QCD Factorization for Semi-inclusive Deep Inelastic Scattering

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Abstract. In this talk, we will present a QCD factorization theorem for the semi-inclusive deep-inelastic scattering with hadrons in the current fragmentation region detected at low transverse momentum.

Introduction

In recent years, there has been considerable experimental and theoretical interest in semi-inclusive hard processes. Rigorous theoretical studies in this direction started from the classical work on semi-inclusive processes in e^+e^- annihilation by Collins and Soper [1], where a QCD factorization was proved, and non-perturbative transverse-momentum-dependent (TMD) parton distributions and fragmentation functions were introduced [1, 2]. In the past few years, gauge properties of the TMD parton distributions have been investigated [3, 4, 5]. More recently, the factorization theorems for the semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan processes have been re-examined in the context of the gauge-invariant definitions [6, 7]. In this talk, I will present the theoretical results of [6].

The main result of [6] is a QCD factorization theorem for the SIDIS cross section at low transverse momentum, accurate up to the power corrections $(P_{h\perp}^2/Q^2)^n$ and to all orders in perturbation theory. For example, the leading spin-independent structure function for SIDIS can be factorized as follows,

$$\begin{aligned} F(x_B, z_h, P_{h\perp}, Q^2) \;=\; & \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp \\ & \times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_T\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho) \\ & \times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) \ , \end{aligned} \tag{1}$$

where μ is a renormalization (and collinear factorization) scale; ρ is a gluon rapidity cut-off parameter; *the μ and ρ dependence cancels among various factors.* In a special system of coordinates in which $x_B\zeta = \hat{\zeta}/z_h$, one has $\zeta^2 x_B^2 = \hat{\zeta}^2/z_h^2 = Q^2\rho$. The physical interpretation of the factors are as follows: q is TMD quark distribution function; $\hat{q}$ is the TMD quark fragmentation function depending on; H represents the contribution of parton hard scattering and is a perturbation series in α_s; and, finally, the soft factor S

comes from soft gluon radiations and is defined by a matrix element of Wilson lines in QCD vacuum.

The Transverse Momentum Dependent Parton Distributions

Consider a hadron, a nucleon for example, with four-momentum P. Let $(xP^+,\vec{k}_\perp)$ represent the momentum of a parton (quark or gluon) in the hadron. In a non-singular gauge (e.g. Feynman gauge), the TMD parton distributions can be defined through the following density matrix [1, 4],

$$\begin{aligned} \mathscr{M}^{\pm}(x,k_\perp,\mu,x\zeta,\rho) &= p^+\int\frac{d\xi^-}{2\pi}e^{-ix\xi^-P^+}\int\frac{d^2\vec{b}_\perp}{(2\pi)^2}e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \\ &\quad\times\frac{\left\langle PS\left|\overline{\psi}_q(\xi^-,\vec{b}_\perp)\mathscr{L}_v^\dagger(\pm\infty;\xi^-,\vec{b}_\perp)\mathscr{L}_v(\pm\infty;0)\psi_q(0)\right|PS\right\rangle}{S^{\pm}(\vec{b}_\perp,\mu^2,\rho)}\ . \end{aligned} \tag{2}$$

The $+(-)$ superscript is appropriate for DIS (Drell-Yan) process [4, 5]. v^μ is a time-like dimensionless ($v^2>0$) four-vector with zero transverse components $(v^-,v^+,\vec{0})$ and $v^-\gg v^+$. Thus the v^μ is a quasi light-cone vector, approaching n^μ. The variable ζ^2 denotes the combination $(2P\cdot v)^2/v^2=\zeta^2$. $\mathscr{L}_v$ is a gauge link along v^μ,

$$\mathscr{L}_v(\pm\infty;\xi)=\exp\left(-ig\int_0^{\pm\infty}d\lambda\, v\cdot A(\lambda v+\xi)\right)\ . \tag{3}$$

Here the non-light-like gauge link is introduced to regulate the light-cone singularities. In the above definition, we have derived a soft factor defined as [6]:

$$S^{\pm}(\vec{b}_\perp,\mu^2,\rho)=\frac{1}{N_c}\langle 0|\mathscr{L}_{\tilde{v}il}^\dagger(\vec{b}_\perp,-\infty)\mathscr{L}_{vlj}^\dagger(\pm\infty;\vec{b}_\perp)\mathscr{L}_{vjk}(\pm\infty;0)\mathscr{L}_{\tilde{v}ki}(0;-\infty)|0\rangle\ , \tag{4}$$

where i,j,k,l are color indices and new quasi light-cone vector $\tilde{v}^\mu=(\tilde{v}^-,\tilde{v}^+,\vec{0})$ has been introduced with $\tilde{v}^-\ll\tilde{v}^+$. The ρ parameter is defined as $\rho=\sqrt{v^-\tilde{v}^+/v^+\tilde{v}^-}\gg 1$.

The TMD parton distribution is defined as such to absorb the collinear divergence in the partonic processes. This has been checked by an explicit calculation at one-loop order [6], where the soft divergence associated with soft gluons in the TMDs has been cancelled out in the total result, and we are left with only the collinear singularity.

Factorization at One-loop Order

To demonstrate the factorization at one-loop order, one needs to calculate the TMDs at one-loop order. Then, we have to show that the SIDIS cross section can be written in terms of these TMDs plus the soft and hard factors.

The semi-inclusive DIS cross section under one-photon exchange is

$$\frac{d\sigma}{dx_Bdydz_hd^2\vec{P}_{h\perp}} = \frac{4\pi\alpha_{\rm em}^2 s}{Q^4}\left[(1-y+y^2/2)x_BF(x_B,z_h,P_{h\perp},Q^2)+\cdots\right]\ , \tag{5}$$

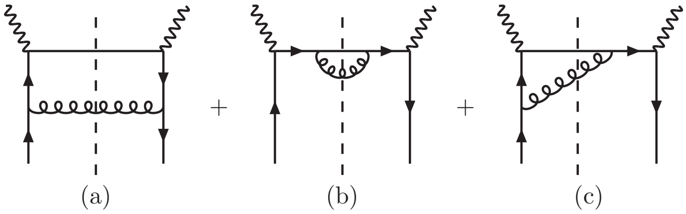

FIGURE 1. One-loop real diagrams for SIDIS.

where F is the spin-independent structure function. In the above equation, we have omitted other terms contributions which may depend on the spin of the hadrons and their factorizations are similar [6].

The one-loop real corrections to the structure function F are shown in Fig. 1. There is no contribution to the hard scattering kernel from any of these diagrams. In Fig. 1a, the soft-gluon radiation generates a transverse-momentum for the struck quark. There is no contribution from the fragmentation function because the contribution from the final state with a gluon in the n^μ direction and a soft quark is power suppressed. Therefore, the diagram must be factorizable into the parton distribution. Similarly for Fig. 1b, which again can be reproduced by the factorization formula with the one-loop fragmentation function and the soft factor S, and the tree-level parton distribution and the hard part. For Fig. 1c and its hermitian conjugate, we find three distinct contributions: where the first term corresponds to a gluon collinear to the initial quark, the second term a gluon collinear to the final state quark, and the third term a soft gluon. All these terms are reproduced by the factorization formula with one-loop parton distribution, fragmentation function, and the soft factor. Similarly, we can show that the virtual diagrams can also be factorized into different pieces in the factorization formula. From the vertex correction to SIDIS cross section, we get the hard factor as

$$H^{(1)}(Q^2,\mu^2,\rho) = \frac{\alpha_s}{2\pi}C_F\left[\left(1+\ln\rho^2\right)\ln\frac{Q^2}{\mu^2} - \ln\rho^2 + \frac{1}{4}\ln^2\rho^2 + \pi^2 - 4\right] . \qquad (6)$$

Therefore we conclude that at the one-loop level, the general factorization formula Eq.(1) holds.

All Order Argument and Discussions

For arguments toward a factorization to all orders, we follow the discussions in [1, 6]. The procedure for this argument is the following. First, for any high order Feynman diagrams, using the power counting rules identifies the leading region contributions [8]. The leading regions clearly separate the soft, collinear, and hard gluons' contributions to the cross section (the cut diagram), where the soft gluons are only attached to the jet functions (parton distributions and/or fragmentation functions); hard gluons are included in the hard part; collinear gluons attached the jet functions to the hard part. On top of that, we can further use the Grammer-Yennie approximation to factorize out the soft factor, which can be expressed as matrix element of Wilson lines [1, 6], as defined above in

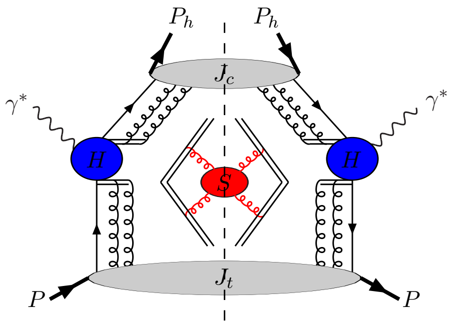

FIGURE 2. All order factorization for SIDIS.

Eq. (4). The Ward Identity will be used to further factorize the collinear gluons from the hard part, which results in a Wilson line (gauge link) association in the definition of the jet functions. The variation of the gauge link gives the Collins-Soper evolution equation for the jet functions [1]. After these procedures, the hard part only contains hard gluons, which can be calculated from perturbative QCD. Once all these being done, we will arrive at the factorization formula for SIDIS as in Eq. (1), and shown in Fig. 2.

Similar factorization formulas can be obtained for other semi-inclusive processes, including back-to-back di-hadron production in e^+e^- annihilation [1], low transverse momentum Drell-Yan [9], and di-jet or di-hadron correlation at hadron colliders [10]. The common feature of these semi-inclusive processes is that they all depend on the TMD parton distributions and/or fragmentation functions. The global analysis of all these processes will definitely provide us a unique picture about the nucleon structure, and reveal the relevant parton orbital motion in the nucleon.

The author is grateful to RIKEN, Brookhaven National Laboratory and the U.S. Department of Energy (contract number DE-AC02-98CH10886) for providing the facilities essential for the completion of his work.

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