much smaller than that of MRST'02. Therefore, the GRV unpolarized strange sea quarks provide a stronger constraint on the polarized ones (see Fig. 3). The impact on the determination of the polarized strange sea density is demonstrated in Fig. 3. As a result, the magnitude of our polarized strange sea density $x|\Delta s(x,Q^2)|$ is larger in the region $x > 0.1$ than those obtained by the other groups. Note also that the magnitude of $x\Delta s$ obtained by the GRSV and BB is smaller than that determined by AAC. We consider the GRSV result to be a consequence of the fact that in their analysis, the GRV positivity constraint is imposed at lower value of Q^2: $Q^2 = \mu^2_{NLO} = 0.4\ GeV^2$, while AAC has used the same requirement at $Q^2 = 1\ GeV^2$. Finally, the different positivity conditions on Δs influence also the determination of the polarized gluon density for larger Q^2 because the evolution in Q^2 mixes the polarized sea quarks and gluons.

To end this discussion, we would like to emphasize that for the adequate determination of polarized strange quarks and gluons at large x, the role of the corresponding unpolarized densities is very important. That is why the latter have to be determined with good accuracy at large x in the preasymptotic (Q^2, W^2) region too. Usually the sets of unpolarized parton densities, presented in the literature, are extracted from the data on DIS using cuts in Q^2 and W^2 chosen in order to minimize the higher twist effects. In order to use the densities for constraining the polarized parton densities they have to be continued to the preasymptotic (Q^2, W^2) region. It is not obvious that the continued unpolarized parton densities would coincide well with those obtained from the data in the region ($Q^2 > 1\ GeV^2$, $W^2 > 4\ GeV^2$) in the presence of the HT corrections to unpolarized structure functions F_1 and F_2. So, a QCD analysis of the unpolarized world data

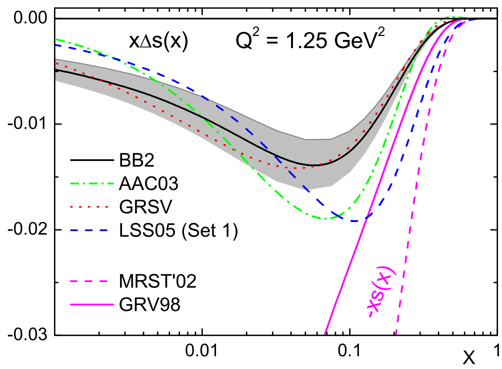

FIGURE 3. Comparison between our NLO($\overline{\rm MS}$) polarized strange sea quark density (Set 1) [3] at $Q^2 = 1.25\ GeV^2$ with those [2] obtained by GRSV ('standard scenario'), BB (ISET=4 or BB2) and AAC (AAC03). The unpolarized MRST02 and GRV98 strange sea quark densities are also shown.

including the preasymptotic $(Q^2,\ W^2)$ region and taking into account HT corrections is needed in order to extract correctly the unpolarized parton densities in the preasymptotic region. Our arguments for the need for a precise determination of the unpolarized densities of strange quarks and gluons in both the asymptotic and preasymptotic regions in Q^2 and W^2, coming from spin physics, could be considered as additional to those discussed in the recent paper [11].

ACKNOWLEDGMENTS

This research was supported by the JINR-Bulgaria Collaborative Grant, by the RFBR (No 05-01-00992, 03-02-16816), by the Bulgarian National Science Foundation under Contract Ph-1010 and by the Royal Society of Edinburgh Auber Bequest.

REFERENCES

1. EMC, J. Ashman et al., *Phys. Letters* **B 206**, 364 (1988) ; *Nucl. Phys.* **B 328**, 1 (1989).
2. M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, *Phys. Rev.* **D 63**, 094005 (2001); J. Blumlein, H. Bottcher, *Nucl. Phys.* **B 636**, 225 (2002); AAC, M. Hirai et al., *Phys. Rev.* **D 69**, 054021 (2004).
3. E. Leader, A.V. Sidorov and D.B. Stamenov, *J. High Energy Phys.* **06**, 033 (2005).
4. R. Sassot, this Workshop; D. de Florian, G.A. Navarro, and R. Sassot, *Phys. Rev.* **D 71**, 094018 (2005).
5. G. Altarelli, S. Forte, and G. Ridolfi, *Nucl. Phys.* **B 534**, 277 (1998); S. Forte, M. L. Mangano, and G. Ridolfi, *Nucl. Phys.* **B 602**, 585 (2001).
6. C. Bourrely, J. Soffer, and O.V. Teryaev, *Phys. Lett.* **B 420**, 375 (1998).
7. E. Leader, A.V. Sidorov, and D.B. Stamenov, *Eur. Phys. J.* **C 23**, 479 (2002).
8. A.D. Martin, R.G. Roberts, W.J. Stirling, and R.S. Thorne, *Eur. Phys. J.* **C 28**, 455 (2003).
9. V. Barone, C. Pascaud, and F. Zomer, *Eur. Phys. J.* **C 12**, 243 (2000).
10. M. Glück, E. Reya and A. Vogt, *Eur. Phys. J.* **C 5**, 461 (1998).
11. S. Forte, hep-ph/0502073.