

# Jefferson Lab's results on the $Q^2$ -evolution of spin structure functions

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On behalf of the CLAS and E94010 collaborations

# Moments of spin structure functions

$$N^{\text{th}}\text{-moments: } \left\{ \begin{array}{l} \int g_1 x^{n-1} dx \\ \int g_2 x^{n-1} dx \end{array} \right. \quad \text{First moments: } \Gamma_1, \Gamma_2$$

Why are moments interesting ?

Related to particle static properties via sum rules:

- ★  $\Gamma_1^N$ :  $\begin{cases} \text{Ellis-Jaffe sum rule (large } Q^2) \\ \text{Gerasimov-Drell-Hearn (GDH) sum rule (} Q^2=0 \text{)} \end{cases}$
- ★  $\Gamma_1^{p-n}$ : Bjorken sum rule (large  $Q^2$ )
- ★  $\Gamma_2^N$ : Burkhardt–Cottingham (BC) sum rule (any  $Q^2$ )

Higher moments:

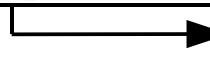
- ★  $d_2$  "sum rule"
- ★ Spin polarizability sum rules

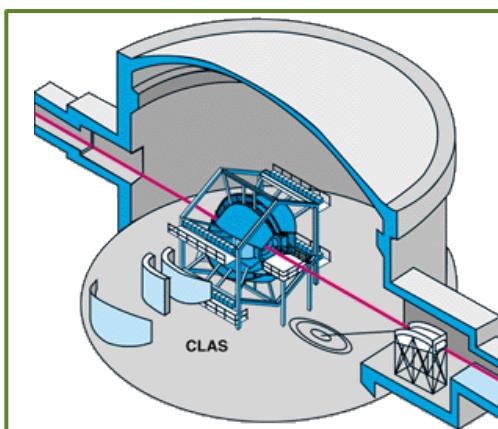
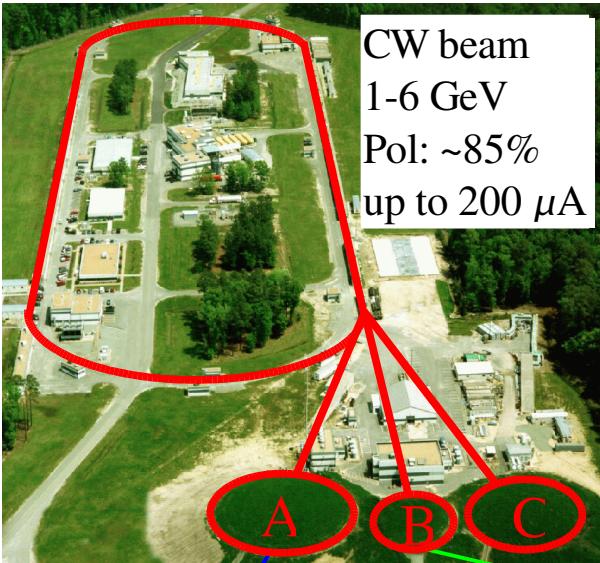
- Some moments are calculable in OPE (large  $Q^2$ , n odd).
- Low  $Q^2$  moments can be calculated using  $\chi pT$ .
- Any  $Q^2$  : Lattice QCD.
- Other (BC sum rule)

⇒ Moments, (via sum rules) can:

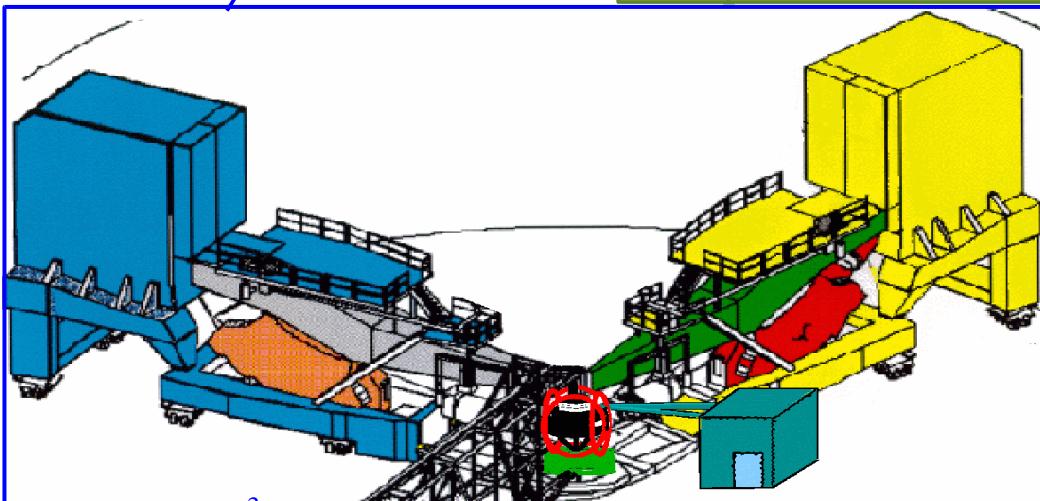
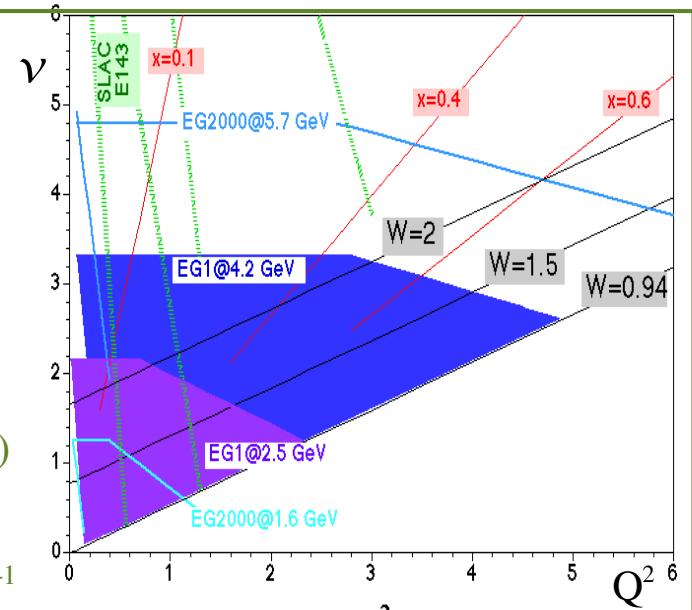
- Check theories (e.g. QCD and Bjorken sum rule)
- Check hypothesis (e.g. EJ Sum Rule)
- Check calculations (e.g. OPE,  $\chi pT$ , Lattice and generalized GDH sum rule)
- Extract quantities (e.g.  $d_2$ , WW, spin pol. sum rules)

In addition,  $Q^2$ -evolution of moments are instrumental to study strong coupling constant, hadron-parton duality...

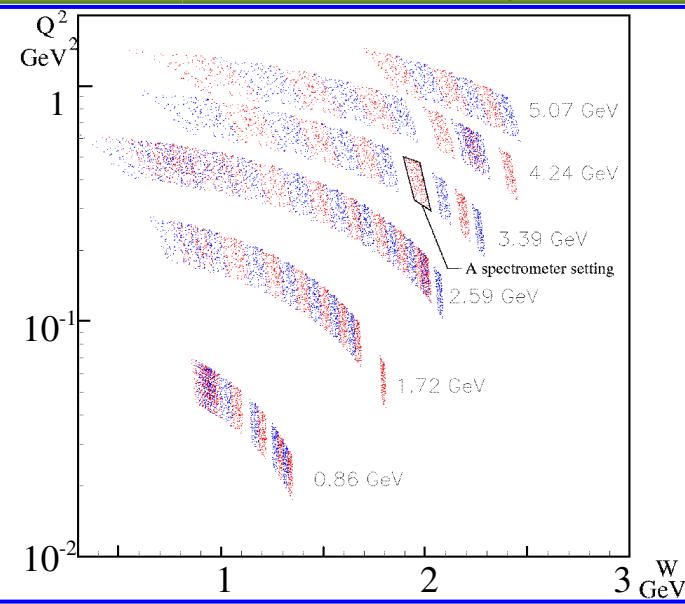
 See N. Liyanage talk, Friday



Pol.  $\text{NH}_2$  &  $\text{ND}_2$  (spin // to beam)  
Pol : 75% ( $\text{NH}_2$ ) or 30% ( $\text{ND}_2$ )  
Accept  $\sim 2.5\pi$   $\mathcal{L}: 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$



Pol.  ${}^3\text{He}$  target (// and  $\perp$  directions) Pol: ~40%  
Accept: 6 mSr,  $\mathcal{L}: 10^{36} \text{ cm}^{-2} \text{ s}^{-1}$



## Extraction of SSF

- CLAS/EG1

Only longitudinal asymmetry:

⇒ Model for  $F_1$  : well measured at low  $Q^2$ (Hall C)

⇒ Model for  $g_2$  : MAID (resonance) &  $g_2^{ww}$ (DIS)

⇒ Model for R : SLAC+Hall C measurements

⇒ Model dependence is minimal.

( $\pm 5\%$  typical uncertainty, dominated by polarization measurement ( $6\%$ ) and dilution factor. Uncertainty at the  $\Delta_{1232}$  larger (dominates for moments).

For deuteron, models not as constraint but statistical is not as good

- Hall A/ E94-010

Measure polarized cross-sections (both // and  $\perp$ )

⇒ Model indep. extraction and no dilution factor

Errors:

polarization measurements:  $\sim 5\%$

cross section:  $\sim 5\%$

Interpolation uncertainty: small (high  $\mathcal{L}$ uminosity)

# $\Gamma_1^p$ from CLAS

$$\text{OPE: } \int g_1^N dx = (\pm 12g_A + \frac{a_8}{36}) C_{NS} + \frac{a_0^1}{9} C_s + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \dots$$

Elastic not included

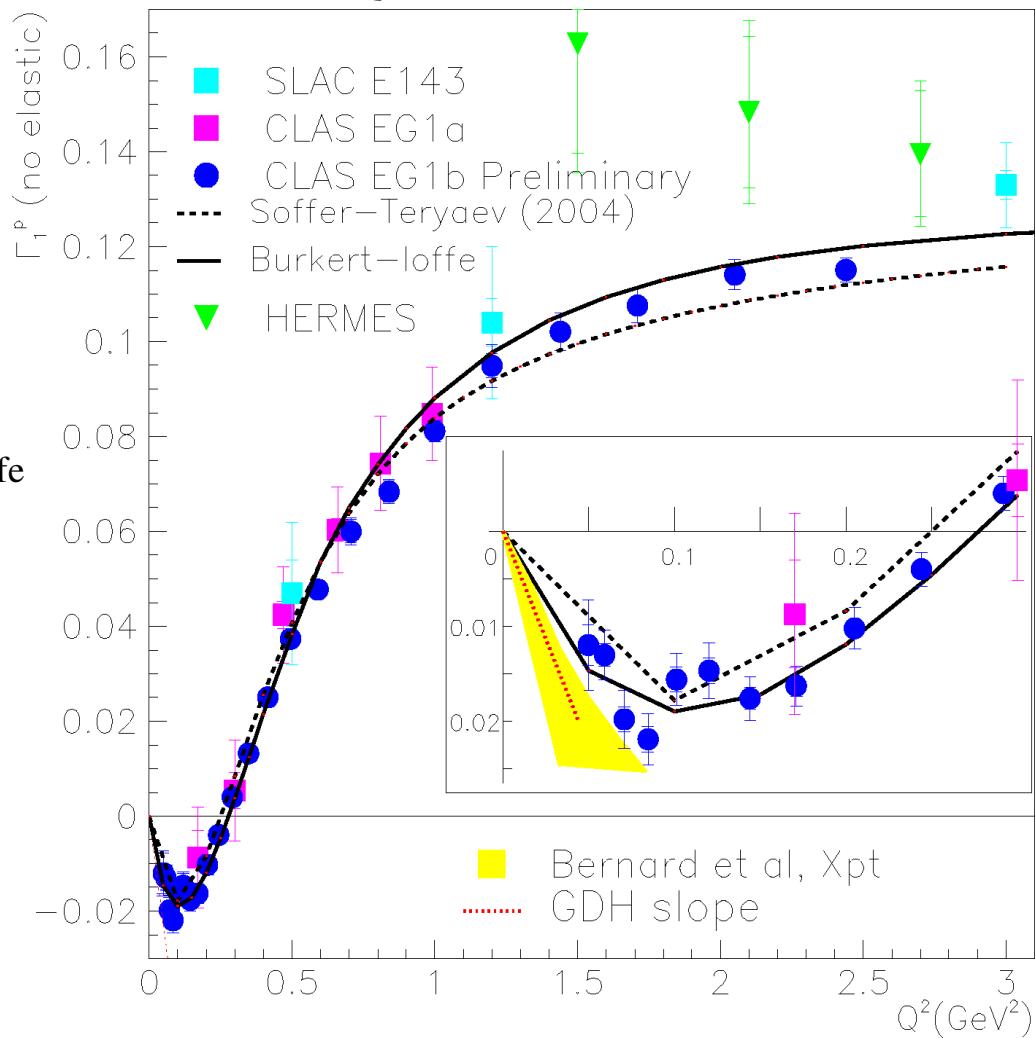
Low-x extrapolation :

EG1a: Bianchi-Thomas

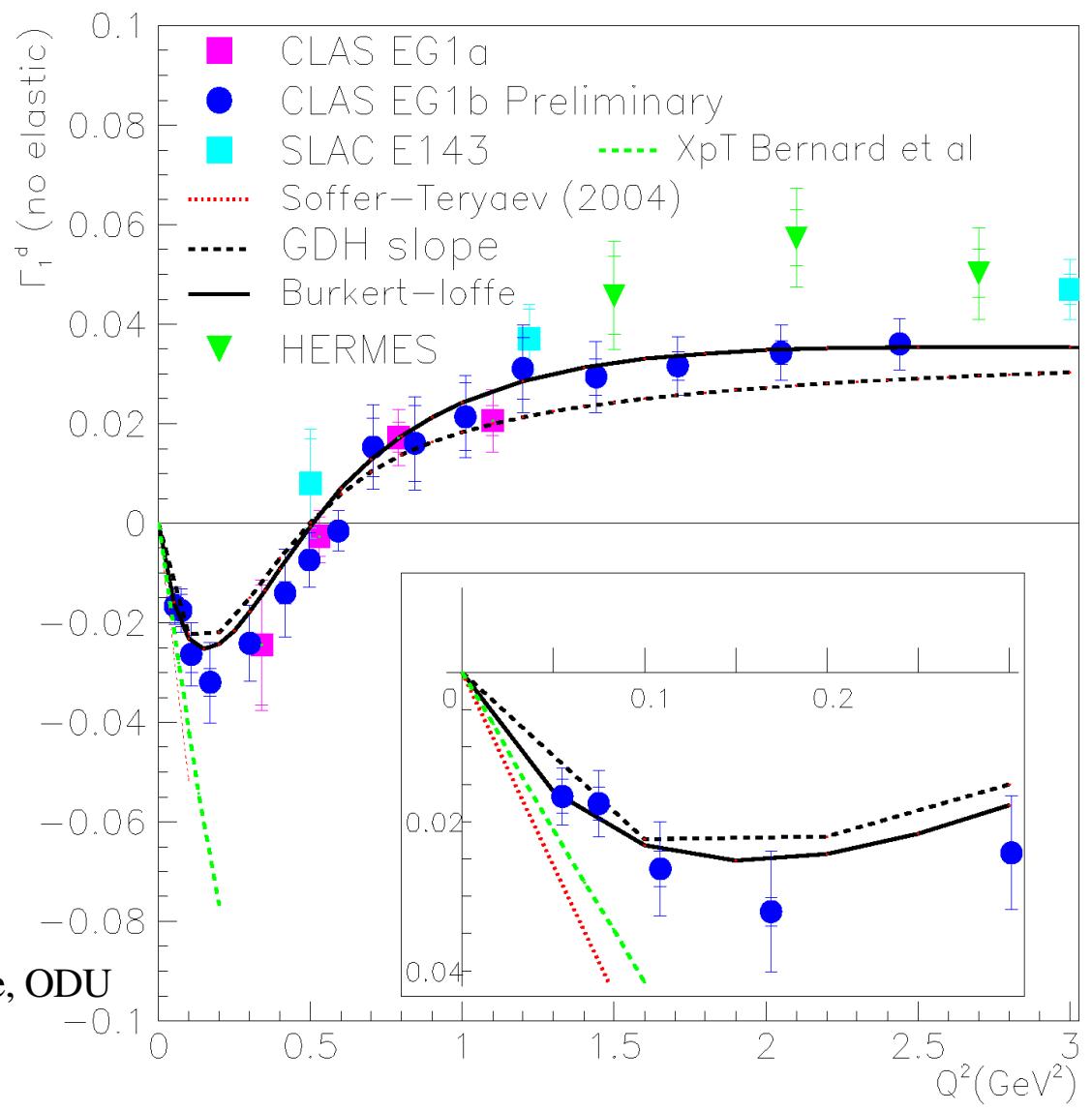
EG1b: EG1 "Model"

Non-res.: DIS fit , res.: Burkert-Ioffe

EG1b data analysis: Y. Prok, U.Va  
 EG1a data: R. Fatemi et al.,  
 PRL.91: 222002, 2003



# $\Gamma_1^d$ from CLAS



EG1b data analysis: V. Dharmawardane, ODU

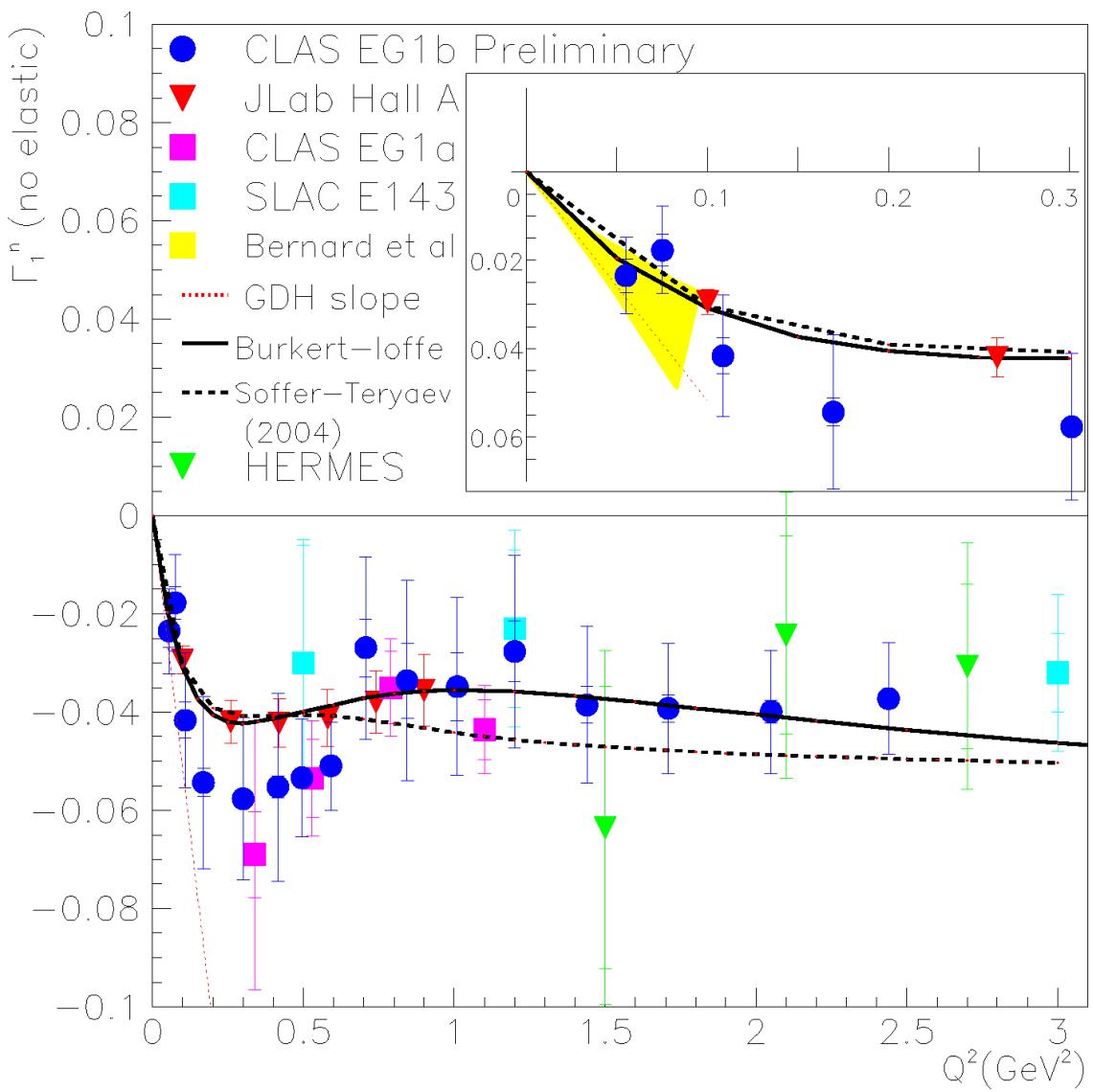
EG1a data: J. Yun et al.,

Phys.Rev.C67:055204,2003

# $\Gamma_1^n$ from Hall A & CLAS

Low-x extrapolation :

Hall A/EG1a: Bianchi-Thomas  
EG1b: "Model"

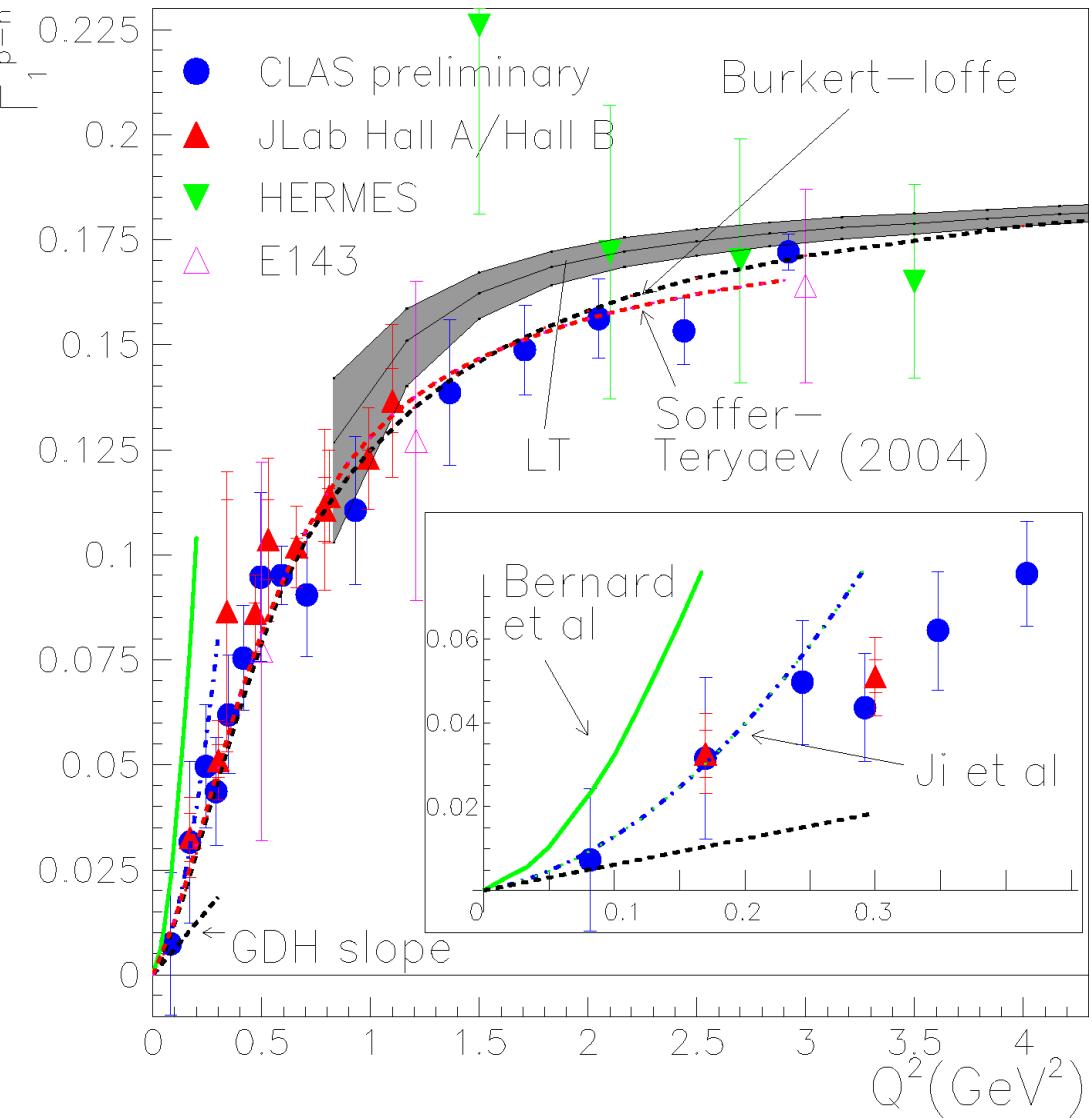


Hall A exp E94-010 data:  
M. Amarian et al., PRL 92, 022301, 2003

# Bjorken sum from Hall A & CLAS

$\Delta_{1232}$  contribution suppressed  
⇒ better check of  $\chi pT$ .

Low-x extrapolation :  
Hall A/EG1a: Bianchi-Thomas  
EG1b: "Model"



Hall A/CLAS data: A. Deur et al.  
PRL 93 212001 (2004)

# Generalized GDH sum from Hall A

$$\text{GDH sum } (Q^2=0): \int_{\nu_{\text{thr}}}^{\infty} \sigma^{1/2} - \sigma^{3/2} \frac{d\nu}{\nu} = \frac{-2\alpha\pi^2\kappa^2}{M^2}$$

$\kappa$ : anomalous magnetic moment

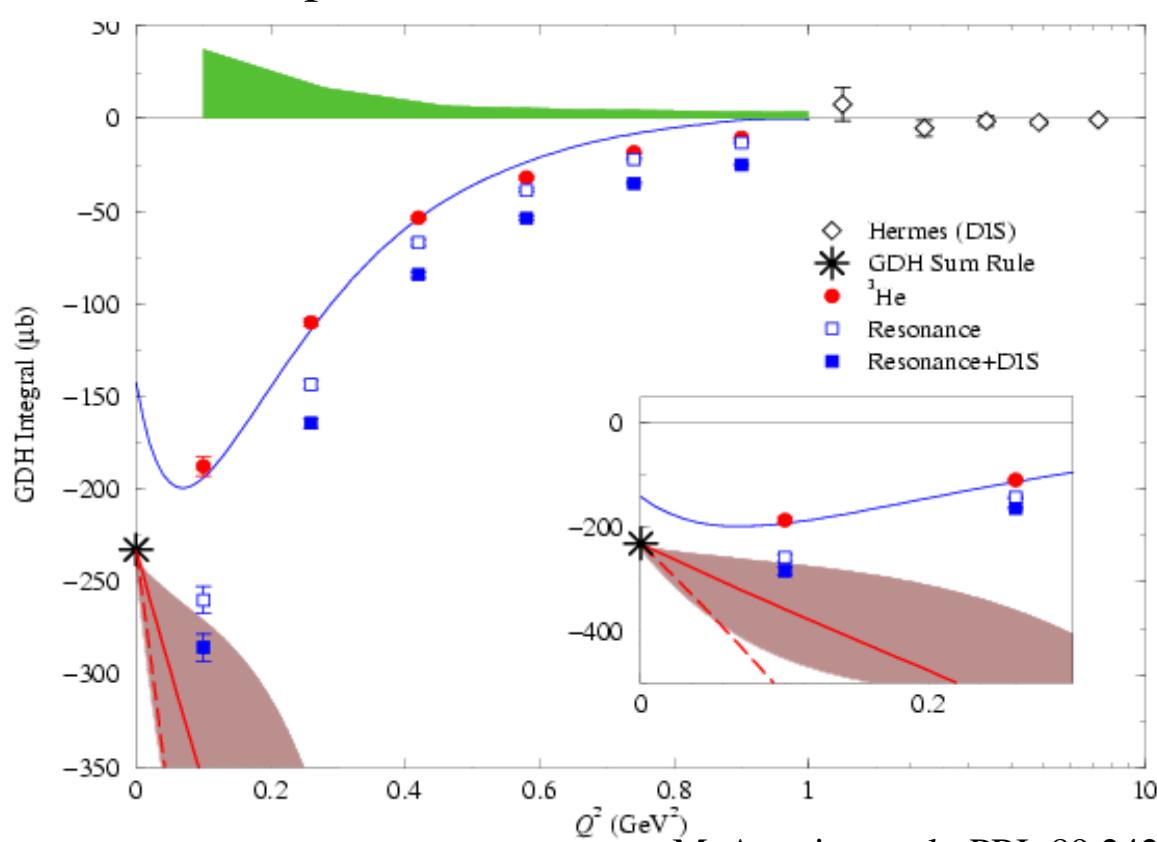
Generalized: Photoproduction  $\rightarrow$  electroproduction

- Bernard et al,  $\chi pT$
- Bernard et al,  $\chi pT$   
 $(\Delta_{1232} \text{ included})$

- - - Ji et al,  $\chi pT$
- Maid model

(Multipole analysis  
of world inclusive and exclusive  
data in the resonance region  
D. Drechsel, S.S. Kamalov, L.Tiator,  
Nucl. Phys. A645 145, 1999)

Low-x extrapolation :  
Bianchi-Thomas



M. Amarian et al. PRL 89 242301 (2002)



- Accurate ( $p$  and  $n$ ) mapping at intermediate  $Q^2$ .
- Connection with DIS data.
- $\chi pT$  not valid beyond  $Q^2 = 0.2 \text{ GeV}^2$ .
- Phenomenological models work well.
- Good leading twist description even at low  $Q^2$  (Bjorken sum).

## Higher Twists

OPE:

$$\int g_1^N dx = (\pm \frac{1}{12} g_A + \frac{a_8}{36}) C_{NS} + \frac{a_0}{9} C_s + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \dots$$

Bjorken sum:

$$\int g_1^{p-n} dx = \frac{1}{6} g_A C_{NS} + \frac{\mu_4^{p-n}}{Q^2} + \frac{\mu_6^{p-n}}{Q^4}$$

Simpler: Low  $Q^2$ : ~no  $\Delta_{1232}$  contribution

Large  $Q^2$ : easier DGLAP evolution, Easier low-x extrapolation, sum rule

Extraction:

- include elastic.
- Consistent low-x extrapolation for world data.
- Fit DIS world data + JLab data for  $Q^2 > X$  using above functional forms.

Wilson coef.  $C_{NS}$  and  $C_s$  known (pQCD)  
 $a_8 (= \Delta\Sigma)$ ,  $g_A$ , and  $a_0$  measured.

$$\frac{\mu_4}{Q^2} = \frac{M^2}{9} (a_2 + 4d_2 + 2f_2)$$

Results (values at  $Q^2 = 1$ ):

Proton:

$$4\mu_4'/9M^2 = f_2 = 0.039^{+0.037}_{-0.043}$$

$$\mu_6'/M^4 = 0.011^{+0.017}_{-0.013}$$

World data+EG1a (no EG1b)

M. Osipenko et al.

Nachtmann  
moment used  
Phys. Lett B609: 258 (2005)

Neutron:

$$f_2 = 0.034 \pm 0.043$$

$$\mu_4'/M^2 = 0.019 \pm 0.024$$

$$\mu_6'/M^4 = -0.019 \pm 0.017$$

Hall A E94-010 data

Z-E Meziani et al.

Phys. Lett. B613 148 (2005)

P-N:

$$f_2 = -0.18 \pm 0.10$$

$$\mu_4'/M^2 = -0.06 \pm 0.02$$

$$\mu_6'/M^4 = 0.09 \pm 0.03$$

EG1a+E94-010 data

A. Deur et al.

PRL 93 212001 (2004)

⇒ Overall, higher twist contribution small at  $Q^2 = 1$ .

From these analysis, we can extract color polarizabilities for the nucleons:

$$\chi_E = (4d_2 + 2f_2)/3$$

$$\chi_B = (4d_2 - f_2)/3$$

$$\chi_E = 0.026 \pm 0.015 \text{ (stat)} \quad {}^{+0.021}_{-0.024} \text{ (sys)}$$

Proton:

$$\chi_B = -0.013 \pm 0.007 \text{ (stat)} \quad {}^{-0.010}_{-0.012} \text{ (sys)}$$

$$\chi_E = 0.033 \pm 0.029$$

Neutron:

$$\chi_B = -0.001 \pm 0.016$$

## Extraction of generalized spin polarizabilities (electromagnetic) using sum rules

Same derivation as GDH sum rule but one higher order in the low energy theorem.  
Better convergence (low-x less of an issue).

Generalized forward spin polarizability:

$$\gamma_0 = \frac{4e^2 M^2}{\pi Q^6} \int_{v_{\text{thr}}}^{\infty} x^2 (g_1 - \frac{4M^2}{Q^2} x^2 g_2) dx$$

Longitudinal-Transverse polarizability:

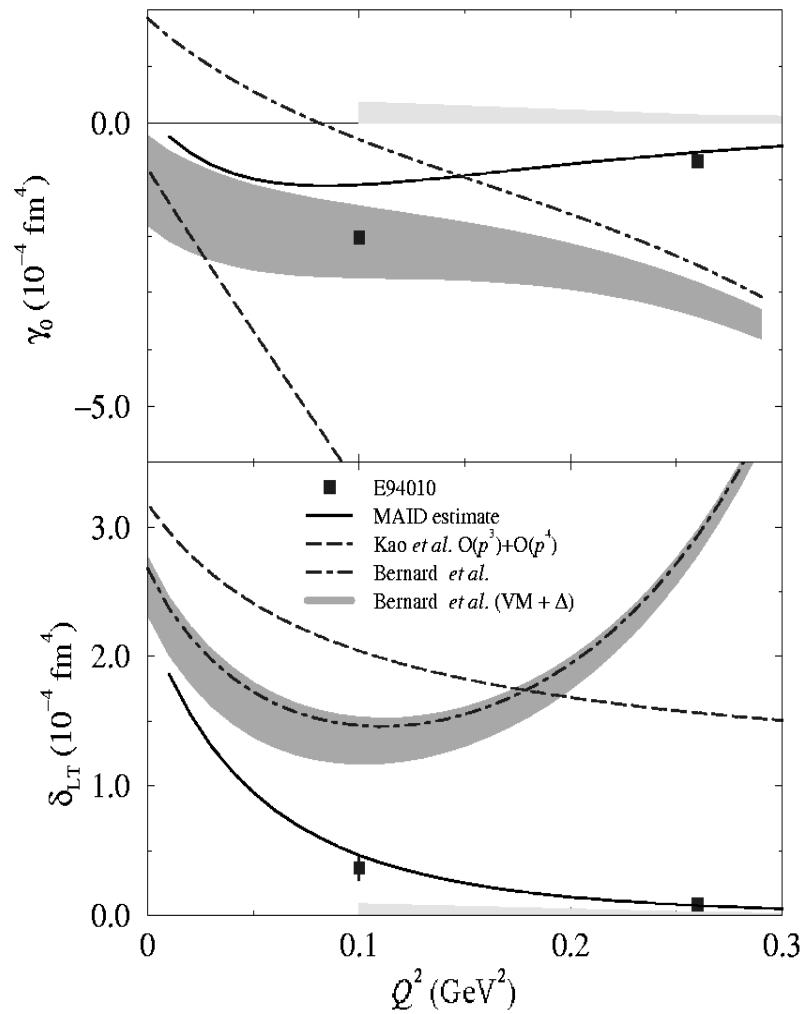
$$\delta_{LT} = \frac{4e^2 M^2}{\pi Q^6} \int_{v_{\text{thr}}}^{\infty} x^2 (g_1 + g_2) dx$$

$\delta_{LT}$  has its  $\Delta_{1232}$  contribution suppressed.

⇒ better check of  $\chi pT$ .

Data from Hall A E94-010. First measurement  
of Generalized spin polarizabilities.

M. Amarian et al., PRL 9 152301 2004



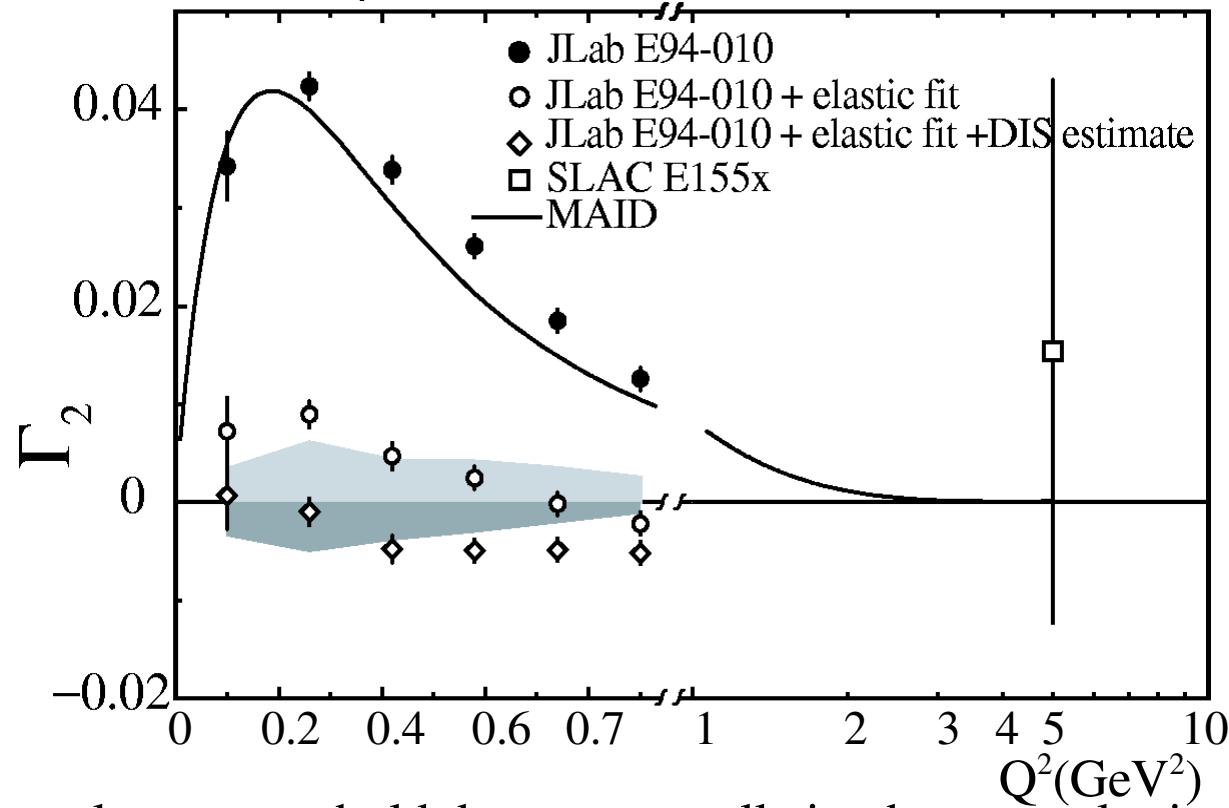
## The Burkhardt–Cottingham sum rule on neutron

$$\text{BC sum rule: } \Gamma_2 = \int_0^1 g_2 dx = 0$$

Based on dispersion relation+nice low-x behavior of  $g_2$  (no OPE  $\Rightarrow$  stands at all  $Q^2$ ).

Not well measured in DIS, but SLAC E155x found a  $3\sigma$  deviation for proton.

Data from Hall A exp E94-010: M. Amarian et al., PRL 92, 022301, 2003



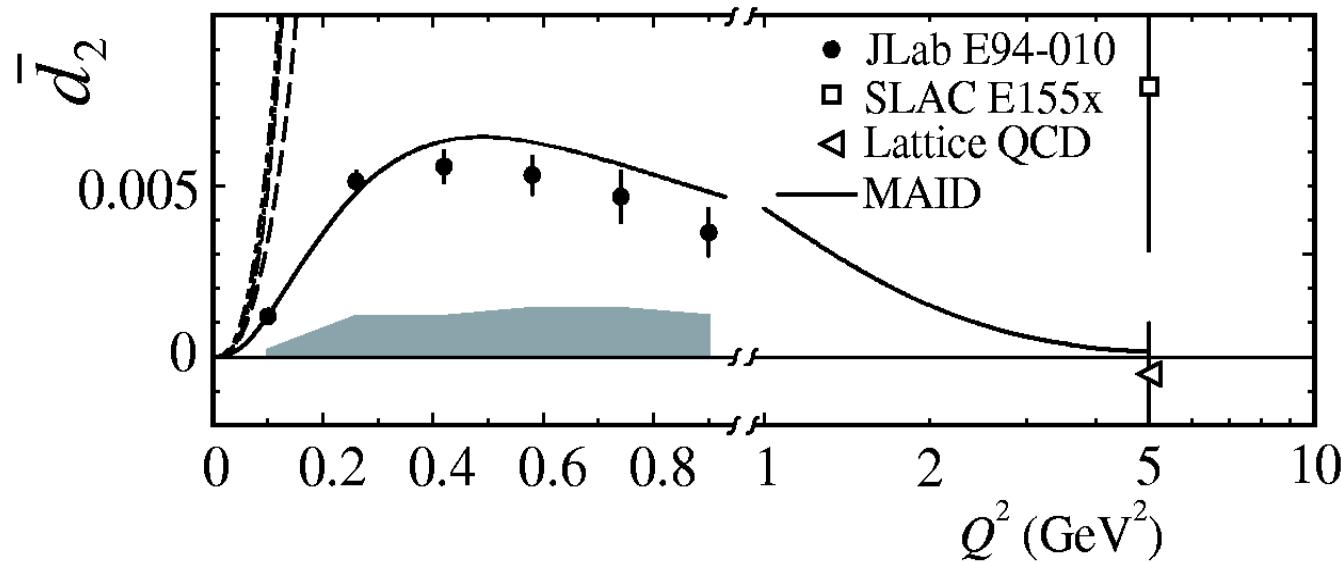
Sum rule seems to hold due to a cancellation between elastic and inelastic contribution.

## The $d_2$ sum on neutron

$$d_2 = \int x^2 (2g_1 + 3g_2) dx = 3 \int x^2 (g_2 - g_2^{\text{ww}}) dx$$

Sensitive to Twist-3 and higher (parton interpretation). No low-x issue

Data from Hall A exp E94-010: M. Amarian et al., PRL 92, 022301, 2003



Precise mapping in the hadron-parton transition. Trend toward the Lattice result.  
Good description from resonance model.

## Summary & perspectives

- JLab data provided an accurate mapping of the hadron-parton transition region.
  - Moments that can be calculated at any  $Q^2$  were presented.
  - OPE analysis reveals that Higher Twist effects are small even at  $Q^2=1$ .
  - Upcoming results at higher  $Q^2$ : CLAS EG1b, Hall A E01-012, Hall C RSS
  - New results at very low  $Q^2$  will complete mapping: Hall A E97-110, *CLAS E03-006*
- 
- 12 GeV JLab Upgrade will allow us to:
    - Improve large-x coverage, minimize low-x issue.
    - Study convergence of sum rules.
    - Increase  $Q^2$  range.