Naturalness of the vacuum energy in holographic theories

- Cosmological constant:

  not a fine-tuning problem!
- Energy-entropy duality and

  holographic quantum statistics
- Not solved, yet ...

C.Balázs and I. Szapudi hep-th/0603133

hep-th/0605nnn

hep-th/0608nnn

The "cosmological constant" problem

Quantum theory

$$\rho = \frac{1}{(2\pi)^3} \int_0^{M_P} \frac{E}{2}\, 4\pi E^2\, dE = \frac{1}{16\pi^2} M_P{}^4$$

Observation (gravity)

$$\rho = \frac{3}{8\pi} H^2 M_P{}^2 = 1.84 \times 10^{-123} M_P{}^4$$

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The vacuum energy problem

Typical solution

Something forbids/cancels/fine-tunes the vacuum energy

Something else (re-)generates a tiny vacuum energy

???

The vacuum energy problem

Holographic solution

Quantum fluctuations generate vacuum energy

Gravity limits the amount of quantum fluctuations arXiv:hep-th/9409089 v2 28 Sep 1994

The World as a Hologram

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ABSTRACT

According to 't Hooft the combination of quantum mechanics and gravity requires the three dimensional world to be an image of data that can be stored on a two dimensional projection much like a holographic image.

The two dimensional description only requires one discrete degree of freedom per Planck area and yet it is rich enough to describe all three dimensional phenomena. After outlining 't Hooft's proposal I give a preliminary informal description of how it may be implemented. One finds a basic requirement that particles must grow in size as their momenta are increased far above the Planck scale. The consequences for high energy particle collisions are described.

The phenomena of particle growth with momentum was previously discussed in the context of string theory and was related to information spreading near black hole horizons. The considerations of this paper indicate that the effect is much more rapid at all but the earliest times. In fact the rate of spreading is found to saturate the bound from causality.

The holographic screen

Event horizon

$$R = 1/H$$

Horizon area of observable Universe

$$A = 4\pi R^2$$

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Entropy limit

Cosmic holography conjecture

$$S \leq \pi R^2$$

Degrees of freedom are limited by the area of the horizon

Temperature of the horizon

Gibbons-Hawking

$$T = \frac{1}{2\pi R}$$

Temperature of an adiabatically expanding system

Holographic solution of the vacuum energy problem

Holographic estimate of the vacuum energy density

$$\rho = \frac{E}{V} = \frac{S\,T}{V} \leq \frac{\pi R^2 \, \frac{1}{2\pi R}}{\frac{4\pi R^3}{3}} = \frac{3}{8\pi}\,\frac{1}{R^2}$$

Exactly the same as the gravitational prediction

$$\rho = \frac{3}{8\pi}\, H^2$$

Gravity is holographic

Schwarzschild black hole

$$E = \frac{R}{2}$$

$$S = \pi R^2$$

$$\rho = \frac{E}{V} = \frac{\frac{R}{2}}{\frac{4\pi R^3}{3}} = \frac{3}{8\pi}\frac{1}{R^2}$$

Gravity must also restrict

$$\rho = \frac{1}{4\pi^2}\int_0^\infty E^3 \, \mathrm{d}E$$

Holographic quantum theory

Gravity → holographic entropy limit

$$S \leq \pi R^2$$

$$\text{if} \quad T = \frac{1}{2\pi R} \quad \text{then} \quad S \leq \frac{1}{4\pi T^2}$$

Quantum system

$$\lim_{T \to 0} S(T) = 0$$

Conflict between holography and the 3rd law!

Energy-Entropy duality

$$\text{If} \quad S = \frac{1}{4\pi T^2} \quad \text{then} \quad \lim_{\frac{1}{T}\to 0} S\left(\frac{1}{T}\right) = 0$$

$$T_D = \frac{1}{T}$$

$$dE = T\,dS = \frac{1}{T_D} dS$$

$$E_D = S, \quad S_D = E$$

$$\boxed{dS_D = \frac{1}{T_D}\, dE_D}$$

Dual thermodynamics

1st law

$$dE = T\,dS - p\,dV + \mu\,dN$$

$$dS_D = \frac{1}{T_D}\,dE_D + \frac{p_D}{T_D}\,dV_D - \frac{\mu_D}{T_D}\,dN_D$$

2nd law

$$dS_D = T\,dS - p\,dV + \mu\,dN \geq 0$$

$$\text{if} \qquad -p\,dV + \mu\,dN \geq 0$$

3rd law

$$\lim_{T_D \to 0} S_D(T_D) = \lim_{\frac{1}{T} \to 0} E\left(\frac{1}{T}\right)$$

$$\text{for a black hole: } E = \frac{1}{8\pi T} \xrightarrow[\frac{1}{T} \to 0]{} 0$$

What does Energy-Entropy duality mean?

$$T = \frac{1}{4\pi R} = \frac{1}{T_D}$$

$$R_D = \frac{1}{4\pi T_D} = \frac{1}{R}$$

T-duality

$$R \leftrightarrow \frac{1}{R}$$

AdS/CFT

black hole $\leftrightarrow$ quantum gas

It from bit!

$$E_D = S, \quad E = S_D$$

Holographic quantum statistics

One dimensional Fermi (Bose) gas

$$E_D = p_D V_D = \frac{V_D}{2\pi} \int_0^\infty f(\epsilon, T_D, \mu_D)\, \epsilon\, d\epsilon$$

$$N_D = \frac{V_D}{2\pi} \int_0^\infty f(\epsilon, T_D, \mu_D)\, d\epsilon$$

$$f(\epsilon) = \left(e^{(\epsilon-\mu_D)/T_D} \pm 1\right)^{-1}$$

$$S_D = \frac{1}{T_D} E_D + \frac{p_D}{T_D} V_D - \frac{\mu_D}{T_D} N_D$$

Quantum effects small if

$$\frac{\mu_D}{T_D} \ll 1$$

Holographic quantum statistics

One dimensional Fermi gas

$$E_D = p_D\,V_D = \frac{4\,\pi^2\,V_D}{3}\left(\frac{1}{16\,\pi\,T^2} - \frac{3\log(2)}{4\,\pi^3\,T^2}\,\mu + O(\mu^2)\right)$$

$$S_D = \frac{4\,\pi^2\,V_D}{3}\left(\frac{1}{8\,\pi\,T} - \frac{3\log(2)}{4\,\pi^3\,T}\,\mu + O(\mu^3)\right)$$

$$N_D = \frac{4\,\pi^2\,V_D}{3}\left(\frac{3\log(2)}{4\,\pi^3\,T} - \frac{3}{8\,\pi^3\,T}\,\mu + O(\mu^2)\right)$$

Quantum effects small if

$$\mu \ll 1$$

Holographic quantum model

Application for black holes

$$E = S_D = \frac{R}{2} + O\left(\frac{1}{R}\right)$$

$$\text{if} \quad V_D = \frac{3}{4\pi^2}$$

$$S = E_D = \pi R^2 - \frac{3}{2}\log(\pi R^2) + O\left(\frac{1}{R}\right)$$

$$\text{if} \quad \mu = \frac{\pi}{16 M^2}$$

$$N = N_D = \frac{6\log(2)}{\pi^2} M$$

2^{nd} law of dual thermodynamics holds

$$dV_D = 0 \quad \text{and} \quad \mu_D \, dN_D \leq 0$$

Holographic quantum model

Reproduced black hole properties

$$E \sim R \sim \frac{1}{T}, \quad dE = T\,dS, \quad E = 2\,T\,S \quad \text{for } \mu = 0$$

$$S = \pi R^2 - \frac{3}{2}\log(\pi R^2) + O\left(\frac{1}{R}\right)$$

$$N = \frac{6\log(2)}{\pi^2}\, M/M_P$$

$$\dim(V_D) = 1 \quad \text{and} \quad V_D = \frac{3}{4\pi^2}\, L_P$$

entropy cutoff $\leftrightarrow$ dual Fermi/Bose distribution

energy cutoff $\leftrightarrow$ dual Fermi/Bose distribution

Conclusions

Degrees of freedom are limited by the horizon →
vacuum energy density small

Vacuum energy density never reaches large values →
the worst fine-tuning problem in the history of science
is *not* a fine-tuning problem!

There is an extra dimension and we live in it

The physical DoF are mapped from/to a holographic screen
defined by the event horizon