(The Tension Between Unification and) A $U(1)_X$ Solution to the μ Problem of the MSSM

David Morrissey

University of Michigan

Based work done in collaboration with
James Wells
[hep-ph/0512019]

University of Wisconsin-Madison, May 15, 2006

The μ Problem of the MSSM

- The minimal supersymmetric standard model (MSSM) has only one dimensionful supersymmetric parameter, μ:

$$W_{MSSM} \supset \mu\, H_u \cdot H_d$$

where $H_u = (\mathbf{1}, \mathbf{2}, \mathbf{1/2})$ and $H_d = (\mathbf{1}, \mathbf{2}, \mathbf{-1/2})$ are the two Higgs boson doublets.

- The μ parameter should be of order $\mu \sim M_{SUSY} \lesssim 1$ TeV.

- $\mu \sim 1$ TeV is technically natural because of supersymmetry.

  But why $\mu \sim 1$ TeV $\sim M_{SUSY}$?

  Why not $\mu \sim M_{GUT}$ or M_{Pl}?

μ and the Lightest MSSM Higgs

- The LEP-II bound on a SM-like Higgs boson is

$$m_h > 114.4 \text{ GeV}.$$

  Except for some loopholes, this also applies to the lightest MSSM Higgs.

- At tree-level in the MSSM,

$$m_h < M_Z \cos 2\beta.$$

- Loop corrections can increase the mass above the experimental limit for $M_{SUSY},\ \mu \gtrsim 1$ TeV.

- However, these parameters are related to M_Z by

$$M_Z^2 \sim \mu^2 - M_{SUSY}^2.$$

  $\Rightarrow$ small fine-tuning problem within the MSSM.

A Singlet Solution

- Both of these difficulties can be solved by replacing the μ term by a new field, S, that develops a vacuum expectation value (VEV),

$$\mu\, H_u \cdot H_d \rightarrow \lambda\, S\, H_u \cdot H_d.$$

- Since $\langle S \rangle$ is determined by M_{SUSY}, this relates μ to the scale of supersymmetry breaking.

- This replacement leads to a new F contribution to the Higgs mass. The tree-level upper bound becomes

$$m_h^2 \leq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta.$$

- The coupling λ runs large in the UV.

  $\lambda(M_Z) \lesssim 0.7$ to avoid a Landau pole below M_{GUT}.

- Even so, the tree-level bound can exceed 110 GeV, removing the need for large loop corrections.

But...

- The new field S should be a singlet under $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$.
- However S should be charged under a new symmetry to forbid the usual μ term as well as new mass terms.
- An uncharged singlet may also develop a large VEV from loops. This can destabilize the gauge hierarchy. [Bagger+Poppitz '93]
- Global symmetries are problematic:
  - Discrete symmetries are plagued by domain walls.
  - Continuous global symmetries lead to axions.

A $U(1)_X$ Resolution

- A safer choice is to protect S with a new $U(1)_X$ gauge symmetry.
- In this case the Goldstone mode is eaten to form a massive Z'.
- There is also an extra D-term contribution to the Higgs mass bound

$$m_h^2 \leq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta + 2g_x^2 v^2 (h_u \cos^2 \beta + h_d \sin^2 \beta)^2,$$

  where h_u and h_d are the $U(1)_X$ Higgs charges.
- We will focus on this possibility.

A Tension with Unification?

- One of the attractive features of the MSSM is that it is consistent with gauge unification.
- For the $U(1)_X$ gauge theory to be consistent, we must assign charges in an anomaly free way.
- In general, this requires new matter fields.
- Adding new matter to the MSSM can disrupt unification.
- Is it possible to solve the μ problem with a gauged $U(1)_X$ and preserve gauge unification?

Assumptions

1. All the terms in the MSSM superpotential (except the μ term) appear in the superpotential of the new model.
   The usual μ term is forbidden by the $U(1)_X$ gauge symmetry, and is replaced with $\lambda\, S\, H_u \cdot H_d$.

2. $U(1)_X$ charges are family-universal.
   This avoids new flavour-changing effects.

3. The exotic matter needed to cancel $U(1)_X$ anomalies consists of G_{SM} singlets, or has the G_{SM} quantum numbers of complete $SU(5)$ multiplets.
   This is the simplest way to preserve gauge unification.

Charges

- The terms in the superpotential are

$$W = y_u\, Q\cdot H_u\, U^c - y_d\, Q\cdot H_d\, D^c - y_e\, L\cdot H_d\, E^c$$
$$+ \lambda\, S\, H_u\cdot H_d + (exotics).$$

- The $U(1)_X$ charges of these fields must satisfy

$$q + u + h_u = 0 \qquad q + d + h_d = 0$$
$$l + e + h_d = 0 \qquad s + h_u + h_d = 0.$$

- The action of any $U(1)_X$ on the MSSM fields is determined by the values of the seven $U(1)_X$ charges $q,\ u,\ d,\ l,\ e,\ h_u,\ h_d$.

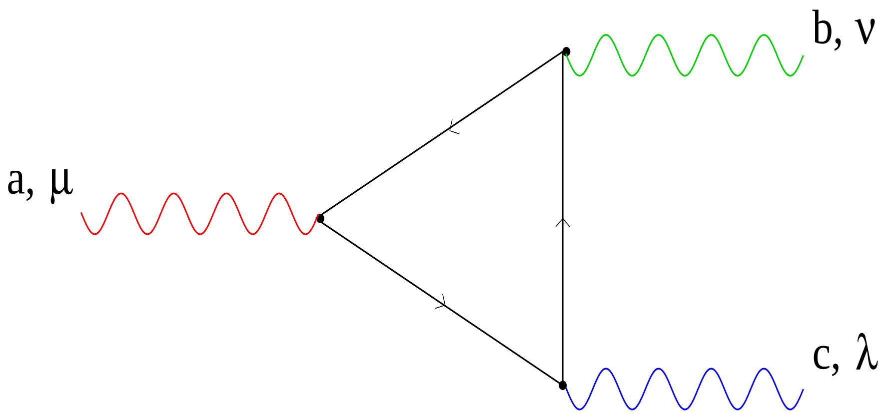
- Gauge invariance implies three independent conditions on them.
  $\Rightarrow$ we can decompose any $U(1)_X$ into four basis $U(1)$'s.

- A convenient set is: $U(1)_Y$, $U(1)_{B+L}$, $U(1)_\psi$, $U(1)_\chi$.

  ($E_6 \to SO(10) \times U(1)_\psi \to SU(5) \times U(1)_\chi \times U(1)_\psi$)

Gauge Anomalies

- For gauge anomaly cancellation, it is sufficient for the triple gauge boson triangle diagrams to vanish:



$$\propto \quad \Sigma_r\, tr\left(t_r^a \left\{t_r^b, t_r^c\right\}\right)$$

- Conditions:

$SU(3)_c^2\, U(1)_X$ $\quad$ $SU(2)_L^2\, U(1)_X$

$U(1)_Y^2\, U(1)_X$ $\quad$ $U(1)_Y\, U(1)_X^2$

$U(1)_X^3$ $\quad$ $U(1)_X\,(gravity)^2$

- The last two conditions can be satisfied by adding only G_{SM} singlets.

G_{SM} Exotics Required

- Suppose all exotics are G_{SM} singlets.
- The $SU(3)_c^2 \times U(1)_X$ anomaly condition is then

$$0 = n_g(2q + u + d)$$

- Gauge invariance of $y_u\, Q\cdot H_u\, U^c$ and $y_d\, Q\cdot H_d\, D^c$ implies

$$2q + u + d + (h_u + h_d) = 0.$$

- Together, these conditions imply $h_u + h_d = 0$,
  so the $U(1)_X$ symmetry does not protect the μ term.
- New coloured particles are needed to avoid this conclusion.

Gauge Unification

- The gauge couplings of the MSSM unify near $M_{GUT} \simeq 2 \times 10^{16}$ GeV.

  The MSSM matter fields fill out complete $SU(5)$ multiplets,

$$\begin{aligned} \overline{\mathbf{5}} &= (D^c, L) = (\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}) \\ \mathbf{10} &= (Q, U^c, E^c) = (\mathbf{3}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{6}}) \oplus (\overline{\mathbf{3}}, \mathbf{1}, -\frac{\mathbf{2}}{\mathbf{3}}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) \end{aligned}$$

- Adding new coloured particles can disrupt these nice features.

- Unification will be preserved if the new coloured particles are members of complete $SU(5)$ multiplets.

Non-Universal Charges Required!

- The most natural possibility is for the components of the exotic multiplets to all have the same $U(1)_X$ charge. (*i.e.* $SU(5) \times U(1)_X$)

- The linear mixed anomaly conditions become

$$\begin{aligned} SU(3)_c^2 \times U(1)_X : &\quad 0 = n_g(2q + u + d) + M \\ SU(2)_L^2 \times U(1)_X : &\quad 0 = n_g(3q + l) + (h_u + h_d) + M \\ U(1)_Y^2 \times U(1)_X : &\quad 0 = n_g(q + 8u + 2d + 3l + 6e) + 3(h_u + h_d) + 5M \end{aligned}$$

where $M = \sum_a (5_a + \overline{5}_a) + 3 \sum_b (10_b + \overline{10}_b) + \ldots$

- These three linear conditions, together with gauge invariance, form a degenerate system with $(h_u + h_d) = 0$.

$\Rightarrow$ universal $SU(5)$ multiplets can't protect the μ term.

Options

- Maybe the μ term isn't so bad after all? [Giudice+ Masiero '88]

- EMSSM: $\mathbf{27} \oplus \mathbf{27} \oplus \mathbf{27} \oplus (\mathbf{1}, \mathbf{2}, 1/2) \oplus (\mathbf{1}, \mathbf{2}, -1/2)$ [King,Moretti,Nevzorov '05]

  ($\mathbf{27} \in E_6$)
  This is anomaly free and unifies, but has a μ-like problem

  for the extra pair of doublets.

- Non-family universal charge assignments.
  [Demir,Kane,Wang '05; Erler,Langacker,Li '02; . . .]
  FCNC's may be an issue.

- Consider instead adding a single "$\mathbf{5} \oplus \overline{\mathbf{5}}$" with non-universal charges within the multiplets:

$$\mathbf{5} = (\mathbf{1}, \mathbf{2}, 1/2, L) \oplus (\mathbf{3}, \mathbf{1}, -1/3, D)$$
$$\overline{\mathbf{5}} = (\mathbf{1}, \mathbf{2}, -1/2, \overline{L}) \oplus (\overline{\mathbf{3}}, \mathbf{1}, 1/3, \overline{D})$$

with $D \neq L$, $\overline{D} \neq \overline{L}$.

An Example with Non-Universal Charges

- Adding a single "$\mathbf{5} \oplus \overline{\mathbf{5}}$" of exotics and some extra G_{SM} singlets, it is possible to protect the μ term with the $U(1)_X$ while cancelling anomalies and preserving gauge unification.

- For example, imposing $SO(10)$ relations on the MSSM matter charges, $q = u = e = d = l$, the solution to the anomaly equations is

$$\begin{aligned} q &= -(\bar{L} + L)/4(n_g - 1) \\ (h_u + h_d) &= (\bar{L} + L)/(n_g - 1) \\ (h_u - h_d) &= 0 \\ (\bar{D} + D) &= n_g(\bar{L} + L)/(n_g - 1) \\ (\bar{D} - D) &= -(n_g - 1)(\bar{L} - L)/n_g. \end{aligned}$$

  Several G_{SM} singlets with non-zero $U(1)_X$ charges are needed as well.

- The only allowed $U(1)_X$ in this case is $U(1)_\psi \in E_6$ (on MSSM fields). For $SU(5)$ compatible charges, the only possible $U(1)_X$ is a combination of the $U(1)_\psi$ and $U(1)_\chi$ subgroups of E_6.

A GUT Interpretation?

- For the non-universal exotic multiplets we have considered, it is difficult to embed the model simply in a GUT.

- We must think of the exotic "multiplets" as coming from distinct split GUT representations,

$$\overline{\mathbf{5}}_a \oplus \overline{\mathbf{5}}_b = (D^c_a, L_a)_{Q_a} \oplus (D^c_b, L_b)_{Q_b} \to (D^c_a, L_b) = \text{non-universal ``}\overline{\mathbf{5}}\text{''}.$$

- This can arise in an extra-dimensional scenario with $SU(5)$-breaking orbifold boundary conditions. [Hall+Nomura '01]

- However, with split multiplets, the unification of gauge couplings with only MSSM matter would appear to be a lucky accident.

Summary

- Adding a $U(1)_X$ gauge symmetry to the MSSM can solve the μ problem and increase the lightest Higgs boson mass;

$$\mu\, H_u \cdot H_d \longrightarrow \lambda\, S\, H_u \cdot H_d.$$

- New coloured exotic matter is needed to forbid the original μ term and cancel $U(1)_X$ anomalies.

- If the new matter has the form of complete GUT multiplets, in order to preserve gauge unification, at least one of the multiplets must have non-universal $U(1)_X$ charges.

- A single non-universally charged "$\mathbf{5} \oplus \overline{\mathbf{5}}$" (and some singlets) is sufficient to cancel anomalies and protect the μ term.

- The need for non-universal charges makes it difficult to embed any such model simply in a GUT, and implies some degree of tension with unification.

Singlet Tadpoles

- Singlet tadpole supergraphs can be quadratically divergent.
- With supersymmetry breaking insertions, these can generate effective superpotential and bosonic potential operators.
  For $X = M_{\mathsf{Pl}}(1 + \tilde{m}\theta^2 + \tilde{m}^*\overline{\theta}^2 + |\tilde{m}|^2\theta^4)$,

$$\int d^4\theta \, \frac{X}{M_{\mathsf{Pl}}^2}\Lambda^2 \, \widehat{S} \longrightarrow \int d^2\theta \, \frac{\tilde{m}^*\Lambda^2}{M_{\mathsf{Pl}}} \, \widehat{S} + \frac{|\tilde{m}|^2\Lambda^2}{M_{\mathsf{Pl}}} \, S$$

- For $\Lambda \sim M_{\mathsf{Pl}}$, the expectation value of S tends to be much larger than $\tilde{m} \sim \mathsf{TeV}$.

$U(1)_x$ Decomposition

- A convenient basis is $U(1)_Y$, $U(1)_{B+L}$, and the $U(1)_\psi$ and $U(1)_\chi$ subgroups of E_6.

- If we define $U(1)_X$ by the charges $\{d, e, h_u, h_d\}$, the charge of a field ϕ_i is

$$\begin{aligned} Q_x^i \;=\; & \frac{2}{5}(-3d + e + 2h_u - 3h_d)\, Q_Y^i \\ & + \frac{1}{2}(-3d - e + h_u - 3h_d)\, Q_{B+L}^i \\ & - \frac{1}{4}(h_u + h_d) 2\sqrt{6}\, Q_\psi^i \\ & + \frac{1}{20}(6d - 2e + h_u + h_d) 2\sqrt{10}\, Q_\chi^i . \end{aligned}$$

- The operator $H_u \cdot H_d$ is only charged under $U(1)_\psi$.

- With MSSM fields alone, only $U(1)_Y$ and the $U(1)_{B-L}$ combination of $U(1)_\chi$ and $U(1)_{B+L}$ is anomaly free.

- E_6 Charges

$\mathbf{27}$	G_{SM}	$2\sqrt{6}\,U(1)_\psi$	$2\sqrt{10}\,U(1)_\chi$
Q	$(\mathbf{3},\mathbf{2},1/6)$	1	-1
L	$(\mathbf{1},\mathbf{2},-1/2)$	1	3
U^c	$(\bar{\mathbf{3}},\mathbf{1},-2/3)$	1	-1
D^c	$(\bar{\mathbf{3}},\mathbf{1},1/3)$	1	3
E^c	$(\mathbf{1},\mathbf{1},1)$	1	-1
N^c	$(\mathbf{1},\mathbf{1},0)$	1	-5
H	$(\mathbf{1},\mathbf{2},-1/2)$	-2	-2
P^c	$(\bar{\mathbf{3}},\mathbf{1},1/3)$	-2	-2
H^c	$(\mathbf{1},\mathbf{2},1/2)$	-2	2
P	$(\mathbf{3},\mathbf{1},-1/3)$	-2	2
S	$(\mathbf{1},\mathbf{1},0)$	4	0

- $E_6 \to SO(10)\times U(1)_\psi \to SU(5)\times U(1)_\chi \times U(1)_\psi$.

$$\mathbf{27} = \begin{cases} \mathbf{16}\oplus\mathbf{10}\oplus\mathbf{1}, & \text{under } SO(10) \\ \overline{\mathbf{5}}\oplus\mathbf{10}\oplus\mathbf{1}\oplus\mathbf{5}\oplus\overline{\mathbf{5}}\oplus\mathbf{1}, & \text{under } SU(5) \end{cases}$$