The DIS heavy flavor coefficient functions in the Mellin space

S.Alekhin (Protvino, Russia)
J. Blümlein (DESY)

Phys. Lett. B594, 299 (2004)

The massless QCD calculations of the DIS cross sections based on the Mellin-space technique has certain advantages:

- Straightforward use of the higher-order calculations, which are commonly derived in terms of the Mellin moments
- Integro-differential evolution equations are reduced to the ordinary differential equations, which can be solved very fast numerically or even analytically in some cases
- The convolution of the PDFs with the coefficient functions are reduced to the algebraic product of the corresponding functions in the Mellin space

Account of the heavy-quarks contribution to the DIS cross section within the Mellin-technique is hampered as compared to the massless case due to the threshold behavior of the coefficient functions, which does not allow to derive analytical expressions in the Mellin space.

(Witten 76)
(Laenen-Riemersma-Smith-van Neerven 93)

The aim of this study is to develop an efficient tool allowing to expand the Mellin technique for the calculation of the massive quark production too.

The heavy-quark contribution to the DIS structure functions is

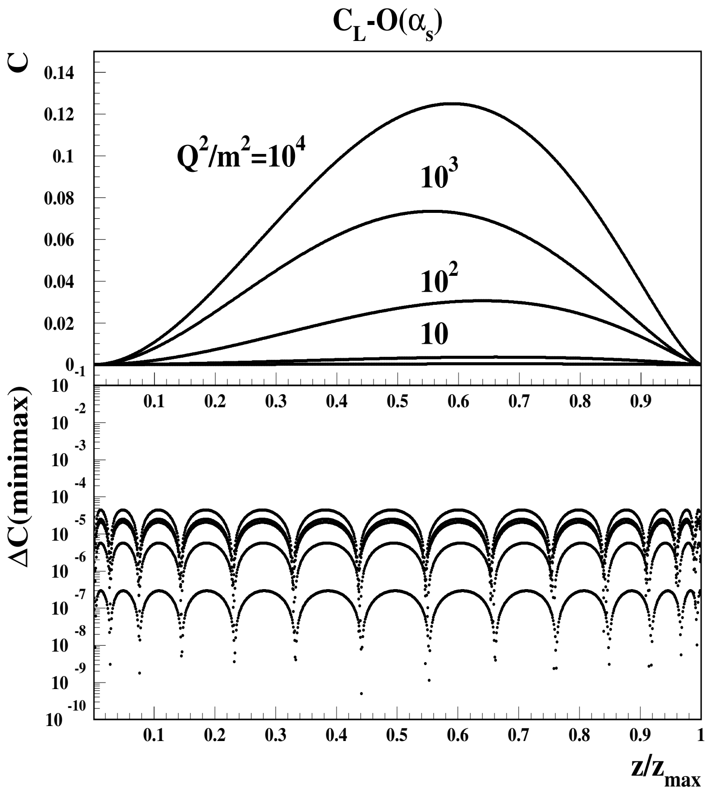
$$H_k(x, Q^2, m^2) = \frac{Q^2}{\pi m^2} e_Q^2 \int_x^{z_{\max}} \frac{dz}{z}\, h_g\left(\frac{x}{z}, \mu^2\right) c_{g,H_k}(\eta, \xi)$$

$$\xi = \frac{Q^2}{m^2},\ \eta = \frac{s}{4m^2} - 1,\ z = \frac{\xi/4}{1 + \eta + \xi/4}$$

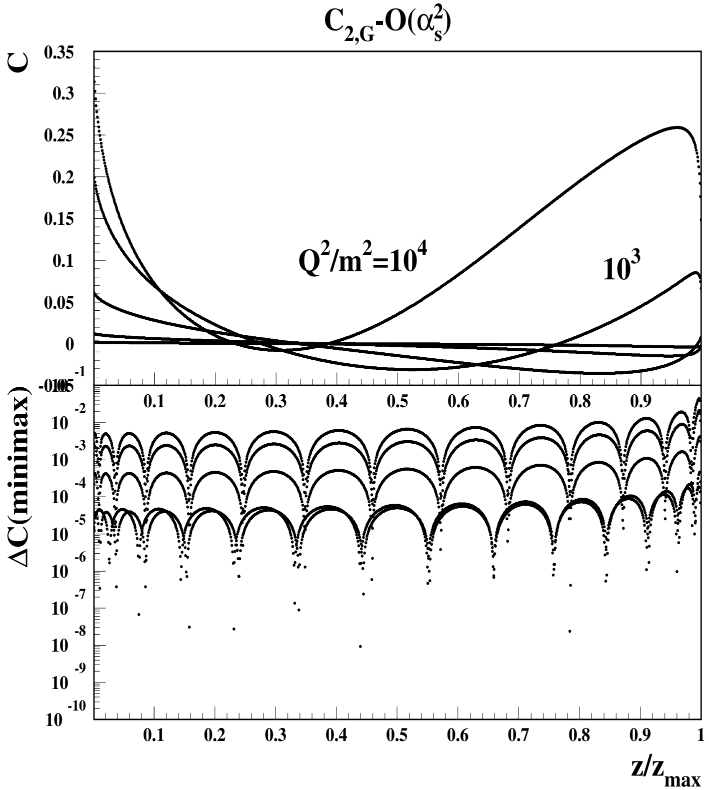
with the $O(\alpha_{\rm s})$ contributions to the coefficient function $c_{g,H_k}(\eta, \xi)$ calculated analytically or given by the $\xi - \eta$ grids.

As a first step the coefficient functions are approximated by the polynomials of z in the range $[0, z_{\max}]$ using the MINIMAX procedure, which minimizes maximal deviation of the polynomial from the approximated function. This technique was successfully used before for analytic continuation of the Mellin transforms for the harmonic sums.

(Blümlein-Kurth 99-03)



For the 15-th order polynomials the MINIMAX precision approximation for the $O(\alpha_{\rm s})$ contribution to c_{F_L} is better than 10^{-4} in the whole realistic kinematical range.



Some coefficient functions rising at large x were multiplied by additional factor of $(z - z_{\max})^k$ in order to improve precision. For the $O(\alpha_s^2)$ gluonic contribution to c_{F_2} the best value of $k = 0.4$.

	max. absolute errors of the MINIMAX-polynomials					
Wilson Coeff.	κ	$\xi = 1$	$\xi = 10$	$\xi = 10^2$	$\xi = 10^3$	$\xi = 10^4$
$c^{(0)}_{F_L,g}$	0.	2.1E-5	4.5E-5	2.5E-5	5.7E-6	2.9E-7
$c^{(0)}_{F_2,g}$	0.5	1.4E-1	8.3E-3	3.0E-3	1.0E-3	3.8E-4
$c^{(0)}_{g_1,g}$	0.	1.1E-3	6.7E-4	2.4E-4	8.3E-5	3.0E-5
$c^{(1)}_{F_L,g}$	0.	4.1E-5	5.0E-5	1.4E-5	8.9E-6	6.9E-7
$\overline{c}^{(1)}_{F_L,g}$	0.	2.3E-5	5.0E-5	1.2E-6	1.8E-6	1.5E-7
$c^{(1)}_{F_L,q}$	0.	1.4E-5	2.2E-5	4.3E-6	3.3E-7	4.4E-7
$\overline{c}^{(1)}_{F_L,q}$	0.	6.0E-7	2.1E-6	3.7E-7	3.7E-8	3.1E-8
$d^{(1)}_{F_L,q}$	0.	4.0E-6	2.6E-6	6.1E-7	1.5E-6	6.3E-7
$c^{(1)}_{F_2,g}$	0.4	5.6E-2	2.6E-2	3.9E-3	1.0E-3	8.5E-4
$\overline{c}^{(1)}_{F_2,g}$	0.	8.9E-4	5.3E-3	1.9E-3	6.6E-4	2.3E-4
$c^{(1)}_{F_2,q}$	-0.5	2.6E-3	1.2E-3	2.2E-4	2.2E-5	6.3E-6
$\overline{c}^{(1)}_{F_L,q}$	0.	3.2E-4	1.3E-4	2.2E-5	1.8E-6	7.1E-7
$d^{(1)}_{F_2,q}$	0.	1.3E-4	5.1E-5	8.1E-6	1.0E-4	5.8E-4

The above accuracies suffice for all practical applications.

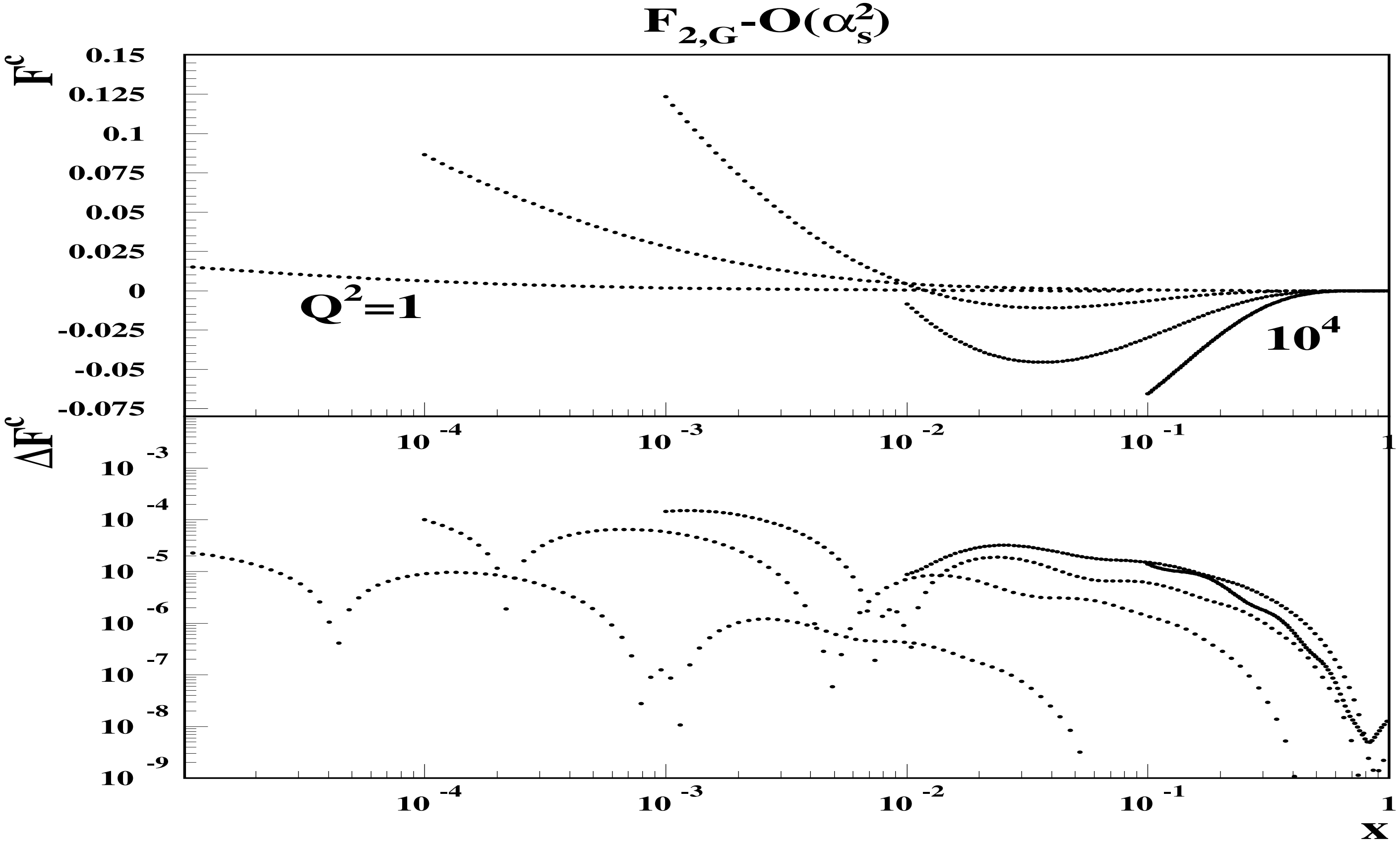
With the Mellin transform of the coefficient functions given as the polynomial of N the x-space heavy-quark structure functions are calculated using the contour integral of the product of the Mellin-space coefficient functions and analytical continuation of the PDFs moments in the complex-moments plane:

$$F_i(x) = \frac{1}{\pi}\int_0^{\infty} dz \mathsf{Im}\left[e^{i\Phi}x^{-c(z)}F(c(z))\right], \;\; c(z) = c_0 + ze^{i\Phi}, \quad \Phi \simeq (3/4)\pi \, .$$

Typical precision of the total procedure including the MINIMAX approximation of the coefficient functions and the contour integration is better than 10^{-4} for the realistic kinematics that is well below the accuracy of the available and foreseen heavy quark production data.

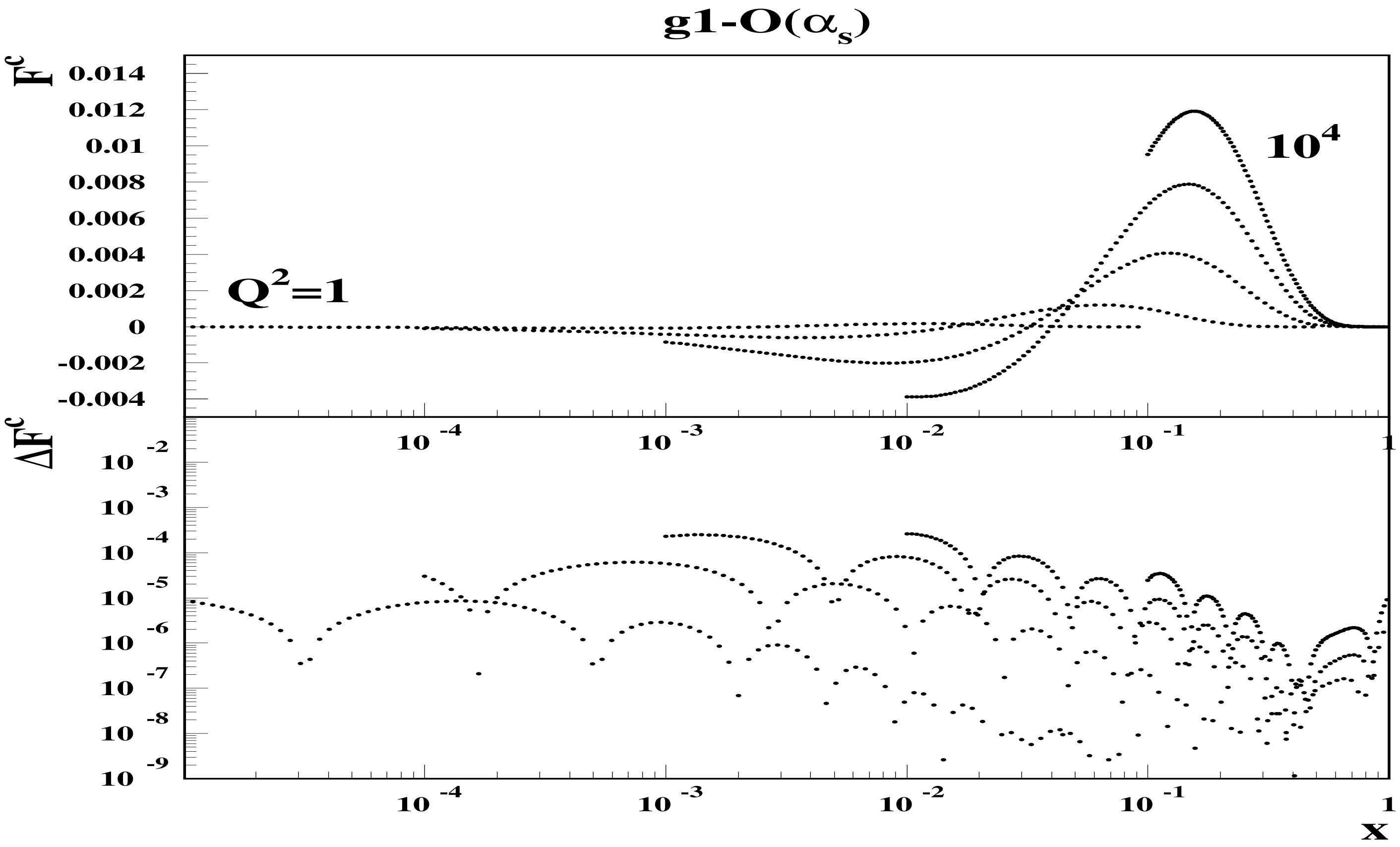
Examples of worst recovery precision

($O(\alpha_s^2)$ gluonic contribution to F_2)



Examples of worst recovery precision (cont'd)

($O(\alpha_\mathrm{s})$ contribution to g_1)



Summary

The MINIMAX procedure provides efficient tool for the semi-analytical derivation of the Mellin-space coefficient functions in the DIS heavy-quark production. With the 15-th order polynomial one can obtain the precision of recovery of the $O(\alpha_{\rm s})$ and $O(\alpha_{\rm s}^2)$ contributions to the structure functions better that 10^{-4} with the speed performance relevant for many practical applications.