Neutrino Masses and Discretized Gravity on the Hyperbolic Disk

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Based on: F. Bauer, T. Hällgren, **GS**, work in progress

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- Introduction

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- Hyperbolic disk

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- Discretized gravity

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- Summary & Conclusions

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Introduction

Outstanding problem in particle physics ...

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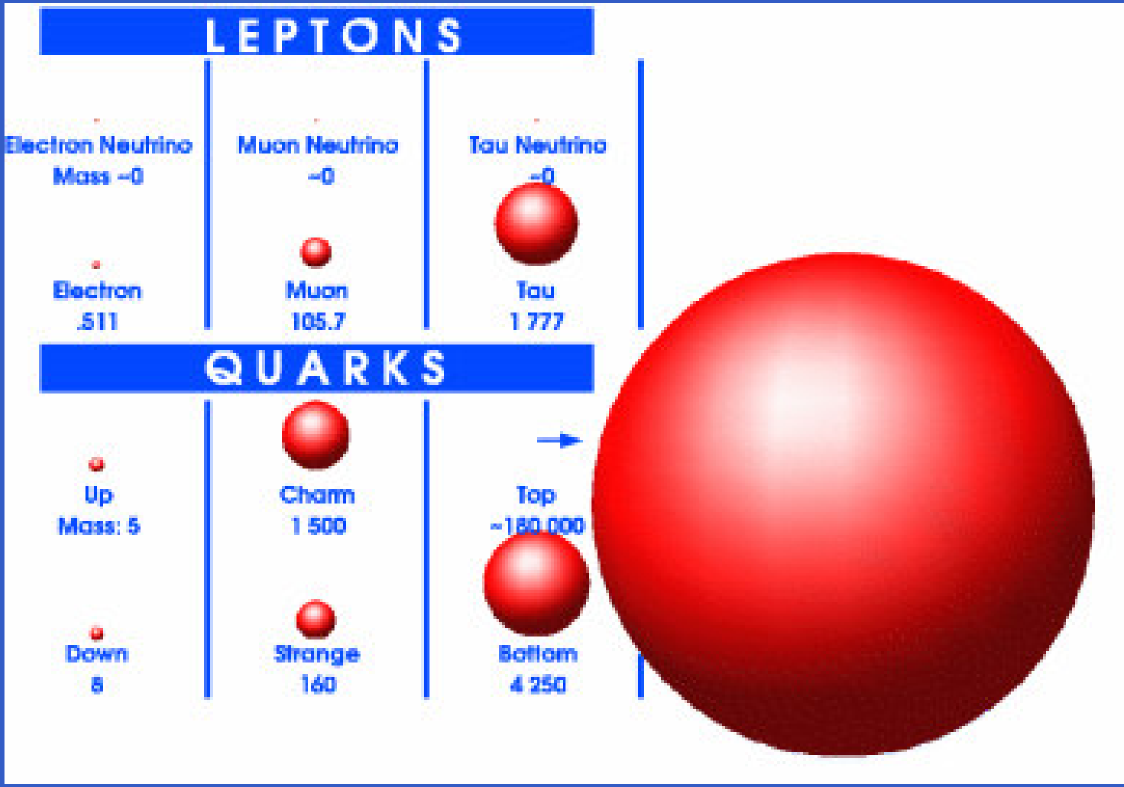
Outstanding problem in particle physics ...

Origin of observed fermion mass hierarchy



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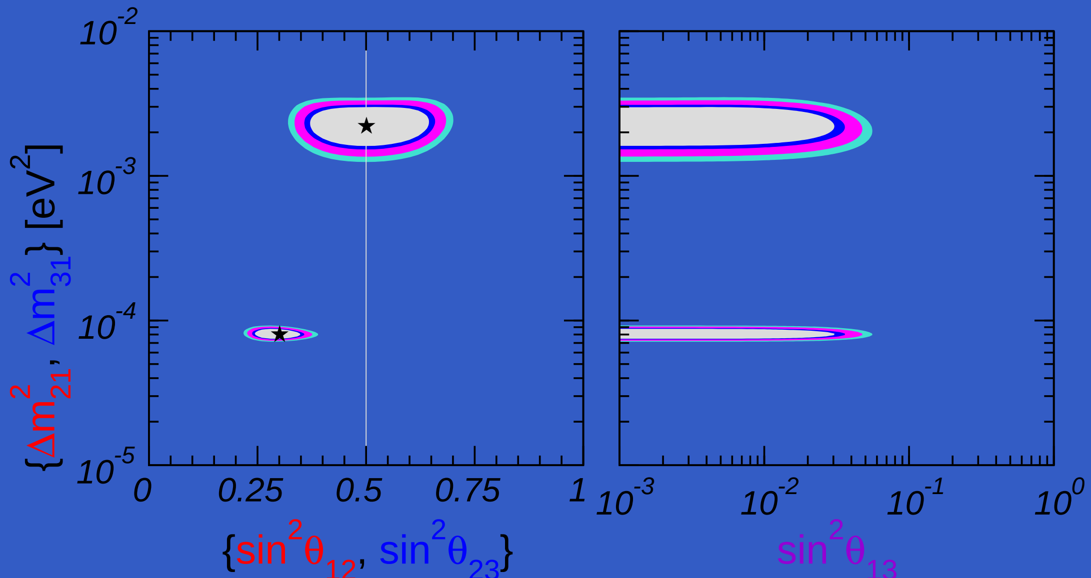
Introduction

Neutrino oscillations $\longrightarrow$ New Physics beyond SM required

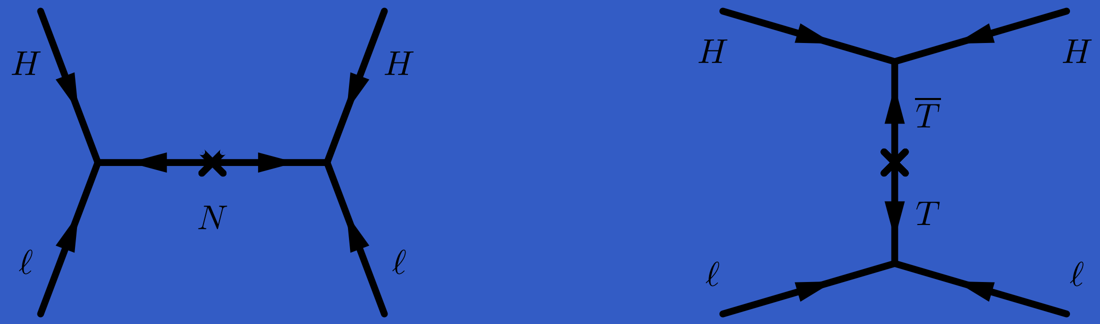


Absolute neutrino mass scale: $10^{-2} \ldots 10^{-1}$ eV $(\ll M_{EW})$

Origin of small neutrino masses?

Introduction

Most popular: neutrino masses from seesaw (type I,II)



Seesaw mechanisms via singlet (left) and triplet (right) exchange.

- Seesaw mechanism $\rightarrow$ Majorana neutrinos ($\leftrightarrow 0\nu\beta\beta$)
- Nature of neutrino mass (Dirac/Majorana) unkown
- Mechanism for small Dirac-ν masses?

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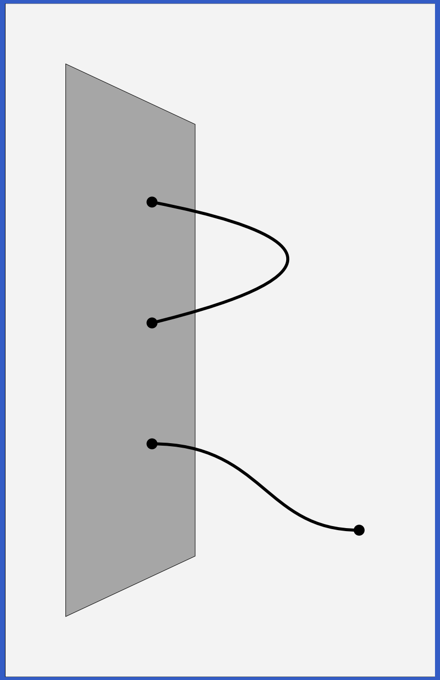
Introduction

Small Dirac-ν masses in ADD scenario   Arkani-Hamed, Dimopoulos, Dvali



- SM on D3-brane + RH νs in bulk
- Yukawa coupling volume-suppressed
- large bulk: light νs & weak gravity
- attractive mechanism

Arkani-Hamed, Dimopoulos, Dvali, March-Russell
Dienes, Dudas, Gherghetta, Smirnov, Mohapatra

Serious constraints from experiment & astrophysics

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Introduction

Constraints avoided in 5D Randall-Sundrum models

$\longrightarrow$ Dirac-ν masses from AdS_5

Grossman, Neubert

Huber, Shafi, Gherghetta

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Introduction

Constraints avoided in 5D Randall-Sundrum models

$\longrightarrow$ Dirac-ν masses from AdS_5 — Grossman, Neubert; Huber, Shafi, Gherghetta

Type IIB string compactifi cations — Verlinde, Klebanov, Strassler; Giddings, Kachru, Polchinski

Throats w/ topologies $\sim AdS_5 \times S^5$, $AdS_5 \times S^2 \times S^3 \ldots$

$\longrightarrow$ new possibilities in model building — Dimopoulos, Kachru, Kaloper; Lawrence, Silverstein; Uranga et al., Csaki et al.

Lattice gravity as useful tool to study warped spacetime

Arkani-Hamed, Georgi, Schwartz, Randall, Thabyapillai, Gallichio, Yavin

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Introduction

Goal: Generation of naturally small Dirac-ν masses from

(a) large volume limit of

(b) 6D compactfi ed curved space

(c) within the context lattice gravity

(d) avoiding all experimental bounds

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Hyperbolic Disk

6D GR compactifi ed to 4D on disk with constant curvature

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \frac{1}{1-er^2}\mathrm{d}r^2 - r^2\mathrm{d}\varphi^2$$

- e: curvature radius, $e < 0 \leftrightarrow$ hyperbolic space
- (r, φ): Euclidean polar coordinates

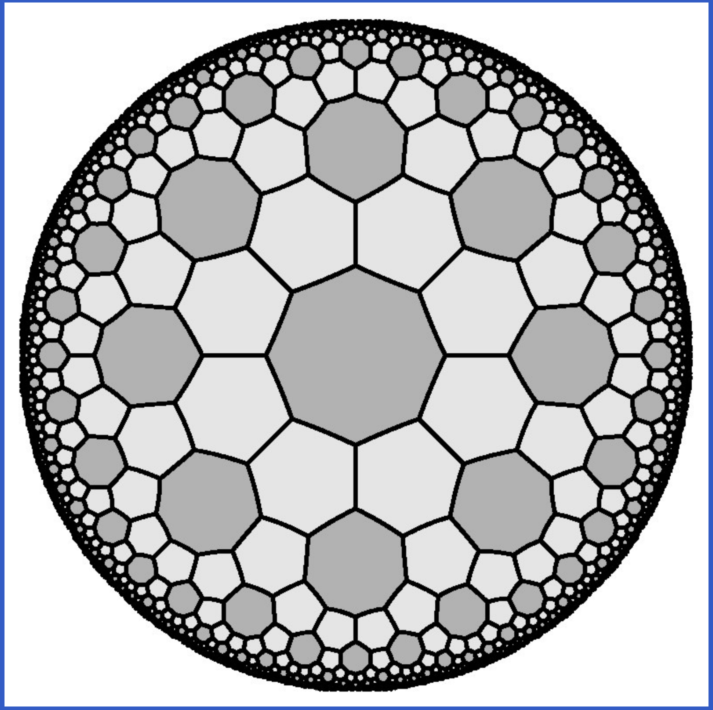
Transformation: $r \to r' \equiv r/(1+er^2/4)$

$\to$ Poincaré hyperbolic metric

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \frac{4}{(1-er^2)^2}(\mathrm{d}r^2 + r^2\mathrm{d}\varphi^2)$$

Hyperbolic Disk

Semi-regular tessellation $\{6, 6, 8\}$ of the Poincaré disk



radius= $\frac{1}{\sqrt{e}} \log \frac{1+\sqrt{e}r}{1-\sqrt{e}r}$ circumference= $\frac{2\pi}{\sqrt{e}} \sinh(\sqrt{e}\hat{r})$

Hyperbolic Disk

6D action splits into

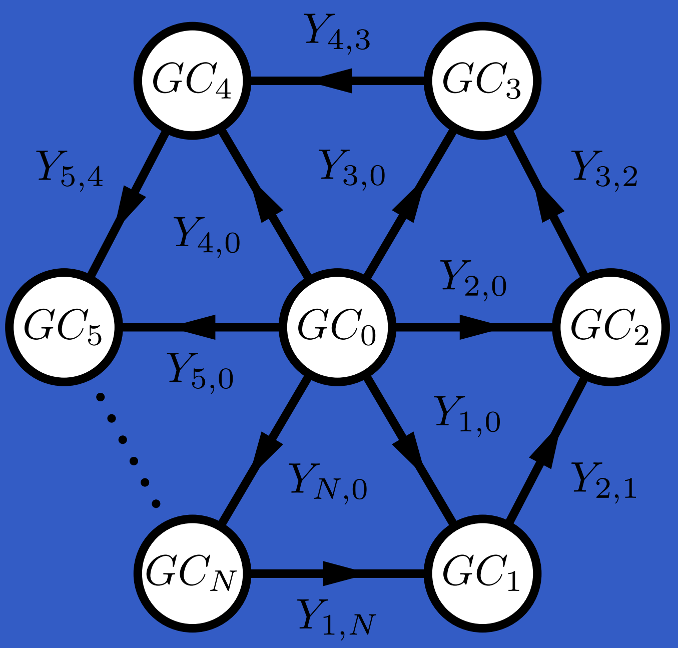
$$\begin{aligned}\mathcal{S} &= M_6^4 \int \mathrm{d}^6x \sqrt{|g|}(R - 2\Lambda) \\ &= M_6^4 \int \mathrm{d}^6x \sqrt{|g|} \bigg(R_{\mathrm{4D}} - \frac{1}{4} g^{55} \partial_r g_{\mu\nu} (g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta}) \partial_r g_{\alpha\beta} \\ &\qquad - \frac{1}{4} g^{66} \partial_\varphi g_{\mu\nu} (g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta}) \partial_\varphi g_{\alpha\beta} \bigg)\end{aligned}$$

M_6: 6D Planck scale

Hyperbolic Disk

Coarse grained discretization

Arkani-Hamed, Georgi, Schwartz



$(N$+1) GCs connected by $2N$ bi-vectorial link fi elds Y^{μ}

Hyperbolic Disk

Replace derivatives by differences

$$\begin{aligned} \partial_\varphi g_{\mu\nu} &\rightarrow m[(\partial_\mu Y^\alpha_{i,i+1})(\partial_\nu Y^\beta_{i,i+1})g^{i+1}_{\alpha\beta} - g^i_{\mu\nu}] \\ \partial_r g_{\mu\nu} &\rightarrow m_*[(\partial_\mu Y^\alpha_{0,i})(\partial_\nu Y^\beta_{0,i})g^i_{\alpha\beta} - g^0_{\mu\nu}] \end{aligned}$$

Inverse proper radial and angular lattice spacings

$$m_* = \sqrt{1 - er^2}/r \quad m = N/(2\pi r)$$

Curvature radius

$$e = \left(\frac{2\pi m}{N}\right)^2 \left(1 - \left(\frac{m_*}{m}\right)^2 \left(\frac{N}{2\pi}\right)^2\right)$$

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Hyperbolic Disk

Replace derivatives by differences

$$\partial_\varphi g_{\mu\nu} \quad\rightarrow\quad m[(\partial_\mu Y^\alpha_{i,i+1})(\partial_\nu Y^\beta_{i,i+1})g^{i+1}_{\alpha\beta} - g^i_{\mu\nu}]$$
$$\partial_r g_{\mu\nu} \quad\rightarrow\quad m_*[(\partial_\mu Y^\alpha_{0,i})(\partial_\nu Y^\beta_{0,i})g^i_{\alpha\beta} - g^0_{\mu\nu}]$$

Inverse proper radial and angular lattice spacings

$$m_* = \sqrt{1-er^2}/r \quad m = N/(2\pi r)$$

Matching: $M_4^2 = M_{6D}^4 m_*^{-1} m^{-1}$ and $M_{\rm Pl}^2 = (N+1)M_4^2$

Expand $g^i_{\mu\nu} = \eta_{\mu\nu} + h^i_{\mu\nu}$ fi rst inunitary gauge $Y^\mu_{i,j} \equiv x^\mu$

Hyperbolic Disk

$\longrightarrow$ Fierz-Pauli mass terms

$$\mathcal{L}_{\text{FP}} = M_4^2 \sum_{i=1}^{N} \Big[m_*^2 (h^i_{\mu\nu} - h^0_{\mu\nu})(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta})(h^i_{\alpha\beta} - h^0_{\alpha\beta}) + m^2 (h^{i+1}_{\mu\nu} - h^i_{\mu\nu})(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta})(h^{i+1}_{\alpha\beta} - h^i_{\alpha\beta}) \Big]$$

$$M_g^2 = m_*^2 \begin{pmatrix} N & -1 & -1 & -1 & \cdots \\ -1 & 1 & 0 & 0 & \cdots \\ -1 & 0 & 1 & 0 & \cdots \\ -1 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + m^2 \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & 2 & -1 & 0 & \cdots \\ 0 & -1 & 2 & -1 & \cdots \\ 0 & 0 & -1 & 2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

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Hyperbolic Disk

Spectrum

$$M_0^2 = 0 \quad M_n^2 = m_*^2 + 4m^2 \sin^2 \frac{\pi n}{N} \quad M_N^2 = (N+1) m_*^2$$

Eigenstates

$$\begin{aligned} H_{\mu\nu}^0 &= \frac{1}{\sqrt{N+1}}(1, 1, 1, \ldots, 1), \\ H_{\mu\nu}^n &= \frac{1}{\sqrt{N}}(0, 1, e^{\mathrm{i}\frac{2n\pi}{N}}, e^{\mathrm{i}\frac{4n\pi}{N}}, \ldots, e^{\mathrm{i}\frac{2(N-1)n\pi}{N}}) \\ H_{\mu\nu}^N &= \frac{1}{\sqrt{N(N+1)}}(N, -1, -1, \ldots, -1) \end{aligned}$$

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Hyperbolic Disk

Spectrum

$$M_0^2 = 0 \quad M_n^2 = m_*^2 + 4m^2 \sin^2 \frac{\pi n}{N} \quad M_N^2 = (N+1) m_*^2$$

- N quasi-degenerate states $\longrightarrow$ signal @ collider
- flat limit of multi-throat geometry Kim
- Correction to Newton's law only @ $r_{\rm crit} \simeq m_* \log N$
- large-N limit: large circumference $\simeq$ AE possible
- $m_* \gtrsim 100$ MeV: all experimental bounds avoided

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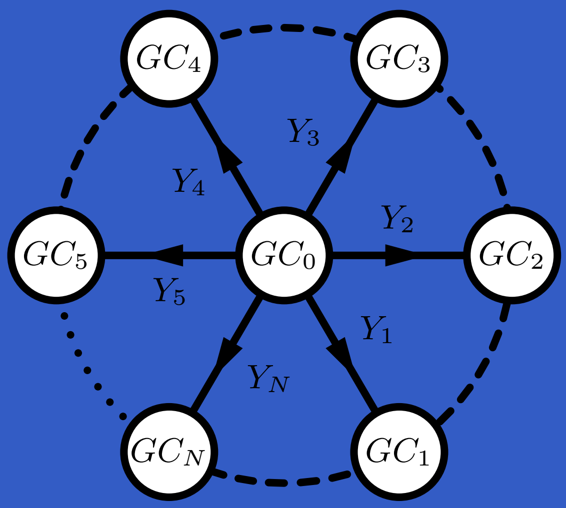
Neutrino Masses

4D RH SM singlet Dirac-ν $\Psi_i = (\nu_{Ri}, \overline{\nu^c_{Ri}})$ on each site i

N intersecting intervals $[0, R_i]$ in two-site limit

Csaki et al.
Kim



Multi-throat configuration.

Neutrino Masses

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N intersecting intervals $[0, R_i]$ in two-site limit

Csaki et al.
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Impose Neumann and Dirichlet BCs

$$\left.\frac{\partial \nu_R}{\partial y_i}\right|_{y_i=0,R_i} = 0 \qquad \nu^c_R|_{y_i=0,R_i} = 0$$

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Neutrino Masses

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N intersecting intervals $[0, R_i]$ in two-site limit — Csaki et al. Kim

Discretized kinetic term for $[0, R_i]$ — Hill, Pokorski, Wang, Skiba, Smith

$$\mathcal{L}^{\Psi}_{(0,i)} = m_*(\nu_{iR}\nu^c_{iR} - \nu_{0R}\nu^c_{iR}) + \mathrm{h.c.}$$

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Contribution from Wilson-Dirac action on circle — Hill, Leibovich

$$\mathcal{L}_{(i,i+1)} = m \cdot \nu_{iR}(\nu^c_{(i+1)R} - \nu^c_{iR}) + \mathrm{h.c.}$$

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Neutrino Masses

4D RH SM singlet Dirac-ν $\Psi_i = (\nu_{Ri}, \overline{\nu^c_{Ri}})$ on each site i

N intersecting intervals $[0, R_i]$ in two-site limit

Csaki et al.
Kim

Combine $\mathcal{L}^{\Psi}_{(0,i)} \cup \mathcal{L}^{\Psi}_{(i,i+1)} \longrightarrow$ total Lagrangian

$$\mathcal{L}^{\Psi}_{\text{disk}} = \sum_{i=0}^{N} \overline{\Psi}_i \not{\partial} \Psi_i + \sum_{i=1}^{N} [\mathcal{L}_{(0,i)} + C \cdot \mathcal{L}_{(i,i+1)}]$$

C: suitable parameter

Neutrino Masses

Total RH Dirac-ν mass matrix

$$M_D = m_* \begin{pmatrix} 0 & -1 & -1 & \cdots & -1 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} - Cm \begin{pmatrix} -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & 0 & \ddots & \ddots & 0 \\ 0 & \vdots & \ddots & -1 & 1 \\ 1 & 0 & \dots & 0 & -1 \end{pmatrix}$$

Choose C such that $M_D M_D^\dagger = M_g^2$

$\longrightarrow$ spectrum and eigenstates like for gravitons $\hat{H}^n_{\mu\nu} \leftrightarrow \hat{\nu}_{nR}$

Neutrino Masses

Adding SM fi elds on site $i = 1$ & B–L conserved in the bulk
$\longrightarrow$ local Yukawa interaction

$$\mathcal{S}_{\text{int}} = \int \mathrm{d}^4x \, f_\alpha \ell_\alpha \epsilon H \nu_{1R} \approx \int \mathrm{d}^4x \, f_\alpha \frac{\langle H \rangle}{\sqrt{N}} \nu_\alpha \hat{\nu}_{0R}$$

- Dirac mass term suppressed by $\sqrt{N} = \sqrt{Rm}$
- analog of ADD-type Dirac neutrino masses
- Dirac neutrino masses $\sim 10^{-2}$ eV for large N
- $m_* \gtrsim 100$ MeV $\longrightarrow$ all experimental bounds avoided

Strong Coupling

Expanding links in EFT

$$Y^{\mu}_{j,i}(x_\mu) = x^\mu + \pi^{ji}_\mu \quad \text{with} \quad \pi^{ji}_\mu = A^\mu_{ji}(x_\mu) + \partial^\mu \phi_{ji}(x_\mu)$$

$$\begin{aligned} \rightarrow \quad \mathcal{L}_{\rm disk} \quad &= \quad \mathcal{L}_{\rm FP} + M_4^2 \Big[h^0_{\mu\nu} \Box h^0_{\mu\nu} + \textstyle\sum_{n,k=1}^{N} \Big(H^n_{\mu\nu} \Box H^{N-n}_{\mu\nu} \\ &- \quad H^n_{\mu\nu} \Box (m_*^2 \Phi_{N-n} + m^2 (1 - e^{-i2\pi\cdot n/N}) \tilde{\Phi}_{N-n}) \\ &+ \quad \frac{m_*^2}{\sqrt{N}} \Box \Phi_n \Box \Phi_k \Box \Phi_{N-n-k} \\ &+ \quad \frac{m^2}{\sqrt{N}} \Box \tilde{\Phi}_n \Box \tilde{\Phi}_k \Box \tilde{\Phi}_{N-n-k} \Big) \Big] \end{aligned}$$

Scalar Goldstones $\Phi_n \leftrightarrow Y_{(0,i)}$ and $\tilde{\Phi}_n \leftrightarrow Y_{i,i+1}$

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Strong Coupling

Expanding links in EFT

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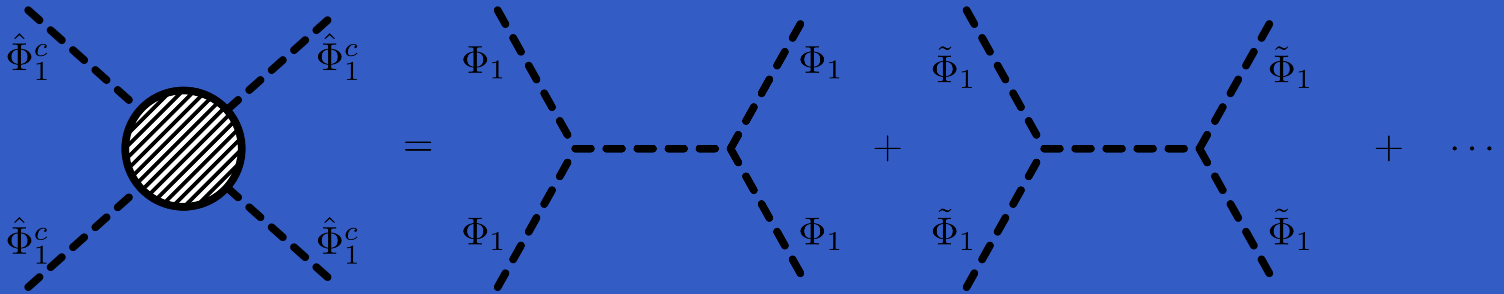
Eaten Goldstones $\hat{\Phi}^c_n \approx M_4 (m_*^2 \Phi_n + m^2 \frac{2\pi n}{N} \tilde{\Phi}_n)$ for $n \ll N$

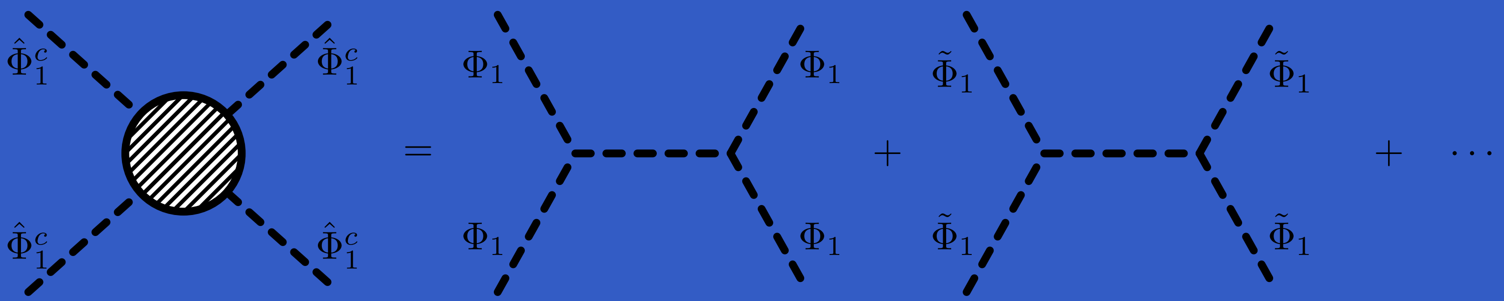
Strong Coupling



Derivative coupling

$$\frac{m^2}{\sqrt{N}}\Box\tilde{\Phi}_n\Box\tilde{\Phi}_k\Box\tilde{\Phi}_{-n-k} \to \frac{m^8}{N^{7/2}M_4 m_*^{12}}\Box\hat{\Phi}^c_n\Box\hat{\Phi}^c_k\Box\tilde{\Phi}^c_{-n-k}$$

Strong Coupling



Strong coupling scale: $\Lambda_{\rm disk} = (M_{\rm Pl} m_*^4)^{1/5}$

- strong coupling scale of single graviton with mass m_*
- $\Lambda_{\rm disk}$ N-independent $\rightarrow$ UV/IR connection avoided

Summary & Conclusions

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- Refi ned scenario: work in progress