Minimal Trinification

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Based on work with S. Wiesenfeldt and S. Willenbrock
(appearing in Phys. Rev. D73)

Earlier work by Achiman, Stech 1979; de Rújula, Georgi, Glashow 1984;
Babu, He, Pakvasa 1986.

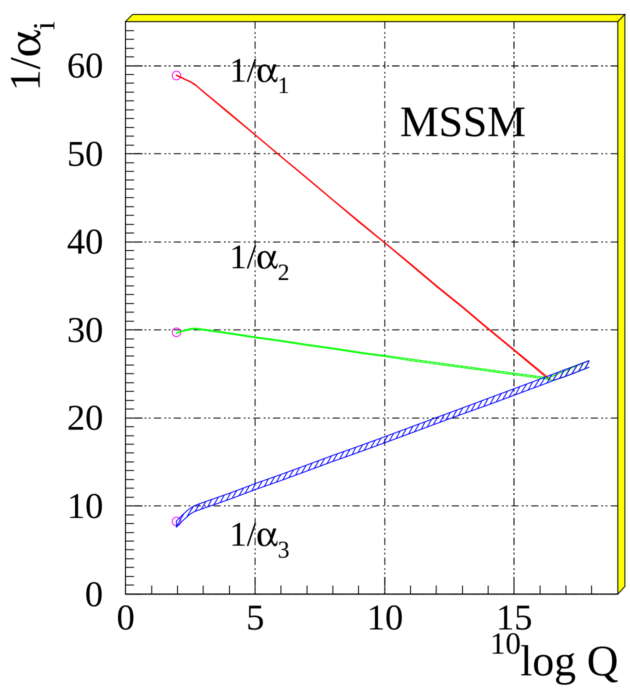
Motivation

SM gauge couplings converge at a high scale M but below M_{Pl}.

$\rightarrow$ Embed SM in a more fundamental theory (Grand Unified Theory);

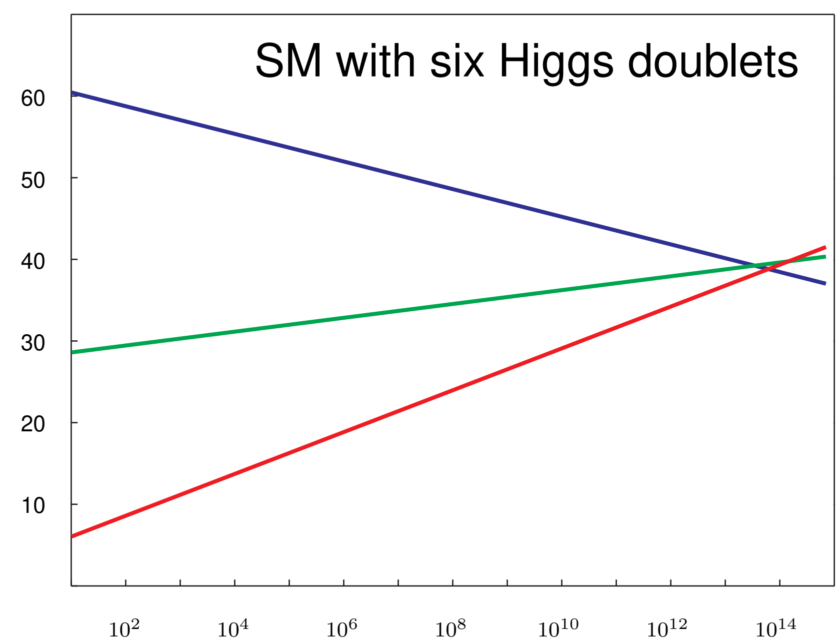
- order of M accounts for small neutrino masses and non-detection of proton decay.

[Amaldi, de Boer, Fürstenau 1991]



$\rightarrow$ SUSY SU(5)

[Willenbrock 2003]



unification without low-energy SUSY

Non-supersymmetric GUTs

SUSY GUTs provide a beautiful framework, low-energy supersymmetry controls the gauge hierarchy.

Nevertheless, unless we focus on SU(5), we do not need low-energy SUSY for unification, and neutrino masses point at groups beyond SU(5).

$\rightarrow$ Models without low-energy SUSY are viable alternatives. (cf. split SUSY)

They require at least five Higgs doublets. [Adler 1998; Willenbrock 2003]

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Trinified model $G_{TR} = SU(3)_C \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3$

- rank 6, subgroup of E_6;
- $\mathbb{Z}_3$ guarantess that gauge couplings coincide at M_u;
- includes five Higgs doublets !

Is the non-supersymmetric trinified model a viable candidate for a more fundamental theory at $M_u \sim 10^{14}$ GeV ?

Trinified Model

Fermions: $(3,3^*,1)\oplus(3^*,1,3)\oplus(1,3,3^*)\equiv\psi_Q\oplus\psi_{Q^c}\oplus\psi_L$

$$\psi_Q=\begin{pmatrix}-d & u & B\end{pmatrix}, \qquad \psi_{Q^c}=\begin{pmatrix}\mathscr{D}^c\\ u^c\\ \mathscr{B}^c\end{pmatrix}, \qquad \psi_L=\begin{pmatrix}(\mathscr{E}) & (E^c) & (\mathscr{L})\\ \mathscr{N}_1 & e^c & \mathscr{N}_2\end{pmatrix}$$

In addition to the 15 SM fermions, there are 12 new fermions,

$$B\,(3,1,-\tfrac{1}{3})\,,\qquad \mathscr{B}^c\,(3^*,1,\tfrac{1}{3})\,,\qquad E^c\,(1,2,\tfrac{1}{2})\,,\qquad \mathscr{E}\,(1,2,-\tfrac{1}{2})\,,\qquad \underbrace{\mathscr{N}_{1,2}\,(1,1,0)}_{\textit{Neutrettos}}$$

$\longrightarrow$ Mixing between $\mathscr{D}^c$ and $\mathscr{B}^c$ as well as $\mathscr{L}$ and $\mathscr{E}$.

Light eigenstates: $d^c=-s_\alpha\,\mathscr{D}^c+c_\alpha\,\mathscr{B}^c$, $L=\ c_\beta\,\mathscr{E}+s_\beta\,\mathscr{L}$;

Heavy eigenstates: $B^c=\ c_\alpha\,\mathscr{D}^c+s_\alpha\,\mathscr{B}^c$, $E=-s_\beta\,\mathscr{E}+c_\beta\,\mathscr{L}$ and E^c.

Similarly, $\mathscr{N}_1$ and $\mathscr{N}_2$ mix to $N_1=s_\beta\,\mathscr{N}_1-c_\beta\,\mathscr{N}_2$ and $N_2=-c_\beta\,\mathscr{N}_1-s_\beta\,\mathscr{N}_2$.

Breaking of G_{TR}

Breaking to G_{SM} by a pair of $\Phi_L\,(1,3,3^*) = \begin{pmatrix} (\phi_1) & (\phi_2) & (\phi_3) \\ S_1 & S_2 & S_3 \end{pmatrix}$ with

$$\langle \Phi_L^1 \rangle = \begin{pmatrix} \begin{pmatrix} \\ \\ \end{pmatrix} & \begin{pmatrix} \\ \\ \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ 0 & 0 & v_1 \end{pmatrix} \text{ and } \langle \Phi_L^2 \rangle = \begin{pmatrix} \begin{pmatrix} \\ \\ \end{pmatrix} & \begin{pmatrix} \\ \\ \end{pmatrix} & \begin{pmatrix} \\ \\ \end{pmatrix} \\ v_2 & 0 & v_3 \end{pmatrix}$$

Yukawa couplings:

$$Y = \psi_{Q^c}\,\psi_Q\,\left(g_1\,\Phi_L^1 + g_2\,\Phi_L^2\right) + \tfrac{1}{2}\,\psi_L\,\psi_L\,\left(h_1\,\Phi_L^1 + h_2\,\Phi_L^2\right)$$

v_1 and v_2 break G_{TR} to different $\mathrm{SU(3)}_{\mathrm{C}} \times \mathrm{SU(2)}_{\mathrm{L}} \times \mathrm{SU(2)}_{\mathrm{R}} \times \mathrm{U(1)}$,
B^c, E^c and $\mathscr{E}$ become heavy.

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$$\langle \Phi^1_{\mathrm{c}} \rangle = \begin{pmatrix} \begin{pmatrix} u_1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ u_2 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ 0 & 0 & v_1 \end{pmatrix} \text{ and } \langle \Phi^2_{\mathrm{c}} \rangle = \begin{pmatrix} \begin{pmatrix} n_1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ n_2 \end{pmatrix} & \begin{pmatrix} n_3 \\ 0 \end{pmatrix} \\ v_2 & 0 & v_3 \end{pmatrix}$$

Yukawa couplings:

For simplicity, we choose $v_3 = n_{1,2,3} = 0$ here.

$$Y = \psi_{Q^c}\,\psi_Q\,\left(g_1\,\Phi^1_L + g_2\,\Phi^2_L\right) + \tfrac{1}{2}\,\psi_L\,\psi_L\,\left(h_1\,\Phi^1_L + h_2\,\Phi^2_L\right)$$

yield the light fermion masses

$$m_u = g_1 u_2\,, \quad m_d \simeq g_1 u_1\, s_\alpha\,, \qquad \tan\alpha = \frac{g_1 v_1}{g_2 v_2}$$

$$m_{\tilde{\nu},N_1} = h_1 u_2\,, \quad m_e \simeq h_1 u_1\, s_\beta\,, \quad m_{N_2} \simeq \frac{h_1^2 u_1 u_2}{\Delta_l}\, s_\beta\,, \quad \tan\beta = \frac{h_1 v_1}{h_2 v_2}$$

$$\Delta_l = \sqrt{h_1^2 v_1^2 + h_2^2 v_2^2}$$

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The active neutrino, $\tilde{\nu}$, together with N_1 forms a Dirac state, whereas the sterile N_2 receives a small Majorana mass.

Correction via one-loop diagrams !

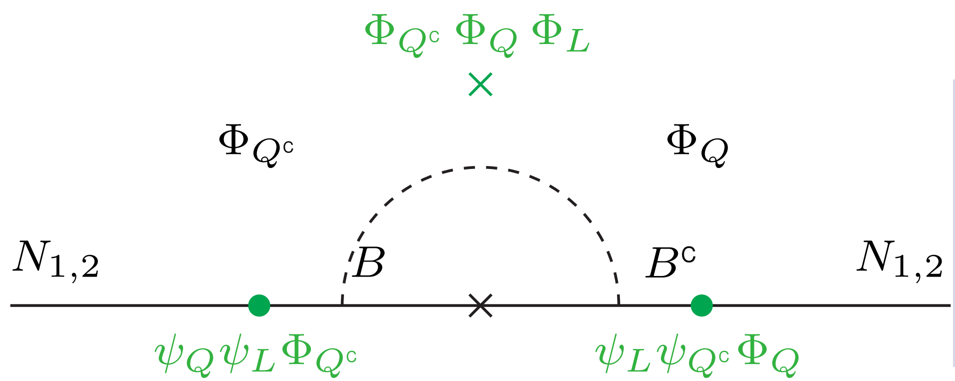
Radiative Seesaw Mechanism

One-loop diagrams appear due to the coupling of the neutral fermions to color-charged Higgs bosons and the cubic couplings of the Higgs fields.

Dominant graphs:

$\Phi_Q\,(3,3^*,1)$

$\Phi_{Q^c}\,(3^*,1,3)$



This mechanism is absent in models with low-energy supersymmetry.

$\rightarrow$ mass matrix for neutrinos $(\tilde{\nu}, N_1, N_2)$

$$M_N \simeq \begin{pmatrix} 0 & -h_1 u_2 & 0 \\ -h_1 u_2 & s_{\alpha-\beta} c_\beta g^2 F_q(B) & (s_{2\beta} s_\alpha - c_\alpha)\, g^2 F_q(B) \\ 0 & (s_{2\beta} s_\alpha - c_\alpha)\, g^2 F_q(B) & c_{\alpha-\beta} s_\beta g^2 F_q(B) \end{pmatrix},$$

$F_q(B) \propto m_B$
(loop integral)

sterile neutrettos obtain masses $\lambda_N \sim F_q(B) \sim \mathcal{O}(M_u)$,

active neutrino is light, $\lambda_\nu \sim \frac{(h_1 u_2)^2}{F_q(B)} = \mathcal{O}\left(\frac{M_{\mathrm{EW}}^2}{M_u}\right) \simeq 0.1$ eV !

Proton Decay

Quarks and leptons in different multiplets. $\rightarrow$ No proton decay via gauge bosons.

Instead, proton decay is mediated by Φ_{Q^c} and Φ_Q.
These dimension-six operators are suppressed by small Yukawa couplings,

$$\frac{g_1 h_1}{M_u^2}\left(s_\beta\; Q\,Q\,Q\,\mathscr{L} + s_\alpha\, u^c u^c \mathscr{D}^c e^c\right), \qquad \tau \simeq \left(\frac{1}{g_1 h_1}\right)^2 \times 10^{28}\ \text{years}$$

$\rightarrow$ Flavor non-diagonal decay dominant, in particular $p \rightarrow \bar{\nu} K^+$.

Assume the following Yukawa matrices
[Lola, Ross 1999]

$$g_1 \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}, \quad \epsilon^2 \simeq \frac{m_c}{m_t},$$

mode	$g_1 h_1$	exp. limit [y]
$\bar{\nu}_\tau K^+$	8×10^{-4}	2.2×10^{33}
$\bar{\nu}_\tau \pi^+$	8×10^{-5}	2.5×10^{31}
$\mu^+ K^0$	1×10^{-5}	1.4×10^{33}

The decay width of $p \rightarrow \bar{\nu} K^+$
is close to the experimental limit.

Summary

- The trinified model, $G_{TR} = SU(3)_C \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3$ is an interesting candidate for non-supersymmetric unification; it includes five Higgs doublets.

- Additional charged fermions appear which mix with the SM fermions; Neutrettos become massive with $M \gg M_{EW}$ via one-loop diagrams.

  At M_{EW}, only the SM fermions remain, no light sterile neutrino.

- Proton decay is mediated by color-charged Higgs bosons.

  The decay mode $p \to \bar{\nu} K^+$ is dominant
  (as familiar from SUSY models with dimension-five operators).

$\to$ Possibility to verify model !

  – no SUSY particles at TeV scale;

  – detection of $p \to \bar{\nu} K^+$ as dominant decay mode
  (future experiments aim to reach 10^{35} years).

- Results are also valid for models, where SUSY is broken at M_u (split SUSY).