QCD Factorization for Semi-Inclusive DIS

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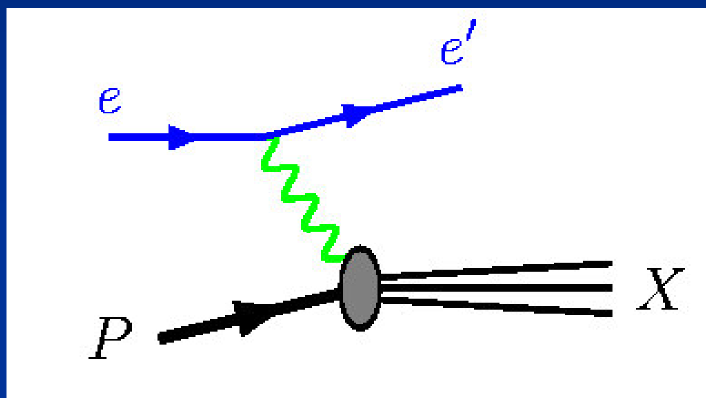
References: Ji, Ma, Yuan, Phys. Rev. D70, 074021;
Phys. Lett. B597, 299;
hep-ph/0503015.

Outline

- Introduction
- The QCD factorization

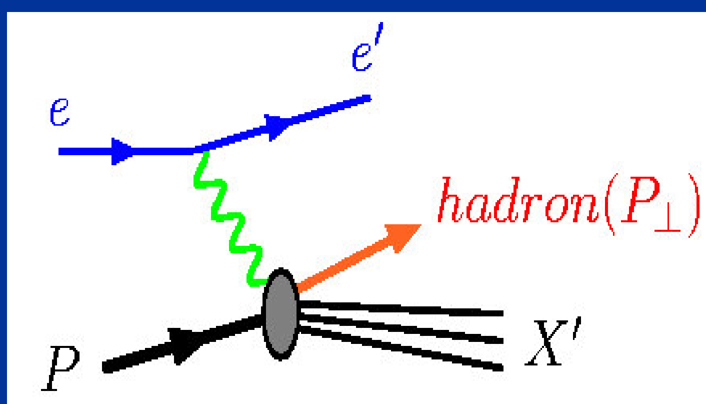
  -- TMD Parton Distributions
- Summary

Inclusive and Semi-inclusive DIS



Inclusive DIS:

Partonic Distribution depending on the longitudinal momentum fraction



Semi-inclusive DIS:

Probe additional information for parto transverse distribution in nucleon

Different $P_?$ Region

- Integrate out $P_?$

  -- normal factorization, similar to inclusive DIS
- Large $P_?$ (ÀΛ_{QCD})

  -- hard gluon radiation, can be calculated from perturbative QCD, factorization similar to inclusive DIS
- Low $P_?$ (»Λ_{QCD})

  -- nonperturbative information: new factorization formula

Wonderful Physics Associated

- Transverse Momentum Dependent (TMD) Parton Distributions
- A way to measure Transversity Distribution, the last **unkown** **leading twist** distribution

  Collins 1993
- The Novel Single Spin Asymmetries
- Quark Orbital Angular Momentum and

  Many others …

Why Worry about Factorization?

- Safely extract nonperturbative information
  Theoretically under control
  No breakdown by un-cancelled divergence

- NLO correction calculable
  Estimate the high order corrections

What to Worry for Factorization?

- Correct definition of TMD parton distributions

  Gauge Invariance?

  Soft divergence gets cancelled
- Hard Part can be calculated perturbatively
- The cross section can be separated into Parton Distribution, Fragmentation Function, Hard and **Soft** factors

Previous Works on Factorization
(basis of the present work)

- Factorization for back-back jet production in e^+e^- annihilation (in axial gauge)
  -- Collins-Soper, NPB, 1981
- Factorization for inclusive processes
  -- Collins, Soper, Sterman, NPB, 1985
  -- Bodwin, PRD, 1985
  -- Collins, Soper, Sterman,
  in *Perturbative QCD*, Mueller ed., 1989

New Context for SIDIS

- The Gauge Invariant definition for the TMDs

  -- Brodsky, Hwang, Schmidt, 2002

  -- Collins, 2002, 2003

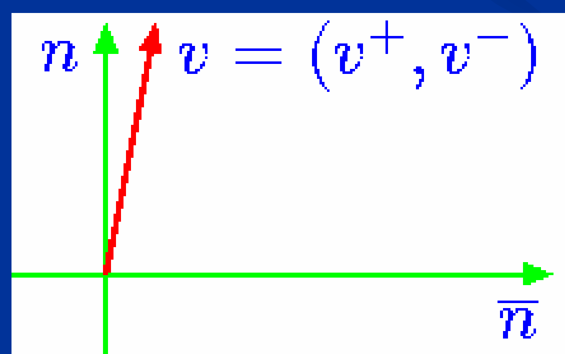
  -- Ji, Belitsky, Yuan, 2002
- Final state interaction induced by the gauge link has novel effects

TMD Distribution: the definition

$$Q(x, k_\perp, \mu, x\zeta) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{k}_\perp} \times \left\langle P \left| \overline{\psi}_q(\xi^-, 0, \vec{b}_\perp) \mathcal{L}_v^\dagger(\infty; \xi^-, 0, \vec{b}_\perp)\gamma^+ \mathcal{L}_v(\infty; 0)\psi_q(0) \right| P \right\rangle$$

Gauge Invariance requires the Gauge Link

$$\mathcal{L}_v(\infty;\xi) = \exp\left(-ig\int_0^\infty d\lambda v\cdot A(\lambda v + \xi)\right)$$



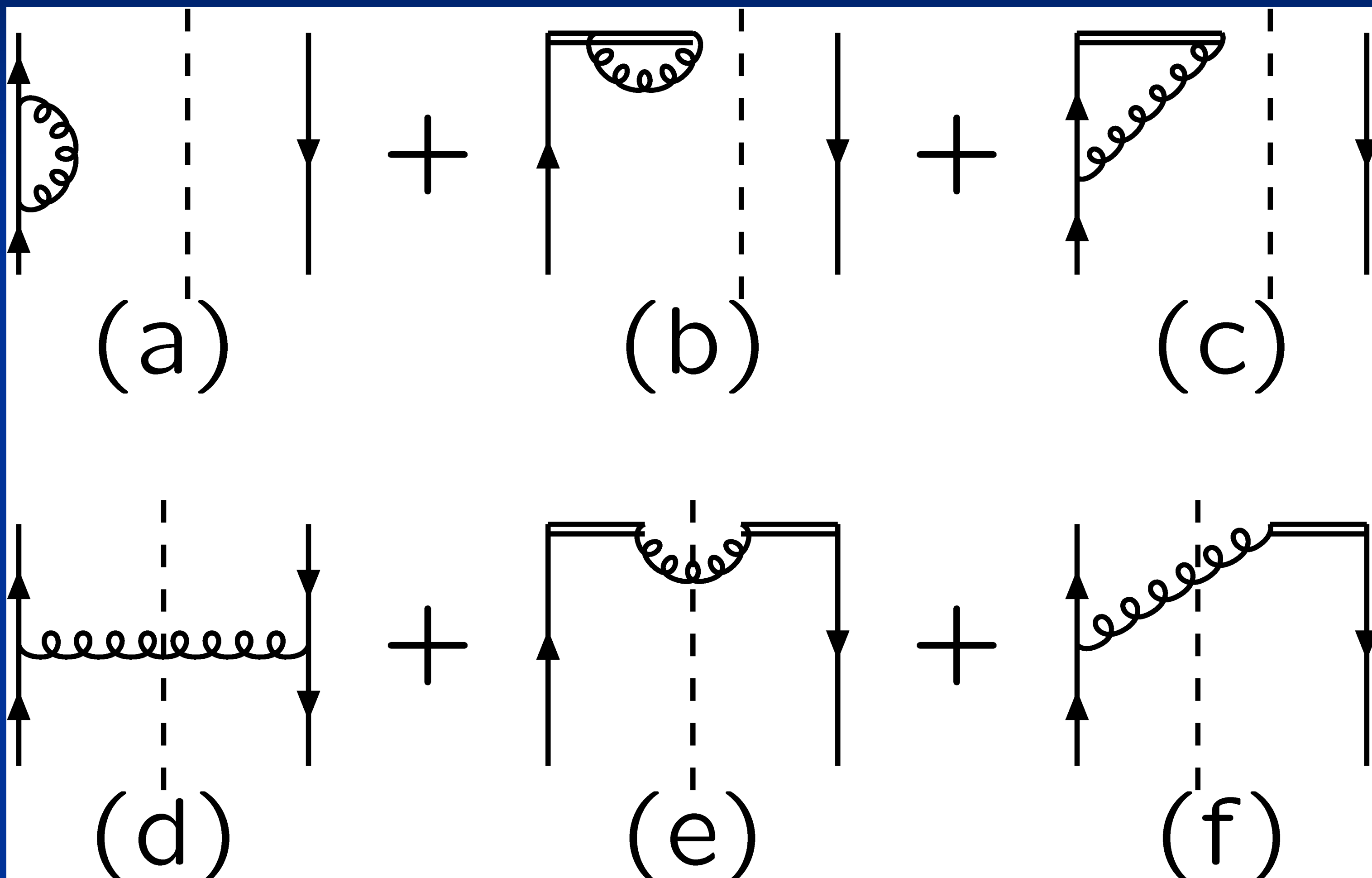
$\zeta^2 = (2v\cdot P)^2/v^2$

v is not along the light-cone to avoid l.c. singularity !!

Illustrate the Factorization
(one-loop order)

- Take an on-shell quark target
- Calculate the TMD Dis. and F.F.
- Separate the cross section into different pieces
- Show the soft divergence is cancelled out

TMD Dis. At One-loop



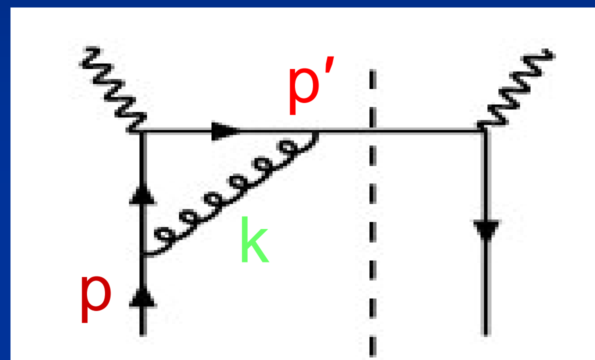
$$q(x,b,\mu,x\zeta,\rho) = \frac{\alpha_s C_F}{2\pi}\left\{\left(\frac{1+x^2}{1-x}\right)_+ \ln\frac{4}{b^2m^2}e^{-2\gamma_E} - \left(\frac{2x}{1-x}\right)_+ - \left(\frac{1+x^2}{1-x}\ln\frac{1}{(1-x)^2}\right)_+ + \delta(x-1)\left[\left(\frac{1}{2}-\ln\rho^2\right)\ln\frac{4}{b^2\mu^2}e^{-2\gamma_E} - \frac{1}{2}\ln^2\left(\frac{\zeta^2b^2}{4}e^{2\gamma_E-1}\right) - \frac{2+\pi^2}{2}\right]\right\}$$

- No soft divergence
- Collinear divergence : **$\ln(m^2)$**
- Double Logarithms: **$\ln^2(\zeta^2 b^2)$**

  --- Collins-Soper equation

One-loop Factorization: virtual

- Vertex corrections (single quark target)

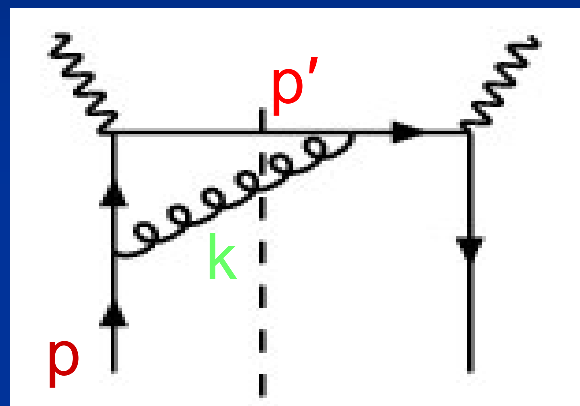


Four possible regions for the gluon momentum k:

1) k is collinear to p (parton distribution)
2) k is collinear to p′ (fragmentation)
3) k is soft (Soft factor)
4) k is hard (pQCD correction)

One-Loop Factorization: real

- Gluon Radiation (single quark target)



Three reagions for the gluon momentum k:

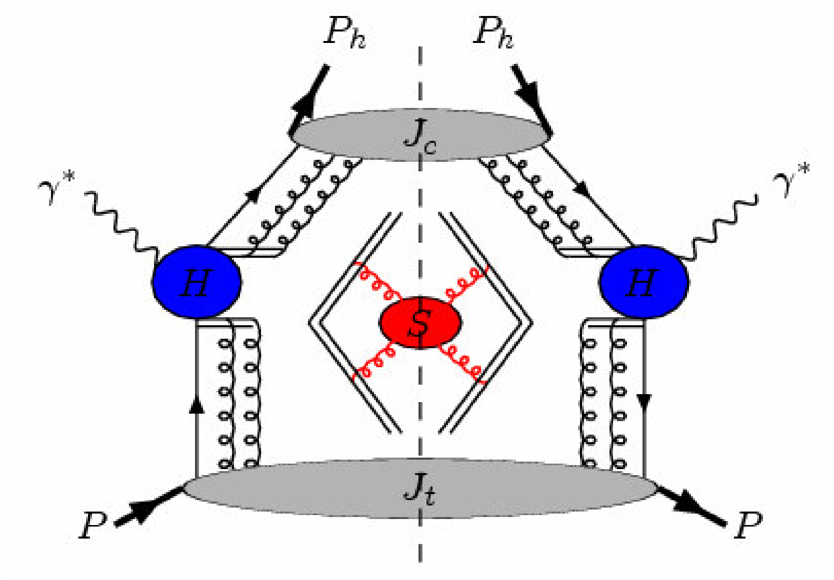
1) k is collinear to p (parton distribution)
2) k is collinear to p′ (fragmentation)
3) k is soft (Soft factor)

All Orders in Perturbation Theory

- Consider an arbitrary Feynman diagram
- Find the singular contributions from different regions of the momentum integrations
- Power counting to determine the leading regions
- Factorize the soft (Grammer-Yennie)and collinear gluons (Ward Identity)
- Factorization theorem.

(Collins, Soper, Sterman, *Perturbative QCD*, Mueller ed.)

Factorization



$$
\begin{aligned}
&F(x_B, z_h, P_{h\perp}, Q^2) \\
&\quad = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp \\
&\qquad \times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_h\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho) \\
&\qquad \times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp})
\end{aligned}
$$

The Factorization Applies to

- Semi-inclusive DIS (polarized and unpolarized)
- Drell-Yan at Low transverse momentum
- Di-hadron production in e+e- annihilation (extract the Collins function)
- Di-jet and/or di-hadron correlation at hadron collider (work in progress)
- Many others, …

Summary

- QCD factorization is valid for the Semi-inclusive Deep Inelastic Scattering
- The cross section can be written in terms of TMD Parton Distribution, Fragmentation Function, Plus the Hard and Soft Factors