Gravitino Phenomenology

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"Gauging Nonlinear Supersymmetry," hep-th/0512078

Why we love Supersymmetry:

Unification of the gauge couplings

Improved UV behavior

solution to the naturalness problem

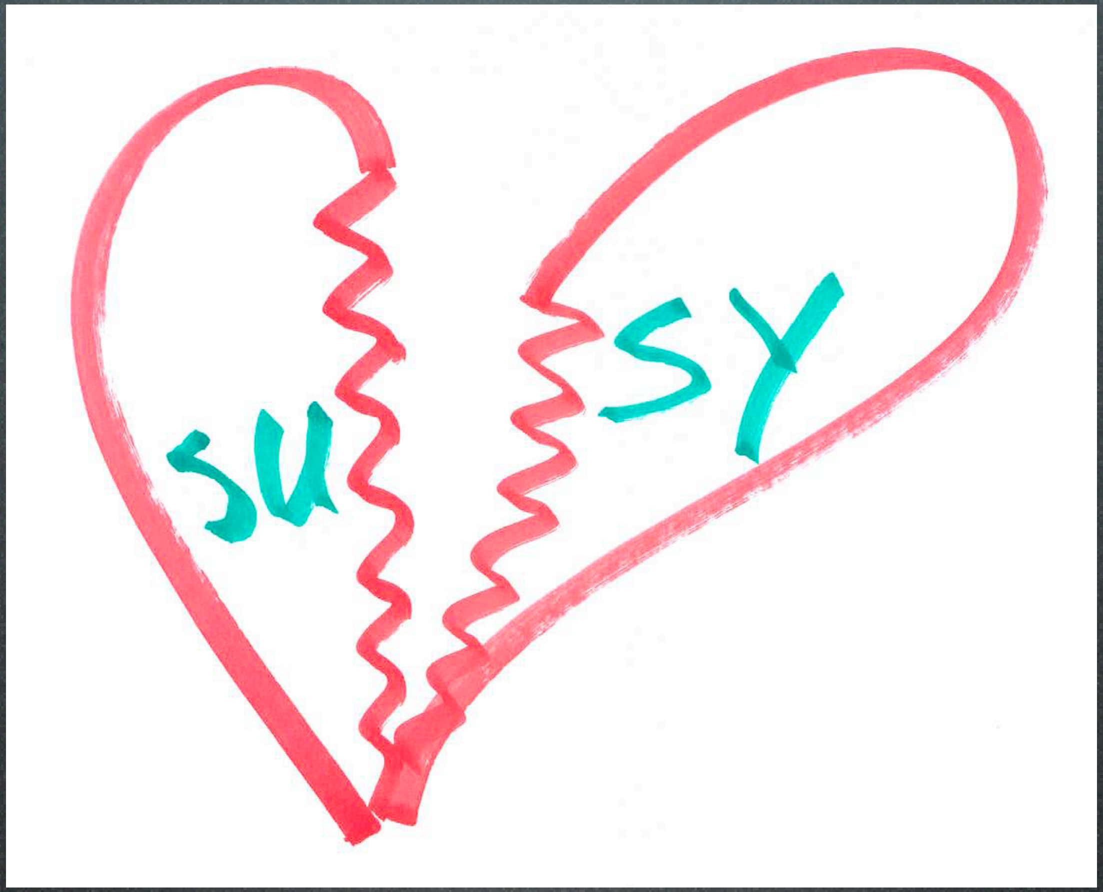
Toy laboratory to study strongly interacting gauge theories

Holomorphy

Neutralino/gravitino dark matter

String theory

One thing is sure:



Supersymmetry, if it exists in Nature, is spontaneously broken

Electroweak

Global

Goldstone boson for each spontaneously broken internal symmetry generator.

Local

Higgs mechanicsm: goldstone bosons are eaten to yield massive vector bosons.

No-Higgs:

Effective Lagrangian:

- Quarks
- Leptons
- Gauge Bosons

Nonlinear realization of electroweak symmetry

Supersymmetry

Global

Goldstone fermion (Goldstino) for each spontaneously broken supersymmetry generator.

Local

Super-Higgs mechanicsm: Goldstinos are eaten to yield massive gravitinos.

No superpartners:

Effective Lagrangian:

- Standard Model particles
- Graviton
- Gravitino

Nonlinear realization of supersymmetry

Objectives

Derivation of the low energy invariant action governing the dynamics of the light degrees of freedom (Gravitino/Goldstino, Graviton, Standard Model particles, perhaps a limited number of additional particles) and then examine some of its consequences.

The invariant action is constructed using the method of nonlinear realizations. The coset method is applied to the case of the local super-Poincaré group.

Note that the nonlinear realization of local symmetry which we construct is achieved using only the graviton and gravitino degrees of freedom. There is no need to include other (superpartner) degrees of freedom which appear in linear realizations of supergravity.

Coset Construction

The central object in the construction is the coset element $\Omega \in \mathcal{SP}_4/SO(1,3)$

Translation generators

Goldstino fields

$$\Omega(x) \quad = \quad e^{ix^\mu P_\mu} e^{i[\lambda^\alpha(x) Q_\alpha + \bar{\lambda}_{\dot{\alpha}}(x) Q^{\dot{\alpha}}]}$$

Broken supersymmetry generators

The super-Poincare algebra:

$$
\begin{aligned}
[M_{\mu\nu}, M_{\rho\sigma}] &= -i\left(\eta_{\mu\rho} M_{\nu\sigma} - \eta_{\mu\sigma} M_{\nu\rho} + \eta_{\nu\sigma} M_{\mu\rho} - \eta_{\nu\rho} M_{\mu\sigma}\right) \\
[M_{\mu\nu}, P_\lambda] &= i\left(P_\mu \eta_{\nu\lambda} - P_\nu \eta_{\mu\lambda}\right) \\
[M_{\mu\nu}, Q_\alpha] &= -\frac{1}{2}\left(\sigma^{\mu\nu}\right)_\alpha^{\ \beta} Q_\beta \\
[M_{\mu\nu}, \bar{Q}_{\dot{\alpha}}] &= \frac{1}{2}\left(\bar{\sigma}^{\mu\nu}\right)_{\dot{\alpha}}^{\ \dot{\beta}} \bar{Q}_{\dot{\beta}} \\
\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu
\end{aligned}
$$

Realization of $\mathcal{SP}_4$ transformations on the Goldstino fields and the coordinates

transformation parameters

$$g(x) = e^{i\epsilon^\mu(x)P_\mu} e^{i\xi^\alpha(x)Q_\alpha} e^{i\bar{\xi}_{\dot{\alpha}}(x)\bar{Q}^{\dot{\alpha}}} e^{\frac{i}{2}\alpha^{\mu\nu}(x)M_{\mu\nu}}$$

Transformations are defined by multiplication from the left:

$$g(x)\Omega(x) = \Omega'(x')h(x)$$

Element of Lorentz stability group

For infinitesimal transformations:

$$\begin{aligned}
x'^\mu &= x^\mu + \epsilon^\mu(x) + i[\xi(x)\sigma^\mu\bar{\lambda}(x) - \lambda(x)\sigma^\mu\bar{\xi}(x)] - \alpha^{\mu\nu}(x)x_\nu \\
\lambda'_\alpha(x') &= \lambda_\alpha(x) + \xi_\alpha(x) + \frac{i}{4}\alpha_{\mu\nu}(x)\,(\sigma^{\mu\nu})_\alpha{}^\beta\,\lambda_\beta(x) \\
\bar{\lambda}_{\dot{\alpha}}(x') &= \bar{\lambda}_{\dot{\alpha}}(x) + \bar{\xi}_{\dot{\alpha}}(x) + \frac{i}{4}\alpha_{\mu\nu}\,(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}}\,\bar{\lambda}^{\dot{\beta}}(x)
\end{aligned}$$

Note that the Goldstino field can be gauged away through a local supersymmetry transformation. (Unitary Gauge)

Construction of the invariant action; covariant building blocks

The invariant action is constructed from the covariantly transforming Maurer-Cartan one forms:

$$\omega \quad \equiv \quad \Omega^{-1}(d + i\hat{E})\Omega = i\left[\omega^m P_m + \omega_Q^\alpha Q_\alpha + \bar{\omega}_{\bar{Q}\dot{\alpha}}\bar{Q}^{\dot{\alpha}} + \frac{1}{2}\omega_M^{mn} M_{mn}\right]$$

In unitary gauge: vielbein (ω^m), Gravitino (ω_Q^α, $\bar{\omega}_{\bar{Q}\dot{\alpha}}$), Spin connection ($\omega_M^{mn}$)

The one-form gravitational field:

$$\hat{E} \quad = \quad \hat{E}^m P_m + \hat{\psi}^\alpha Q_\alpha + \hat{\bar{\psi}}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}} + \frac{1}{2}\hat{\gamma}^{mn} M_{mn}$$

Local Lorentz transformation:

$$\begin{aligned}
\omega'^m(x') &= \omega^n(x)\Lambda_n{}^m(\alpha(x)) \\
\omega'_{Q\alpha}(x') &= D_\alpha^{(\frac{1}{2},0)\beta}(\alpha(x))\omega_{Q\beta} \\
\bar{\omega}'^{\dot{\alpha}}_{\bar{Q}}(x') &= D^{(0,\frac{1}{2})\dot{\alpha}}{}_{\dot{\beta}}(\alpha(x))\bar{\omega}^{\dot{\beta}}_{\bar{Q}} \\
\omega'^{mn}_M(x') &= \omega_M^{rs}(x)\Lambda_r{}^m(\alpha(x))\Lambda_s{}^n(\alpha(x)) - d\alpha^{mn}(x)
\end{aligned}$$

Transformation as a gauge field:

$$\hat{E}'(x') \quad \equiv \quad g(x)\hat{E}(x)g^{-1}(x) - ig(x)dg^{-1}(x)$$

Emergence of Gravity

Vielbein:

$$\omega^m = dx^\mu e_\mu{}^m$$

Spin Connection:

$$\omega^{mn}_M = dx^\mu \omega^{mn}_{M\mu}$$

Tensors

$$T'^m_\mu(x') = G^{-1\nu}_\mu(x) T^n_\nu(x) \Lambda^m_n(\alpha(x))$$

Metric:

$$g_{\mu\nu} = e_\mu{}^m \eta_{mn} e_\nu{}^n$$

Spinors

$$\Psi'_{\alpha_1} = D^{(\frac{1}{2},0)\beta_1}_{\alpha_1}(\alpha(x))\Psi_{\beta_1}$$

Affine connection

$$\nabla_\rho e_\mu{}^m = 0 \longrightarrow \Gamma^\nu_{\sigma\rho} = e^{-1\nu}_n \partial_\rho e_\sigma{}^n - e^{-1\nu}_n \omega^{nr}_{M\rho} e_\sigma{}^s \eta_{rs}$$

Riemann curvature tensor

$$R^{mn} = d\omega^{mn}_M + \eta_{rs}\omega^{mr}_M \wedge \omega^{ns}_M \longrightarrow R^\rho{}_{\sigma\mu\nu} = \partial_\nu \Gamma^\rho_{\sigma\mu} - \partial_\mu \Gamma^\rho_{\sigma\nu} + \Gamma^\lambda_{\sigma\mu}\Gamma^\rho_{\lambda\nu} - \Gamma^\lambda_{\sigma\nu}\Gamma^\rho_{\lambda\mu}$$

Covariant derivatives

$$\nabla_\rho T^{m\nu} \equiv \partial_\rho T^{m\nu} - \omega^{mr}_{M\rho} T_r{}^\nu + \Gamma^\nu_{\sigma\rho} T^{m\sigma}$$

$$\nabla_\rho \Psi_{\mu\alpha} \equiv \partial_\rho \Psi_{\mu\alpha} + \frac{i}{4}\omega^{mn}_{M\rho}(\sigma_{mn})_\alpha{}^\beta \Psi_{\mu\beta} - \Gamma^\nu_{\rho\mu}\Psi_{\nu\alpha}$$

Invariant action

Newton's Constant

Cosmological Constant

Kinetic Term

$$
\begin{aligned}
\Gamma \;=\; \int d^4x \,\det e \,\Big\{ & \Lambda - \frac{M_{\rm Pl}^2}{16\pi} R + Z M_{\rm Pl}^2 \epsilon^{\mu\nu\rho\sigma} \omega_{Q\mu} (\sigma^s e^{-1}_{s\sigma}) \nabla_\rho \bar\omega_{\bar Q\nu} \\
& - \frac{i}{2} Z_m M_{\rm Pl}\, M_S^2 \left[\omega_{Q\mu}{}^{\alpha} \sigma_{\alpha}^{\mu\nu\beta} \omega_{Q\nu\beta} + \bar\omega_{\bar Q\mu\dot\alpha} \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta} \bar\omega_{\bar Q\nu}{}^{\dot\beta} \right] \\
& + M_{Pl}^2\, \omega_{Q\mu} \left[i Z_1 g^{\mu\nu} \sigma^\rho + i Z_2 g^{\mu\rho} \sigma^\nu + i Z_3 g^{\nu\rho} \sigma^\mu \right] \nabla_\rho \bar\omega_{\bar Q\nu} \\
& - \frac{1}{2} Z'_m M_{Pl} M_S^2 \left[\omega_{Q\mu}{}^{\alpha} g^{\mu\nu} \omega_{Q\nu\alpha} + \bar\omega_{\bar Q\mu\dot\alpha} g^{\mu\nu} \bar\omega_{\bar Q\nu}{}^{\dot\alpha} \right] \Big\}
\end{aligned}
$$

Mass Term

Coupling to Matter

Matter fields can be characterized by their Lorentz group transformation properties.

$$M'(x') \equiv \tilde{h} M(x) \qquad \text{with} \qquad \tilde{h} = e^{\frac{i}{2}\alpha_{mn}(x)\tilde{M}^{mn}}$$

The gauge and super-Poincaré covariant derivative of the matter field is secured as

$$\nabla M = [d + \frac{i}{2}\omega_M^{mn}\tilde{M}_{mn} - gA]M$$

A generic nonlinearly realized supergravity and gauge invariant matter field action can be constructed as

$$\Gamma_{\rm matter} = \int d^4x \det e \, \mathcal{L}_{\rm matter}$$

The fully invariant matter field Lagrangian takes the form

$$\mathcal{L}_{\rm matter} = \mathcal{L}_{\rm matter}(M, \nabla_\mu M, \omega_Q, \bar{\omega}_{\bar{Q}}, \nabla_\mu \omega_Q, \nabla_\mu \bar{\omega}_{\bar{Q}}, e_\mu{}^m, R_{\mu\nu\rho\sigma}, F^A_{\mu\nu}),$$

Phenomenology

Standard Model + Gravitino

Catalog the terms in the effective Lagranian by an expansion in the number of the fermionic Mauer-Cartan one-forms in each expression:

$$\mathcal{L}_{\rm eff} \quad = \quad \left[\mathcal{L}_{(0)} + \mathcal{L}_{(1)} + \mathcal{L}_{(2)} + \cdots\right]$$

In Unitary gauge this is tantamount to counting gravitino fields in each interaction.

$$\mathcal{L}_{(0)} \quad = \quad \mathcal{L}_{\rm StandardModel}$$

Lepton doublet

$$\mathcal{L}_{(1)} \quad = \quad \sum_f \frac{c_f}{M_S} \omega^{\alpha}_{Q\mu} (\sigma^{\mu\nu})_{\alpha}^{\ \beta} [l^a_{f\beta} (\nabla_\nu \phi)^b \epsilon_{ab}] + h.c.$$

Gravitino

Higgs doublet

Goldstino

$$\frac{c_3}{M_S^2}\partial_\mu \lambda^\alpha (\sigma^{\mu\nu})_\alpha^{\ \beta}[l_{3\beta}^a (D_\nu \phi)^b \epsilon_{ab}] + h.c.$$

↔

Gravitino

$$\frac{c_3}{M_S}\Psi_\mu^\alpha (\sigma^{\mu\nu})_\alpha^{\ \beta}[l_{3\beta}^a (\nabla_\nu \phi)^b \epsilon_{ab}] + h.c.$$

These interactions preserve lepton number if the Goldstino/Gravitino transforms as an anti-lepton.

Both the Z boson and the Higgs boson can decay into a neutrino and a Goldstino.

Allows the possibility that the Higgs boson decays with a sizable branching ratio.

"Phenomenology of a leptonic goldstino and invisible Higgs boson decays."
I. Antoniadis, M. Tuckmantel, F. Zwirner. **Nucl.Phys.B707:215-232,2005, hep-ph/0410165.**

Conclusions

- Constructed an action that is fully invariant under local Super Poincaré transformations.
- Showed how to couple the Standard Model fields in an invariant way.
- Effective theory therefore describes the Gravitino, the Graviton, and the Standard Model particles.
- Other potentially light degrees of freedom can be included as well. (For example, a light stau*)
- Effective Lagrangian approach only works when the Gravitino is light compared to other superpartners. It provides a meeting ground for specific Ultra Violet theories and experimental/observational results.

***"Gravitino and goldstino at colliders." W. Buchmuller, K. Hamaguchi, M. Ratz, T. Yanagida Contribution to the LHC / LC Study Group report, eds. G. Weiglein, et al. hep-ph/0403203**