Soft resummation for heavy-quark production

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Outline

- Large logs in perturbation theory: parametrical vs. kinematical logs.
- Threshold resummation in inclusive CC DIS at low Q^2.
- The NC case. One particle inclusive observables.
- Conclusions.

- G. Corcella and A.M., hep-ph/0308105 (CC DIS)
- E.Laenen and S-O. Moch, hep-ph/9809550
- P.Nadolsky, N.Kidonakis, F.Olness and C.-P. Yuan, hep-ph/0210082

Large logs in DIS

The perturbative calculations often produce large logs that diverge in certain limits:

1) "Parametrical" of the type: ~ ln(m/Q) ,
2) "Kinematical" of the type: ~ ln(E-E_max) .

For example: m – a mass,

Q – (fixed) hard scale,

E - kinematical variable that is not fixed.

☹ The bad news about those logs:

can be very large and invalidate the perturbative expansion.

☺ There is good news too:

they have universal (albeit different) origin. We can use that (RG) universality to resum them to all orders in the strong coupling.

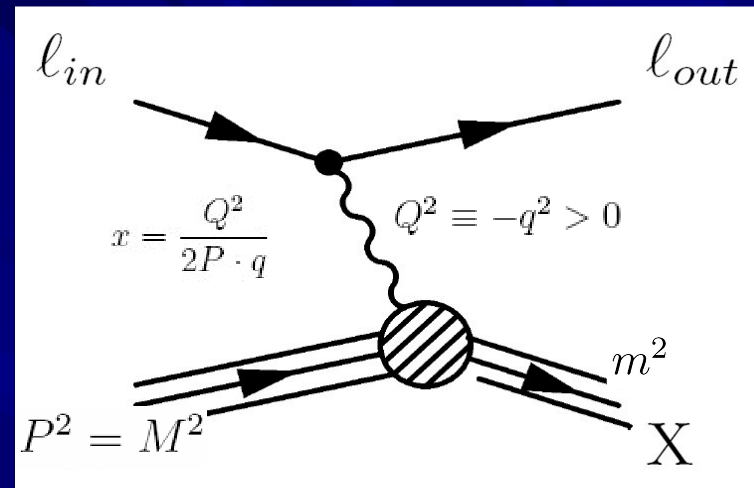
! In this talk we will be interested only in the kinematical logs.

Heavy Quark Production in Inclusive CC DIS

Two mass scales - two limiting cases:

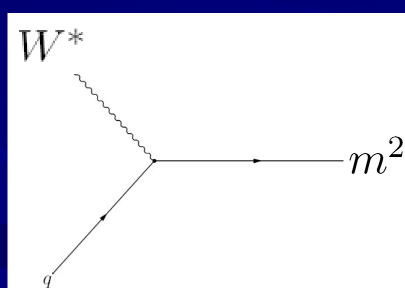
1) Massive: m^2 ≈ Q^2 ,

2) Massless: m^2 << Q^2 .

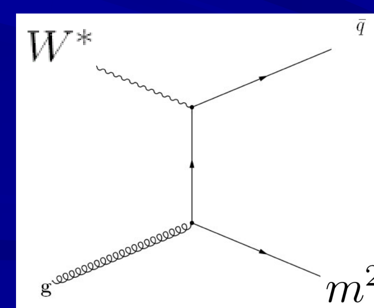


Two ways to produce the heavy flavor when m ≈ Q:

Direct flavor excitation



Gluon splitting



In the inclusive case the limit m/Q->0 is finite !

What do the fixed order calculations have to say?

$$\frac{d^2\sigma^{\nu(\bar\nu)}}{dxdy} = \frac{G_F^2 M E_\nu}{\pi(1+Q^2/M_W^2)^2}\left[y^2 x F_1 + \left(1-\left(1+\frac{Mx}{2E_\nu}\right)y\right)F_2 \pm y\left(1-\frac{y}{2}\right)xF_3\right]$$

For small Q^2 we account for the target mass effects: Nachtmann variable.

$$\begin{aligned}\mathcal{F}_i(x,Q^2) &= \int_\chi^1 \frac{d\xi}{\xi}\left[C_i^q\left(\frac{\chi}{\xi},\mu^2,\mu_F^2,\lambda\right) q_1\left(\xi,\mu_F^2\right)\right.\\ &+ \left. C_i^g\left(\frac{\chi}{\xi},\mu^2,\mu_F^2,\lambda\right) g\left(\xi,\mu_F^2\right)\right], \quad i=1,2,3.\end{aligned}$$

$$x = \frac{\lambda\chi}{1-M^2\lambda^2\chi^2/Q^2}$$

$$\lambda = \frac{Q^2}{Q^2+m^2} \leq 1$$

$$\begin{aligned}C_i^q(z,\mu^2,\lambda) &= \delta(1-z) + \frac{\alpha_S(\mu^2)}{2\pi}H_i^q(z,\mu_F^2,\lambda)\\ C_i^g(z,\mu^2,\mu_F^2,\lambda) &= \frac{\alpha_S(\mu^2)}{2\pi}H_i^g(z,\mu_F^2,\lambda).\end{aligned}$$

$$H_i^q(z\to 1,\mu_F^2,\lambda) = H^{soft}(z,\mu_F^2,\lambda)$$

$$\begin{aligned}H^{soft}(z,\mu_F^2,\lambda) &= 2C_F\left\{2\left(\frac{\ln(1-z)}{1-z}\right)_+ - \left(\frac{\ln(1-\lambda z)}{1-z}\right)_+\right.\\ &+ \left.\frac{1}{4}\left(\frac{1-z}{(1-\lambda z)^2}\right)_+ + \frac{1}{(1-z)_+}\left[\ln\frac{Q^2+m^2}{\mu_F^2}-1\right]\right\}\end{aligned}$$

The “Soft Limit” z->1.

We observe that the limit z->1 is singular. What is the meaning of this singularity?

The limit z->1 is “kinematical” i.e. in that corner of the phase-space only soft gluons can be radiated.

In addition, when the produced final-state quark is …

massive - collinear radiation from it is not singular,

massless – enhancement from collinear radiation from the final quark.

Large “soft” logs appear in the quark initiated coefficient function:

q+W -> Q(m)

But not in the gluon initiated one:

g+W-> q+Q(m)

Therefore, in the following, we are going to resum the large soft logs in the quark initiated contribution.

Interplay between soft logs and quark masses.

$$H^{soft}(z,\mu_F^2,\lambda) = 2C_F\left\{2\left(\frac{\ln(1-z)}{1-z}\right)_+ - \left(\frac{\ln(1-\lambda z)}{1-z}\right)_+ + \frac{1}{4}\left(\frac{1-z}{(1-\lambda z)^2}\right)_+ + \frac{1}{(1-z)_+}\left[\ln\frac{Q^2+m^2}{\mu_F^2} - 1\right]\right\}$$

$$\lambda = \frac{Q^2}{Q^2+m^2} \leq 1$$

$$z = \frac{\hat{x}}{\lambda} = \frac{Q^2+m^2}{2p\cdot q}, \quad 0 \leq z \leq 1$$

z – related to the partonic equivalent of the Bjorken variable.

However: the z->1 behavior depends very strongly on the value of the mass m (through λ).

Since z->1 effects become important also for moderate values of z (0.6 – 0.8), we divide the mass range into:

massive case: m/Q~1 , i.e. λ<<1,

massless case: m/Q<<1 , i.e. λ ≈1.

Soft gluon NLL resummation

We follow the standard methods to evaluate the Sudakov factor:

$$\ln \Delta_N^{\overline{\rm MS}} = \\ = \int_0^1 dz \frac{z^{N-1}-1}{1-z} \left\{ \int_{\mu_F^2}^{K^2(z,m^2)} \frac{dq^2}{q^2} A\left[\alpha_S(q^2)\right] + S\left[\alpha_S(K^2(z,m^2))\right] \right\}.$$

The Sudakov factor has very simple meaning:

It approximates the coefficient function in the soft limit z->1.

Where: $K^2(z,m^2)$ - maximum k_T for the radiated gluon.
It is smooth function of z and m^2:

$$K^2(z,m^2) \cong \begin{cases} Q^2(1-z) & \text{when } m^2 = 0, \\ \mathcal{M}^2(1-z)^2 & \text{when } m^2 \sim Q^2, \end{cases}$$

and $\mathcal{M}^2 = m^2(1+Q^2/m^2)^2$.

The function A[…] is now known to three loops (NNLL) from the three loop anomalous dimensions:

S. Moch, J. Vermaseren, and A. Vogt (2004).

S[…] - soft radiation from a massive particle.

Relation to Other Processes. New results.

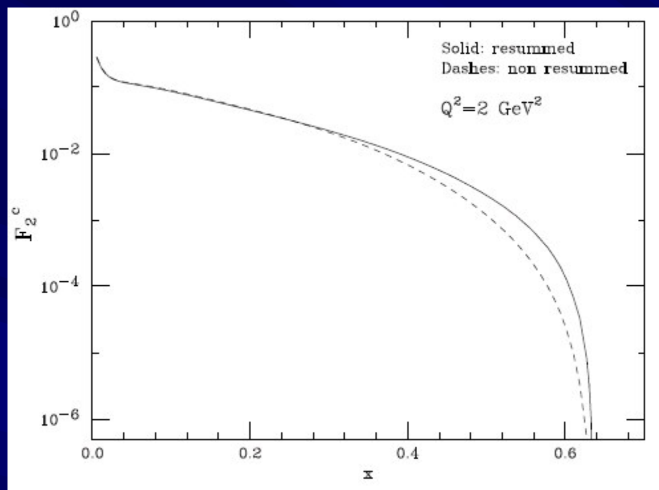
The Sudakov factor …

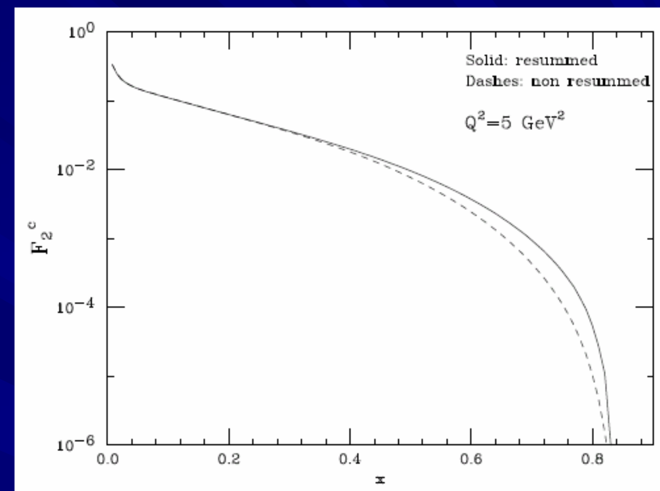
1) coincides with the one for b-fragmentation in top decay up to identification of scales (that is not a coincidence !).
2) can be presently evaluated with NNLL accuracy.

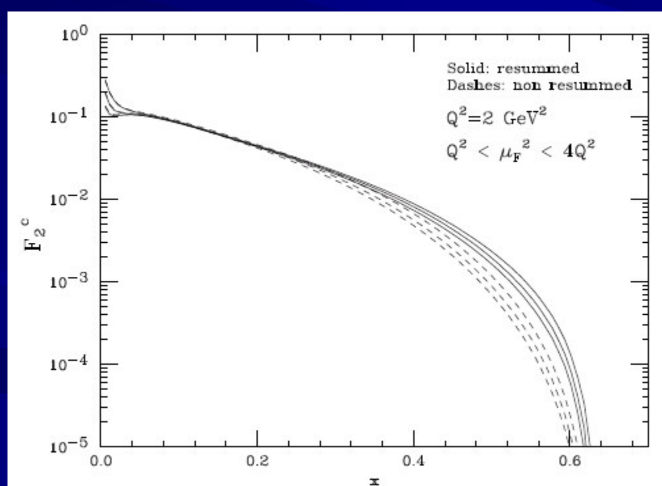
a) The function S[…] is known to \alpha_s^2 from a calculation of the heavy quark fragmentation function (K. Melnikov, A.M. (2004)) (my talk tomorrow…) ,
b) Was then applied by Gardi and Andersen (2005) to the resummation of the photon spectrum in b-> s+ gamma ,
c) We have checked their result with explicit calculation of the photon spectrum at NNLO.

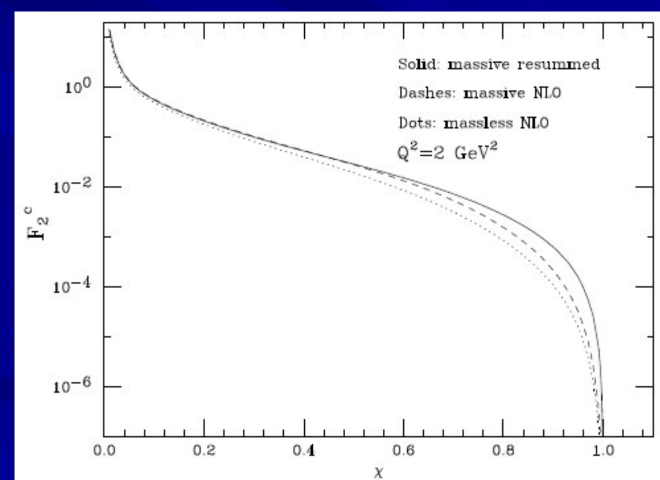
Phenomenological Results for Charm Production in CC DIS at low Q^2.

Results for the structure function F_2(x) in neutrino scattering.









Should be taken into account in the fits of pdf's when data is available. Talk by G. Corcella.

The NC DIS case.

At low Q^2 (i.e. close to the heavy quark threshold) only the gluon splitting is important. However, it is not soft-divergent.

Therefore, no need for the large-x resummation we performed for the CC case (true only for the massive case !).

However, there are other "kinematical" limits where large logs appear. To expose them one has to consider more differential observables.

One-particle inclusive differential cross-sections:

$$g+\gamma^* \to H + X(H) .$$

Being differential w/r to H one can impose constraints on its kinematics:

a) $q_T/Q \to 0$ (Nadolsky et al.) ,

b) $m_X^2 - m^2 \to 0$; $m_{HH}^2 - s \to 0$ (Laenen and Moch) .

In both cases, due to the constraint imposed on the kinematics only soft-gluon emissions are allowed.

Results for the NC DIS Case.

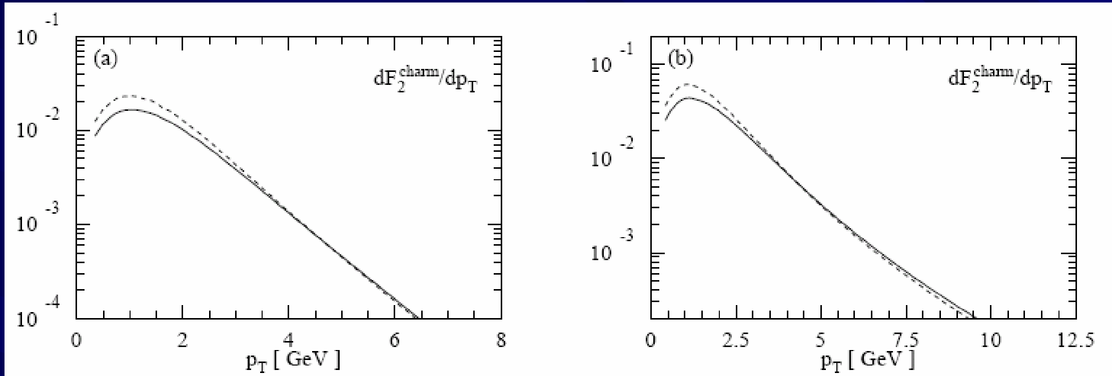


Figure 17: (a): The differential distribution $dF_2^{\mathrm{charm}}/dp_T$ as a function of p_T with the CTEQ4M gluon PDF, $x = 0.01$, $m = 1.6\,\mathrm{GeV}$, $Q^2 = 10\,\mathrm{GeV}$ and scale choice $\mu = \sqrt{Q^2 + 4(m^2 + p_T^2)}$. Plotted are: The exact NLO result (solid line) and the improved NLL approximation at NNLO (dashed line) (exact NLO result plus NLL approximate NNLO result with the damping factor $1/\sqrt{1+\eta}$). (b): Same as Fig. 17a for $x = 0.001$.

Laenen and Moch, hep-ph/9809550

Charm Production:

NLO vs. NNLO from NLL resummation:

The effect at small p_T is visible.

Nadolsky et al, hep-ph/0210082

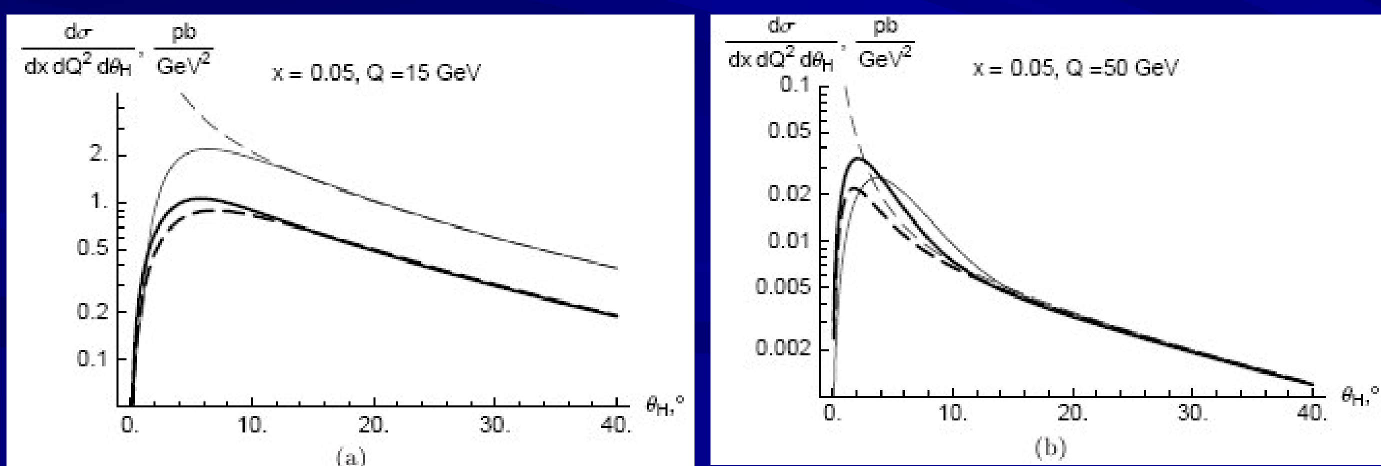


Figure 6: Comparison of the massive and massless cross sections at (a) $Q = 15$ GeV, and (b) $Q = 50$ GeV. The plots show the massive resummed cross section σ_{TOT} (thick solid line); the massless resummed cross section σ_{TOT} (thin solid line); the massive finite-order cross section σ_{FO} (thick dashed line); and the massless finite-order cross section σ_{FO} (thin dot-dashed line).

Bottom Production:

Better agreement at large Q^2 between the massive and the massless results.

Conclusions

1) I have reviewed the origin of large logs in DIS with massive quarks.
2) Of particular interest are “kinematical” logs that appear in corners of the phase space: large-x for the inclusive CC DIS and one-particle inclusive observables in the NC DIS case.
3) I discussed the interplay between the soft limit and the massive/massless transition in the CC DIS case.
4) I discussed updates on the results (in the CC case) and relation to other processes.

Numerics and Applications:

1) We demonstrated the effect of the resummation on the charm structure function for neutrinoproduction at low Q^2 (2 and 5 GeV^2). Important effect above x>0.4
2) I also presented examples for the effect of the resummation in differential observables in NC DIS.