



Measurements of the **Sides** of the **Unitarity Triangle** @ **BaBar**

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On behalf of the **BaBar** Collaboration



PHENO 06
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Overview



- Introduction to CKM Matrix & Unitarity Triangle (UT)
- Measuring the **sides** of the UT @ BaBar
- Interesting and recent measurements @ BaBar (& *Belle...* & *CDF...*)
Due to limited time: I will highlight the most recent analyses... And give you the general status...

Introduction to CKM Matrix & UT

Cabbibo-Kobayashi-Maskawa
(**CKM**) quark mixing matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Relates the quark mass eigenstates and the weak eigenstates.

In other words:

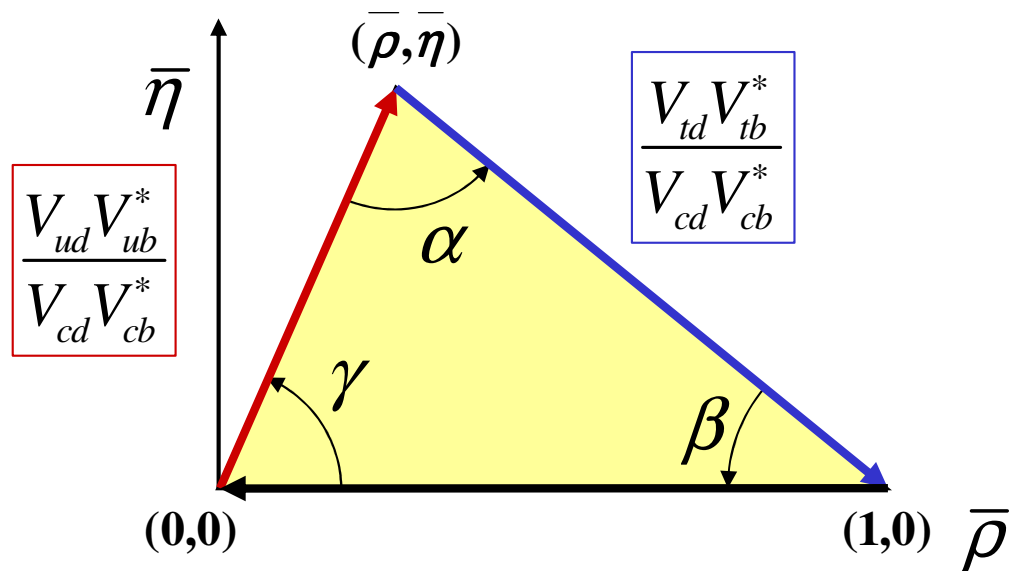
CKM matrix element $|V_{xy}| \sim$ probability of a quark of flavor "x" to become a quark of flavor "y"

One of the unitarity conditions:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Representation:
Unitarity Triangle

ULTIMATE GOAL:
we want to measure the **sides** & **angles** of the unitarity triangle to constrain & test the Standard Model



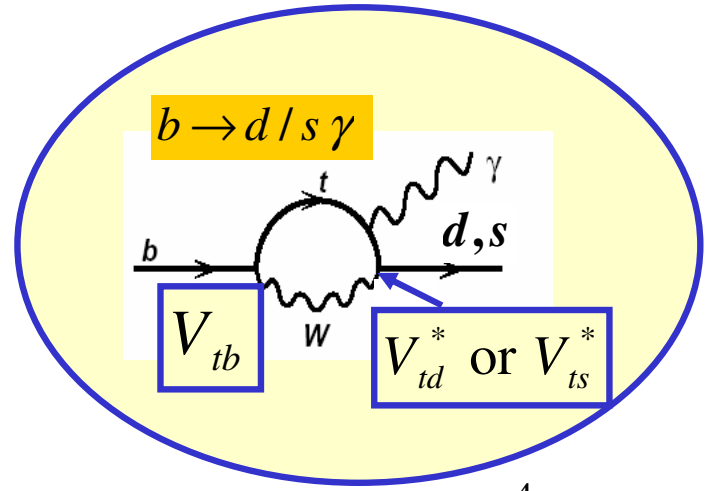
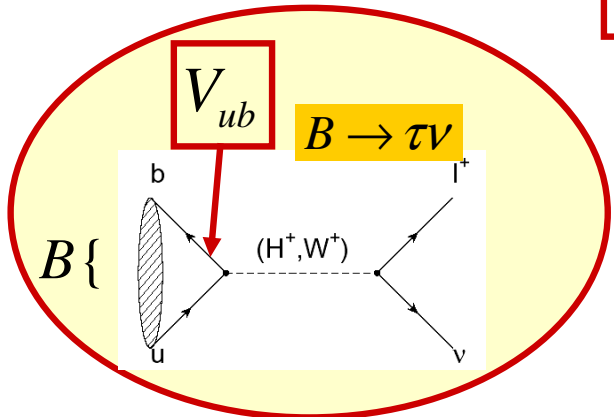
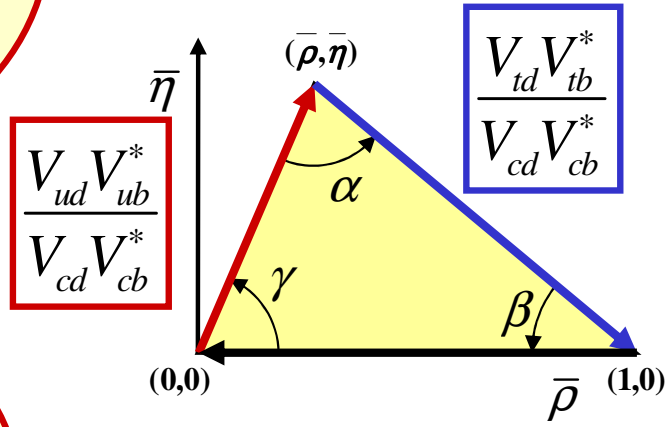
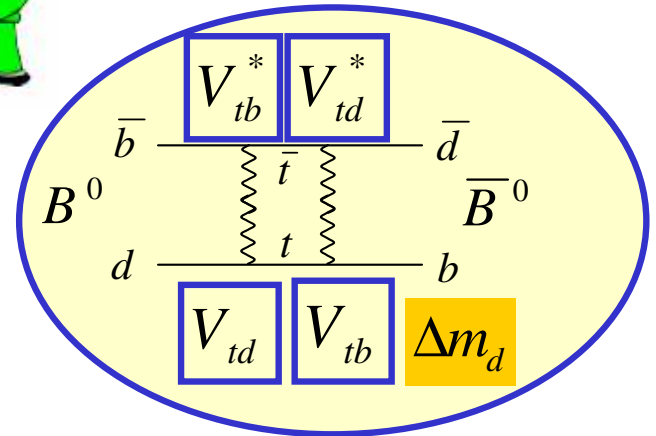
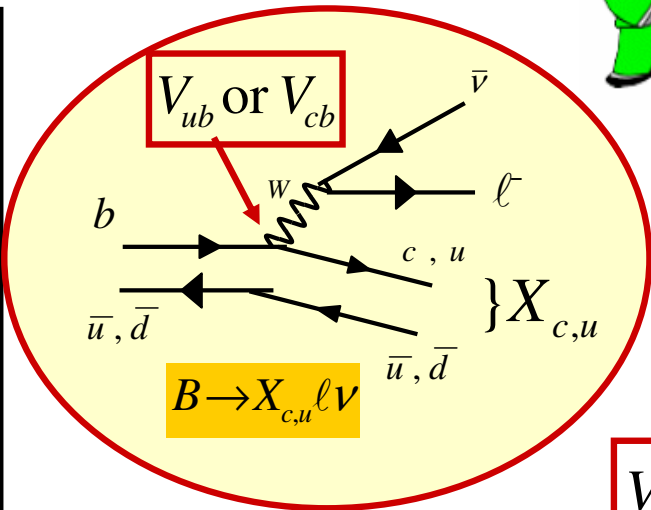
In this talk: measurements of the **sides** of the UT

Info on the **angles**: see Francesco Polci's talk

Measuring the **sides** of the unitarity triangle @ BaBar

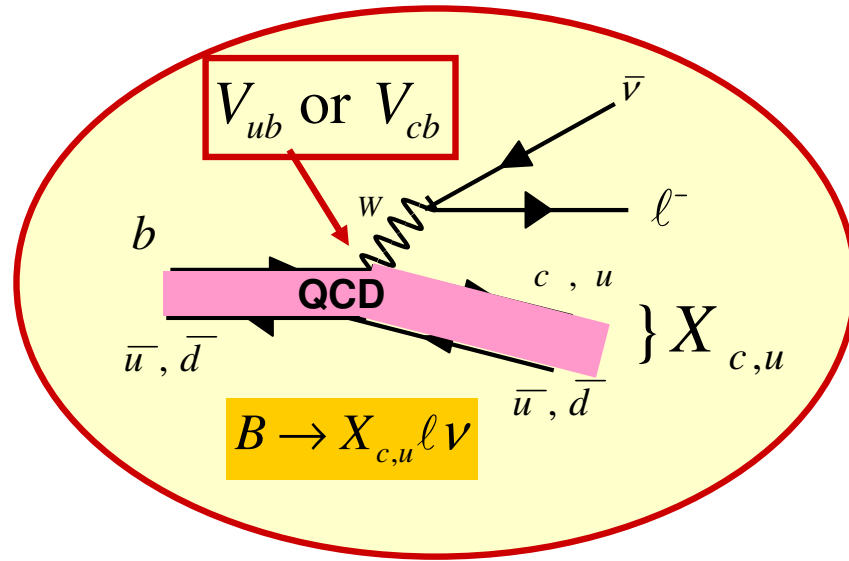


BaBar:
Let's use our
B mesons!!!!





$|V_{ub}|$ and $|V_{cb}|$ from $b \rightarrow ul\nu$ and $b \rightarrow cl\nu$ decays



Common feature: Need help from theory to describe the **QCD part**

3 new BaBar measurements:

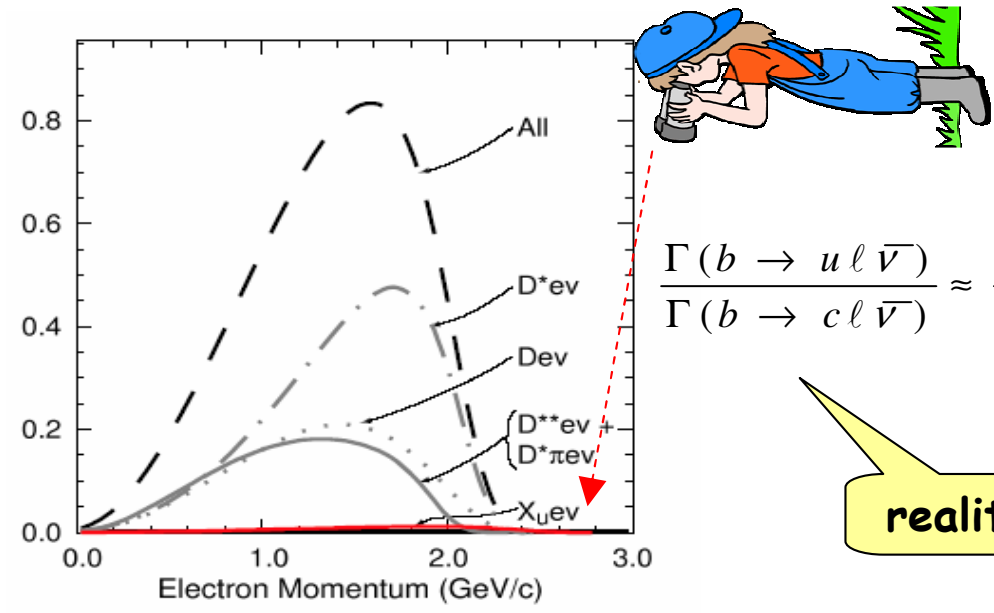
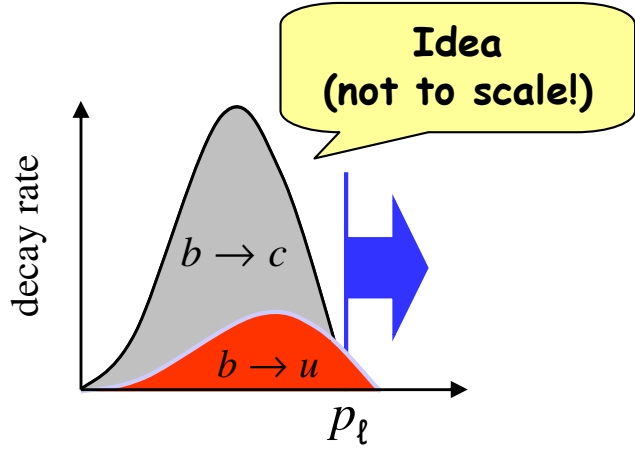
- $|V_{ub}|$ from lepton endpoint PRD73, 012006 (2006)
- $|V_{ub}|$ with reduced model dependence hep-ex/0601046
- $B \rightarrow D^* \ell \nu$ Form Factors hep-ex/0602023





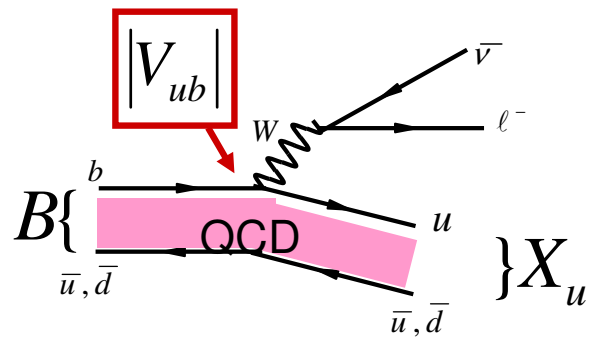
$|V_{ub}|$ From Lepton Endpoint (1)

Sylvie Brunet, Université de Montréal, PHENO 06



$$\frac{\Gamma(b \rightarrow u \ell \bar{\nu})}{\Gamma(b \rightarrow c \ell \bar{\nu})} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}$$

reality



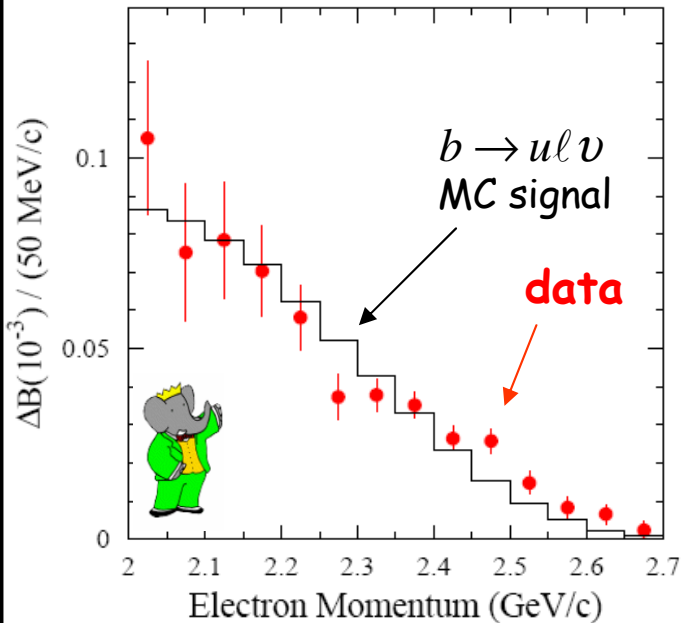
- measure partial branching fraction ΔB
- get predicted partial rate $\Delta \zeta$ from theory

$$|V_{ub}| = \sqrt{\frac{\Delta B(B \rightarrow X_u \ell \nu)}{\Delta \zeta \cdot \tau_B}}$$

theoretical input

$|V_{ub}|$ From Lepton Endpoint (2)

Measured electron spectrum



88×10^6 BB pairs

Best result: electrons with $2.0 < p_l < 2.6 \text{ GeV}$

Compared to previous measurement:

Use larger part of the electron spectrum: 10% \rightarrow 25%

\rightarrow smaller theoretical error

\rightarrow Accurate subtraction of background is needed!

Measure BF :

Uses inputs from other $b \rightarrow q$ analyses to describe the motion of the b -quark in the B meson (“shape function (SF)”)

$$\text{BR}(B \rightarrow X_u e \nu) = \left(2.27 \pm 0.26_{\text{exp}} \pm \left(\begin{array}{c} 0.33 \\ 0.26_{SF} \end{array} \right) \pm 0.17_{\text{theo}} \right) \times 10^{-3}$$

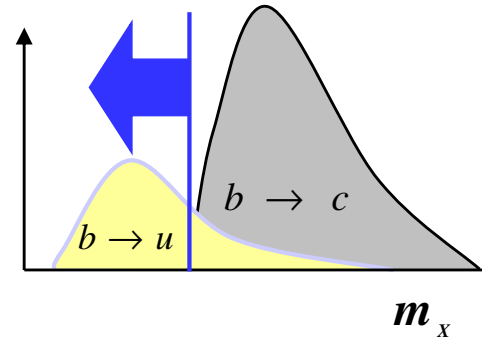
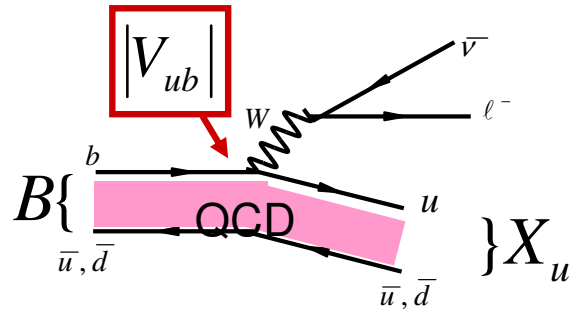
Extract $|V_{ub}|$ from BNLP calculations:

(Bosch-Lange-Neubert-Paz Nucl. Phys. B 699, 335, 2004)

$$|V_{ub}| = \left(4.44 \pm 0.25_{\text{exp}} \pm \left(\begin{array}{c} 0.42 \\ 0.38_{SF} \end{array} \right) \pm 0.22_{\text{theo}} \right) \times 10^{-3}$$



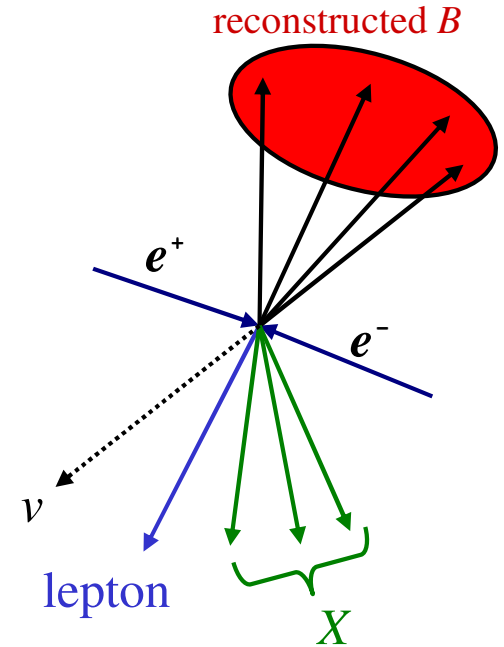
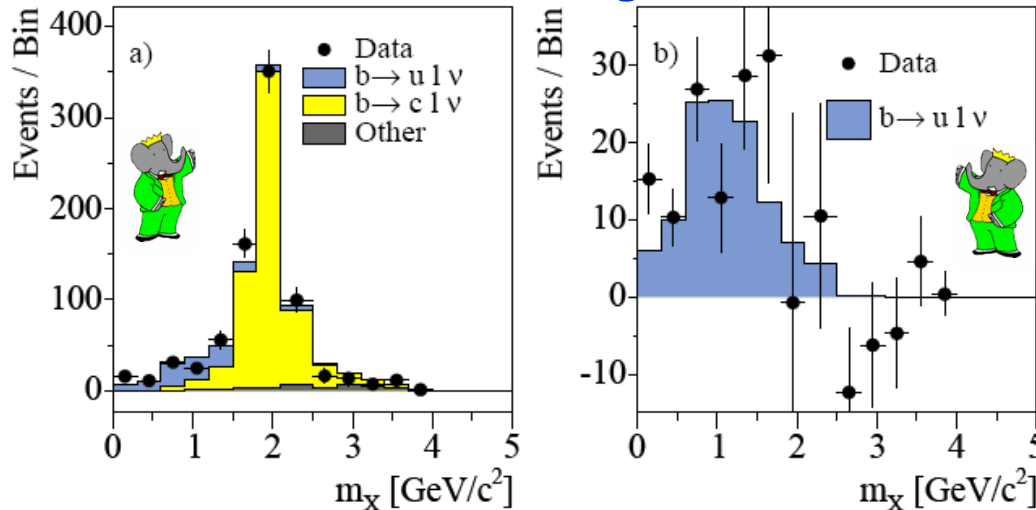
$|V_{ub}|$ From m_x , reduced model dependence (1)



Idea (not to scale!)

- m_x = mass of the hadronic system
- Fully reconstruct one B in hadronic decay mode (clean sample)
- Search for $Xl\nu$ in the other B
- Subtract $X_c l\nu$ background

Before and after background subtraction





$|V_{ub}|$

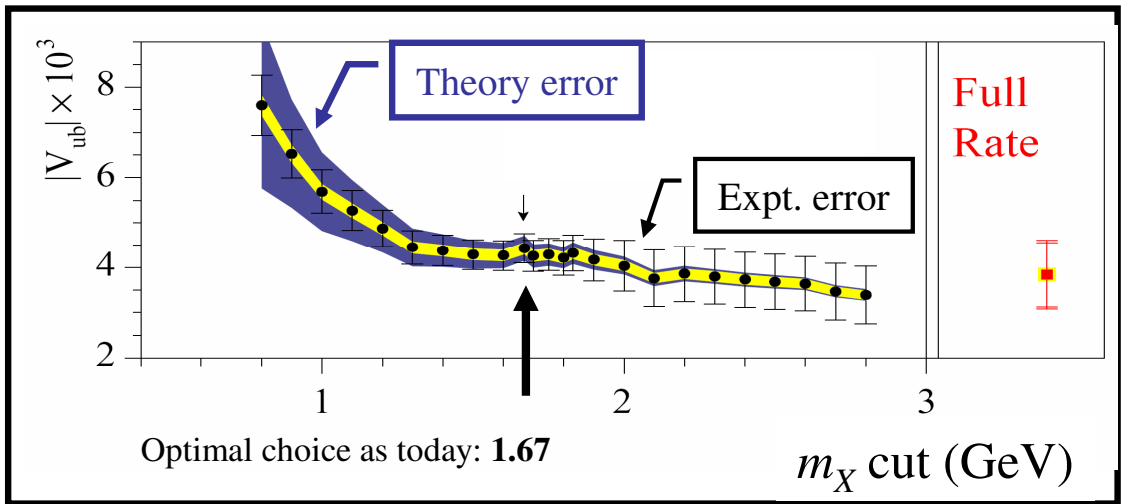
From m_X , reduced model dependence (2)

- Possible to combine $b \rightarrow u\ell\nu$ and $b \rightarrow s\gamma$ so that the SF cancels

$$\Gamma(B \rightarrow X_u \ell \nu) = \frac{|V_{ub}|^2}{|V_{ts}|^2} \int W(E_\gamma) \frac{d\Gamma(B \rightarrow X_s \gamma)}{dE_\gamma} dE_\gamma$$

Weight function

(Following LLR : Leibovich, Low, Rothstein - hep-ph/0005124,0105066)



Loose m_X cut:
Trade SF error \rightarrow Stat. error
 (you don't have to extrapolate to full decay rate; however, you have more background)

$89 \times 10^6 B\bar{B}$ pairs **Acceptance:**

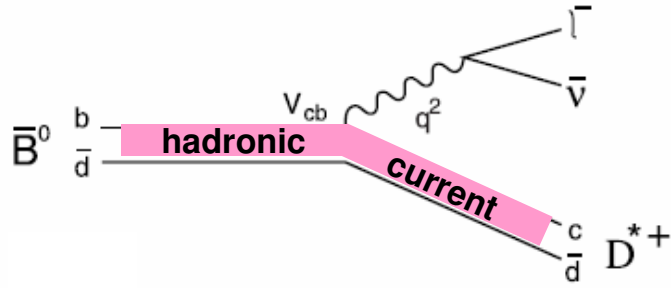
$M_X < 1.67 \text{ GeV: } |V_{ub}| = (4.43 \pm 0.38_{\text{stat}} \pm 0.25_{\text{syst}} \pm 0.29_{\text{theo}}) 10^{-3}$
 $M_X < 2.50 \text{ GeV: } |V_{ub}| = (3.84 \pm 0.70_{\text{stat}} \pm 0.30_{\text{syst}} \pm 0.10_{\text{theo}}) 10^{-3}$



72%
98%



$B \rightarrow D^* \ell \nu$ Form Factors (1)



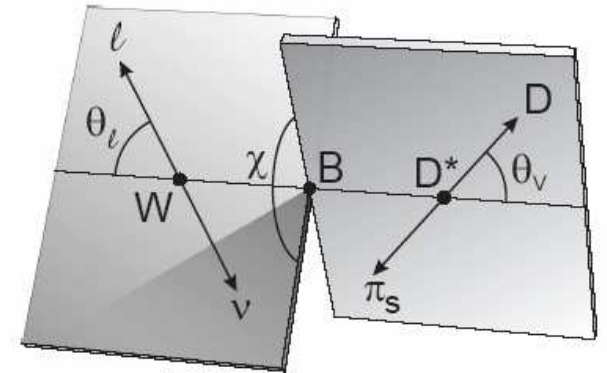
- Hadronic current described by 3 FF.
- Can be parameterized by $R_1 R_2 \rho^2$
- Uses constraints from theory
(Caprini-Lellouch-Neubert Nucl. Phys. B 530, 153, 1998)

Interests of the measurement:

- 1) FF themselves (previous measurement 10 years ago!)
- 2) Reduce $|V_{cb}|$ error
- 3) Reduce $|V_{ub}|$ error (important background)

• Decay can be **experimentally** characterized by 4 kinematic variables: $\theta_\ell \theta_\nu \chi W$
 where $W = D^*$ boost in B rest frame

$$\frac{d\Gamma(B \rightarrow D^* \ell \nu)}{dw d \cos \theta_\ell d \cos \theta_\nu d \chi} = |V_{cb}|^2 f(w, \theta_\ell, \theta_\nu, \chi, \rho^2, R_1, R_2)$$



Extract R_1, R_2 and ρ^2 from multi-dimensional fit to differential decay rate

$B \rightarrow D^* \ell \nu$ Form Factors (2)

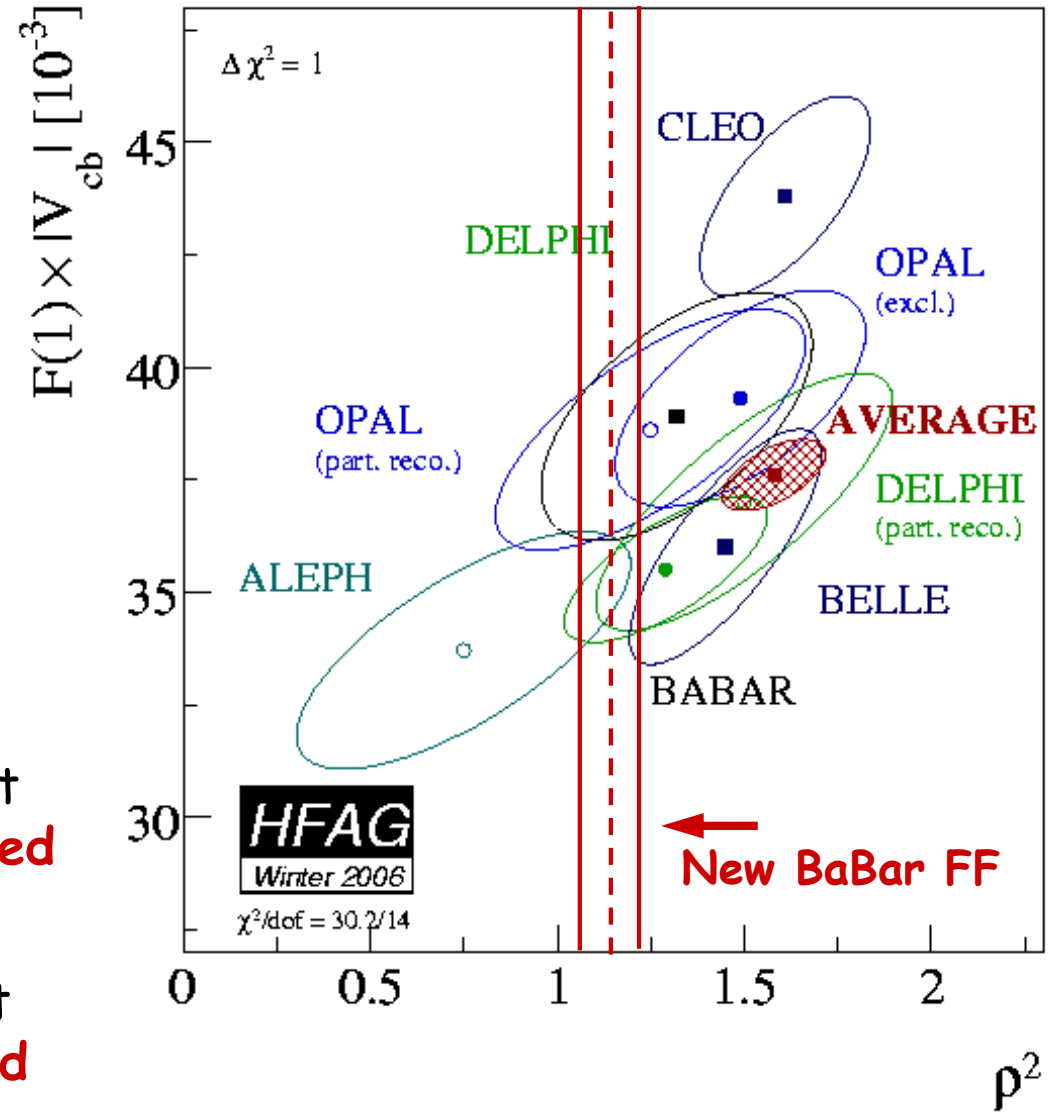
Results:  86×10^6 $B\bar{B}$ pairs

	stat	data	stat	MC	theo
R_1	± 0.060	± 0.035	± 0.027		
R_2	± 0.040	± 0.022	± 0.013		
ρ^2	± 0.059	± 0.030	± 0.035		

- **Factor 5 improvement** on FF uncertainties wrt. previous CLEO measurement (PRL 76 (1996) 3898)!

- Updated a BaBar $|V_{cb}|$ measurement (PRD-RC 051502 (2005)) : syst. error **is reduced by 25%!**

- BaBar lepton endpoint measurement (PRD73, 012006 (2006)) : syst. error **is reduced by 20% on the partial BF!**

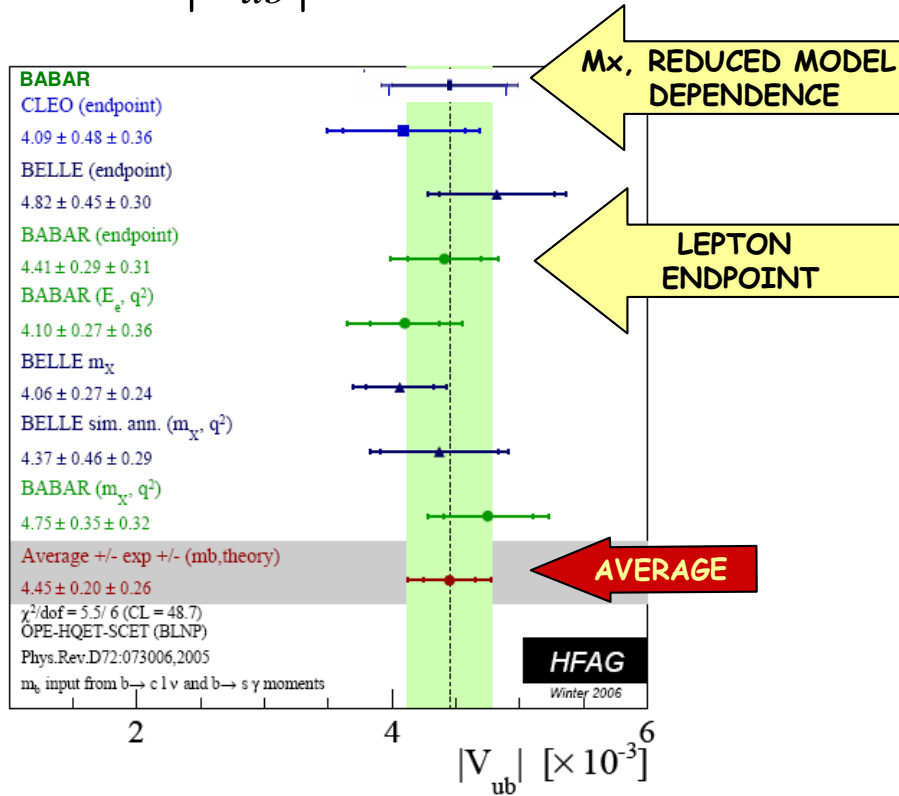




$|V_{ub}|$ & $|V_{cb}|$ Status (HFAG Winter 2006)



$|V_{ub}|$ $\sigma \approx 7\%$



$|V_{cb}|$ $\sigma \approx 2\%$

Best result: Global fit on many inclusive measurements.

Buchmüller, Flächer: hep-ph/0507253

Uses inputs from BaBar, Belle, CLEO, CDF & DELPHI.

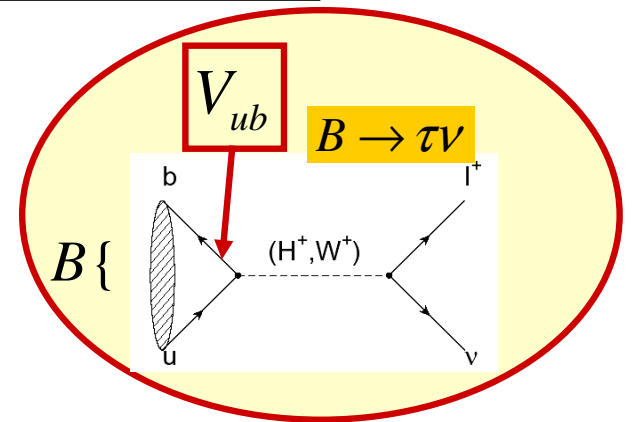
BLNP: Shape Function, PRD72:073006(2005)
 $(4.45 \pm 0.20_{\text{exp}} \pm 0.18_{\text{SF}} \pm 0.19_{\text{theo}}) \times 10^{-3}$

Andersen-Gardi: DGE, JHEP0601:097(2006)
 $(4.41 \pm 0.20_{\text{exp}} \pm 0.20_{\text{theo}}) \times 10^{-3}$
 →NEW THEORETICAL SCHEME!



$B \rightarrow \tau \nu$

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau) = \frac{G_F^2 m_B}{8\pi} m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$



Interests:

- f_B , $|V_{ub}|$, Physics Beyond the SM,
- $BR(B \rightarrow \tau \nu) / \Delta m_{B_d}$ constrains $|V_{ub}|^2 / |V_{td}|^2$

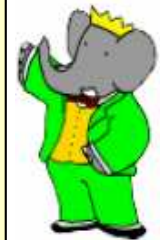


No evidence of signal seen @ BaBar Phys.Rev.D73:057101 (2006)

Reconstruct one B in semileptonic or hadronic modes

$$\text{BF}(B^+ \rightarrow \tau^+ \nu_\tau) = (1.28_{-0.90}^{+0.95}) \times 10^{-4} \quad (< 2.6 \times 10^{-4}, 90\% \text{CL})$$

$$f_B < 0.34 \text{ GeV}, 90\% \text{ CL} \quad (232/88) \times 10^6 \text{ } B\bar{B} \text{ pairs}$$



Belle @ FPCP06 : First evidence ! hep-ex/0604024

Reconstruct one B in hadronic modes

$$\text{BF}(B^+ \rightarrow \tau^+ \nu_\tau) = 1.06_{-0.28}^{+0.34} {}_{-0.16}^{+0.18} \times 10^{-4} \quad (4.2\sigma \text{ significance})$$

$$f_B = 0.176_{-0.023}^{+0.028} {}_{-0.018}^{+0.020} \text{ GeV} \quad 447 \times 10^6 \text{ } B\bar{B} \text{ pairs}$$

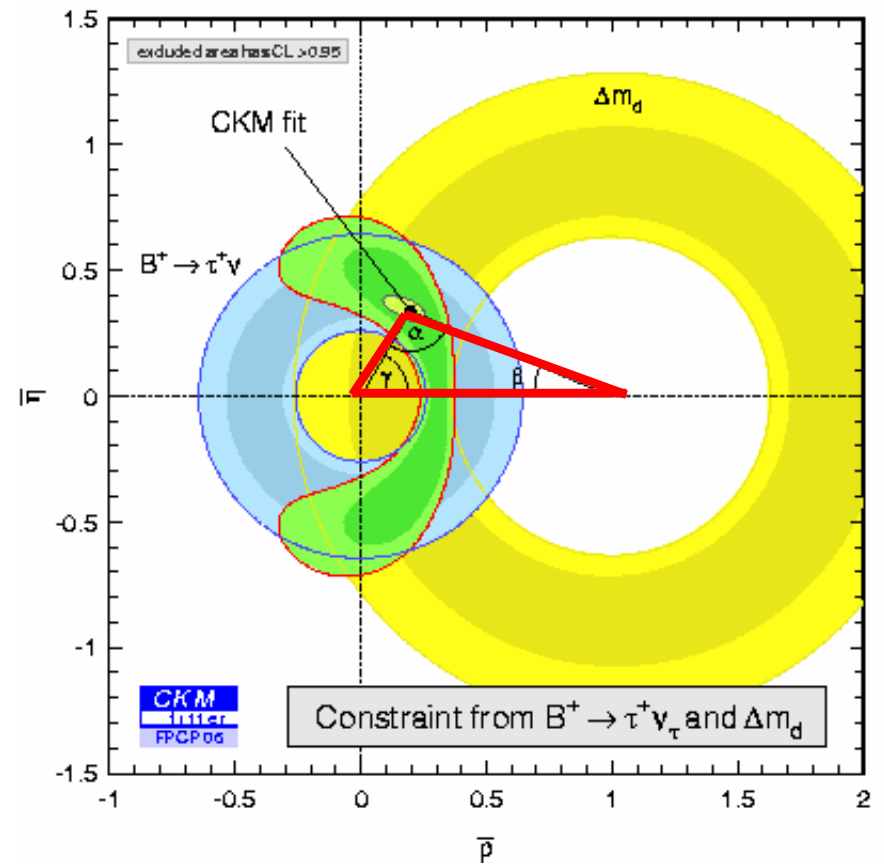
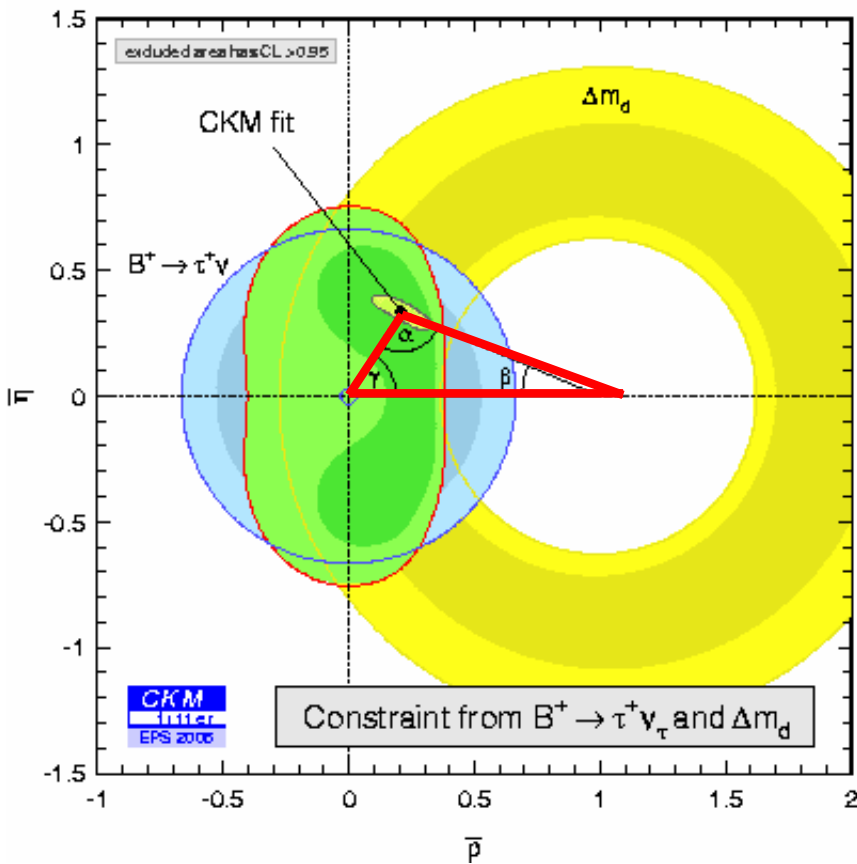


$B \rightarrow \tau \nu$ & Δm_d : Constraints on the UT

EPS 2005

new
 $B \rightarrow \tau \nu$

FPCP 2006



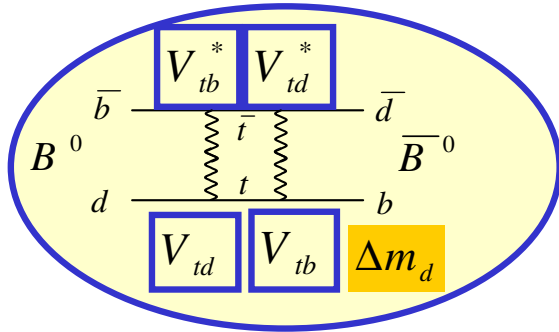


B factories: B_d only



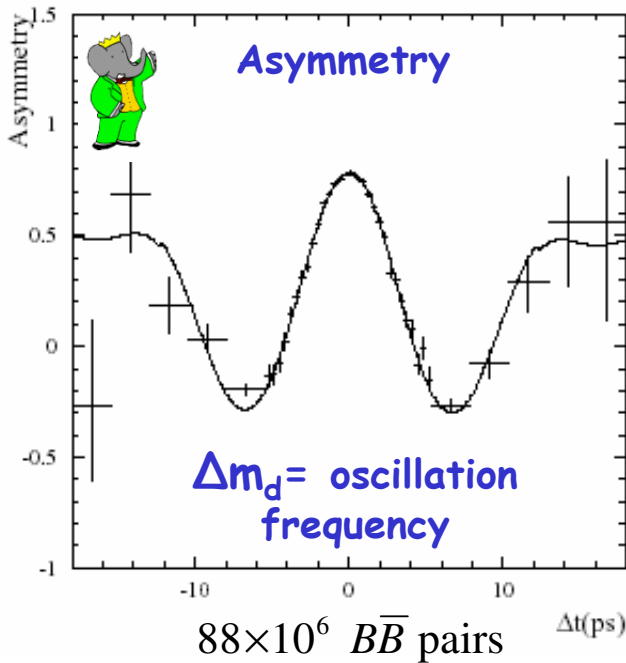
Δm_d and $B^0 \bar{B}^0$ oscillations

$$\text{Prob}(B^0 \bar{B}^0 \rightarrow B^0 B^0 \text{ or } \bar{B}^0 \bar{B}^0, B^0 \bar{B}^0) = \frac{\Gamma}{4} e^{-\Gamma|\Delta t|} (1 \mp \cos(\Delta m_d \Delta t))$$



$$\Delta m_d = \frac{G_F}{6\pi^2} m_{B_d} \hat{B}_{B_d} F_{B_d}^2 \eta_B m_w^2 S_0(x_t) |V_{td}|^2$$

dominant theoretical uncertainties ($\sigma \sim 20\%$)



-Fit the number of mixed/unmixed events in many Δt bins

$$A(\Delta t) = \frac{\mathcal{N}_{\text{unmix}}(\Delta t) - \mathcal{N}_{\text{mix}}(\Delta t)}{\mathcal{N}_{\text{unmix}}(\Delta t) + \mathcal{N}_{\text{mix}}(\Delta t)}$$

$$\Delta m_d = (0.511 \pm 0.007(\text{stat})_{-0.006}^{+0.007}(\text{syst})) ps^{-1} \rightarrow \sigma \approx 1.9\%$$

Can compare to HFAG Average Winter 2006

$$\Delta m_d = 0.507 \pm 0.004 ps^{-1} \rightarrow \sigma \approx 0.8\%$$

Adding the Δm_s ingredient

$$\Delta m_d \sim (\text{lots of QCD}) \times |V_{td}|^2$$

$$\Delta m_s \sim (\text{lots of QCD}) \times |V_{ts}|^2$$

$$\Delta m_s / \Delta m_d \sim (\text{much less QCD}) \times |V_{ts}|^2 / |V_{td}|^2$$

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

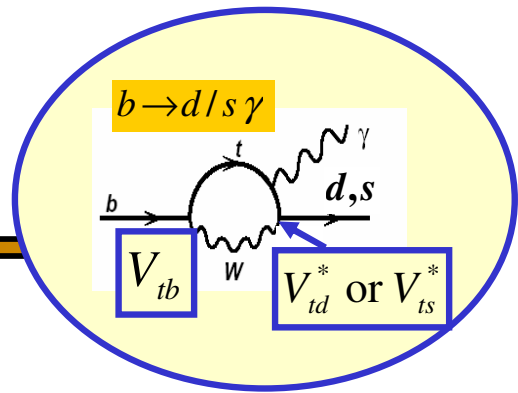
Input	Value	Source	
$m(B_d)/m(B_s)$	0.98320	PDG	
ξ^2	$1.21^{+0.047}_{-0.035}$	Okamoto, Lattice 2005	$\rightarrow \sigma \approx 4\%$
Δm_d	0.505 ± 0.005	PDG	$\rightarrow \sigma \approx 1\%$
Δm_s	$17.330^{+0.426}_{-0.221}$	This analysis	$\rightarrow \sigma \approx 3\%$

Copied from Guillermo Gomez-Ceballos's talk, FPCP 2006

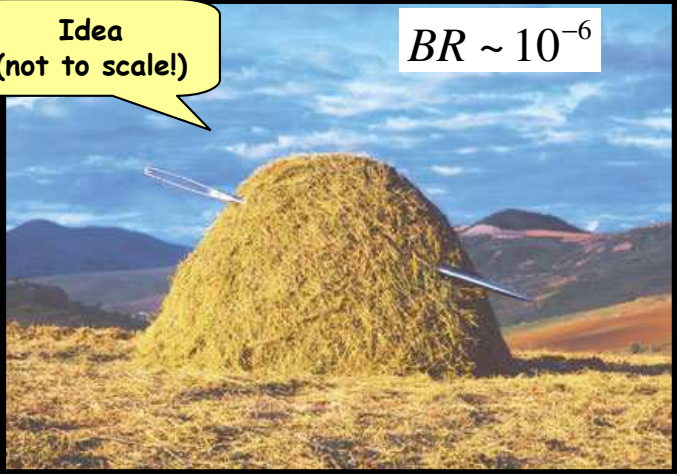
$$|V_{td}|/|V_{ts}| = 0.208^{+0.008}_{-0.007} \rightarrow \sigma \approx 4\%$$



Search for $B \rightarrow \rho / \omega \gamma$



Idea (not to scale!)



$BR \sim 10^{-6}$

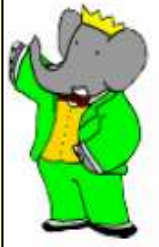
$$\frac{B(B \rightarrow \rho \gamma)}{B(B \rightarrow K^* \gamma)} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \left(\frac{1 - m_\rho^2/m_B^2}{1 - m_{K^*}^2/m_B^2} \right)^3 \zeta^2 [1 + \Delta R]$$

Theoretical inputs

No evidence of signal seen @ BaBar PRL 94 011801 (2005) $211 \times 10^6 \ B\bar{B}$ pairs

$$B(B \rightarrow (\rho, \omega) \gamma) = (0.6 \pm 0.3 \pm 0.1) \times 10^{-6} \ (2.1 \sigma)$$

$$|V_{td} / V_{ts}| < 0.19 \ (90\% CL)$$



Belle @ LP 2005: hep-ex/0506079 : **First observation!** $386 \times 10^6 \ B\bar{B}$ pairs

$$B(B \rightarrow (\rho, \omega) \gamma) = \begin{pmatrix} 1.32 & +0.34 & +0.10 \\ & -0.31 & -0.09 \end{pmatrix} \times 10^{-6} \ (5.1 \sigma)$$

$$|V_{td} / V_{ts}| = 0.199 \begin{matrix} +0.026 & +0.018 \\ -0.025 & -0.015 \end{matrix}$$

exp. theo.



UT fit to all modes $|V_{td} / V_{ts}| = 0.16 \pm 0.02 \rightarrow \sigma \approx 13\%$

Independent of Δm_s CDF: B_s - Mixing (FPCP06): $|V_{td}|/|V_{ts}| = 0.208^{+0.008}_{-0.007} \rightarrow \sigma \approx 4\%$



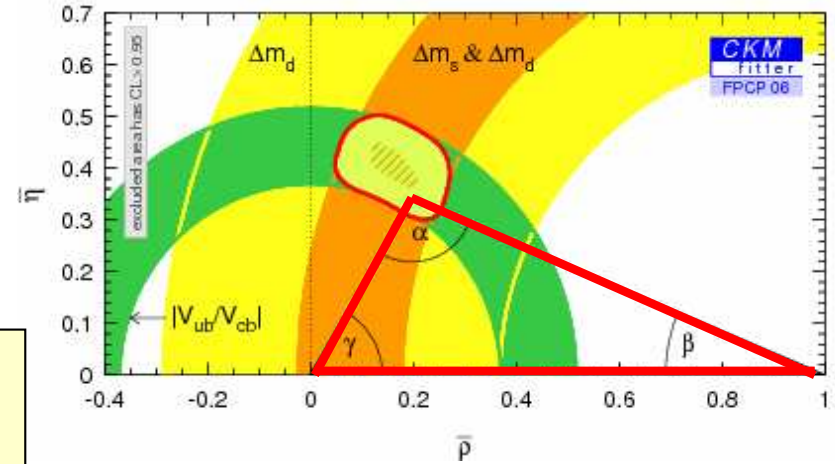
Summary



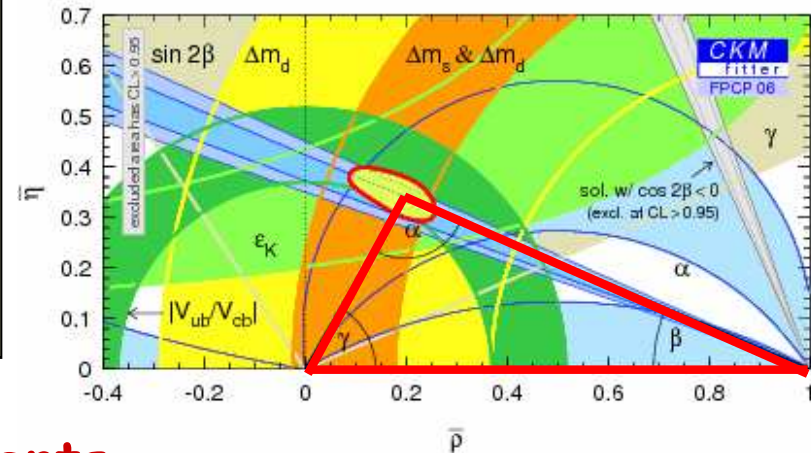
• **B factories:** Can use B mesons to measure the sides of the UT $|V_{ub}|$, $|V_{cb}|$, $|V_{td}|$, $|V_{tb}|$...

- $|V_{ub}|$: $\sigma \sim 7\%$, 2 new BaBar results
- $|V_{cb}|$: $\sigma \sim 2\%$, new BaBar $B \rightarrow D^* \lnu$ FF result
- $B \rightarrow \tau \nu$: ($|V_{ub}|$, $|V_{ub}|/|V_{td}|$) new BaBar result, Belle's first observation, UT constraints improved!
- Δm_d : $\sigma \sim 0.8\%$, combined with Δm_s ($\sigma \sim 3\%$) new results from CDF and D0: $|V_{td}|/|V_{ts}|$, $\sigma \sim 4\%$
- $B \rightarrow \rho/\omega \gamma$: BaBar & Belle "state of the art". Combined with $B \rightarrow K^* \gamma$: $|V_{td}|/|V_{ts}|$, $\sigma \sim 12\%$

Apex fit with the "sides" constraints only



Apex fit with all constraints



Great improvements since last few years, let's continue the good work!

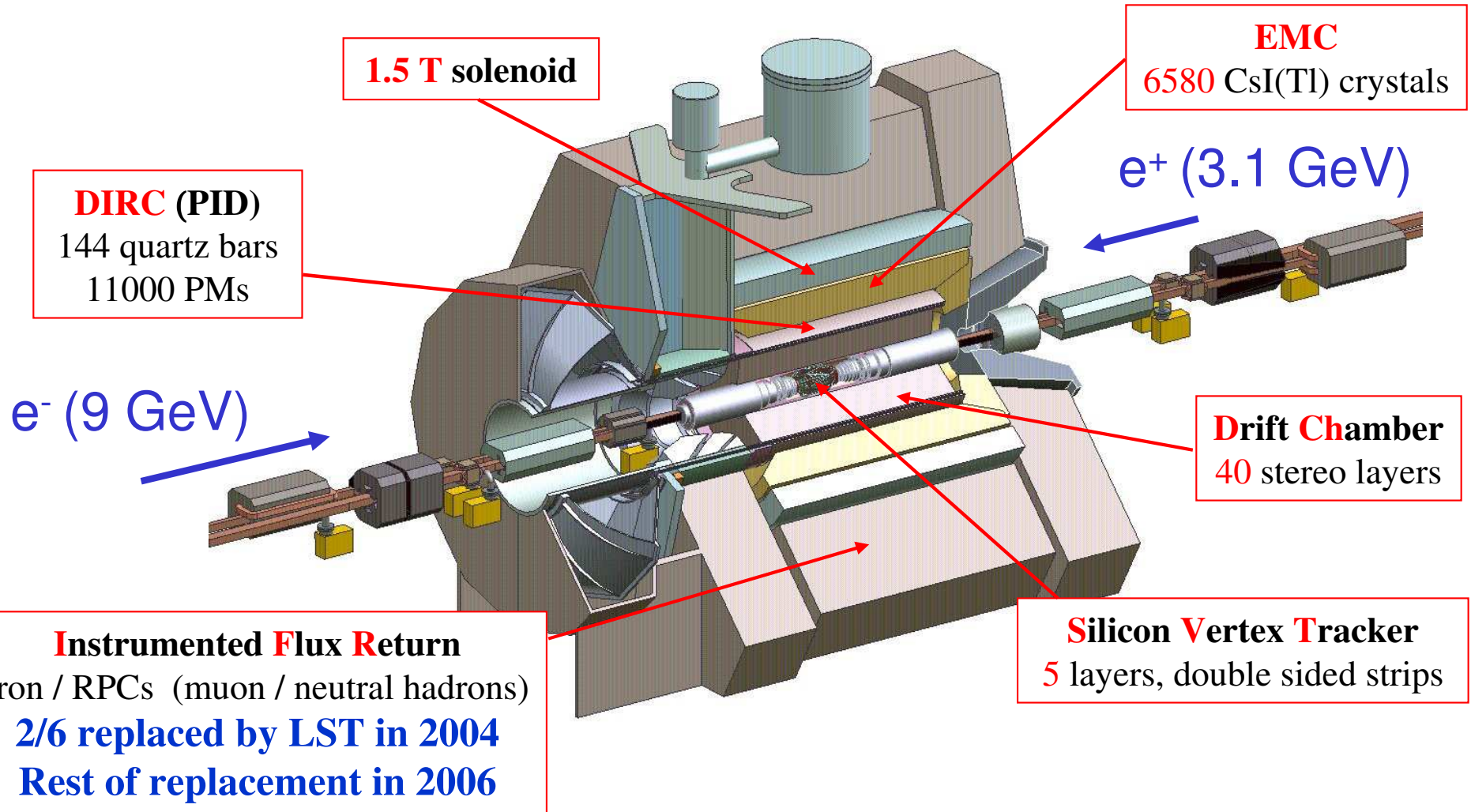


Backup Sides



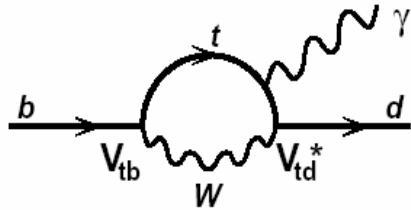


The BABAR Detector





Theoretical inputs... more details (I)



$$\frac{B(B \rightarrow \rho \gamma)}{B(B \rightarrow K^* \gamma)} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \left(\frac{1 - m_\rho^2/m_B^2}{1 - m_{K^*}^2/m_B^2} \right)^3 \zeta^2 [1 + \Delta R]$$

Theoretical inputs

$$b \rightarrow d \gamma \quad / \quad b \rightarrow s \gamma$$

ζ Flavor-SU(3) breaking between ρ and K^*
 $\Delta\zeta/\zeta \approx 10\%$ ΔR Annihilation correction
 ~ 0 , small impact

$$\Delta m_d = \frac{G_F}{6\pi^2} m_{Bd} \hat{B}_{Bd} F_{Bd}^2 \eta_B m_w^2 S_0(x_t) |V_{td}|^2$$

Theoretical inputs

$$\Delta m_d \quad \& \quad \Delta m_s$$

Bag Factor & meson decay constant taken from LQCD, dominates theoretical error (~20%)

Using Δm_s , many uncertainties cancel... theoretical error on $\left| \frac{V_{ts}}{V_{td}} \right|$ (~5%) $\frac{\Delta m_s}{\Delta m_d} = \frac{m_{Bs}}{m_{Bd}} \zeta^2 \left| \frac{V_{ts}}{V_{td}} \right|^2$

Theoretical inputs... more details (II)

$$B \rightarrow D^* \ell \nu \quad FF$$

$$\frac{d\Gamma}{dw} \propto \mathcal{G}(w) \mathcal{F}(w)^2 |V_{cb}|^2$$

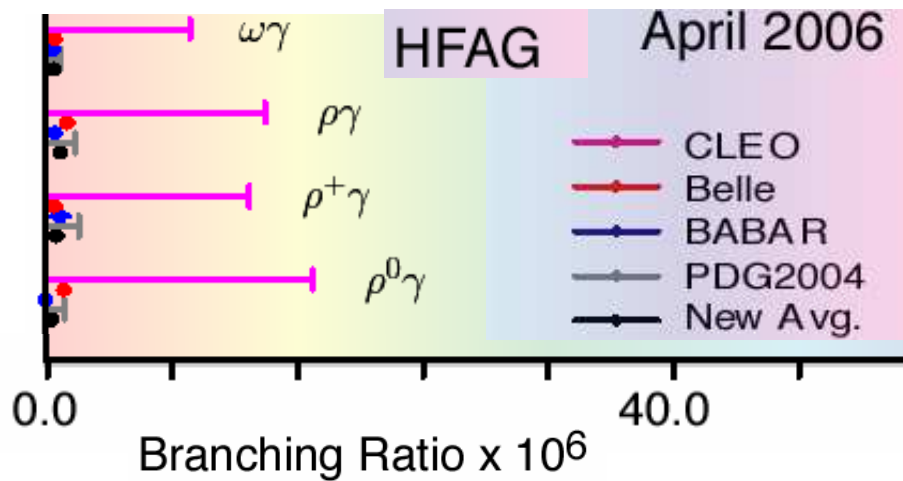
$\mathcal{G}(w)$ known phase space factor
 $\mathcal{F}(w)$ Form Factor (FF)
 $w = D^*$ boost in B rest frame

- $F(1)=1$ in heavy quark limit; lattice QCD says: $F(1) = 0.919$ Hashimoto et al, PRD 66 (2002) 014503
- Shape of $F(w)$ unknown
- Parametrized with ρ^2 (slope at $w = 1$) and form factor ratios
 $R_1, R_2 \sim$ independent on w
 h_{A1} expansion a-la Caprini-Lellouch-Neubert Nucl. Phys. B 530, 153 (1998)

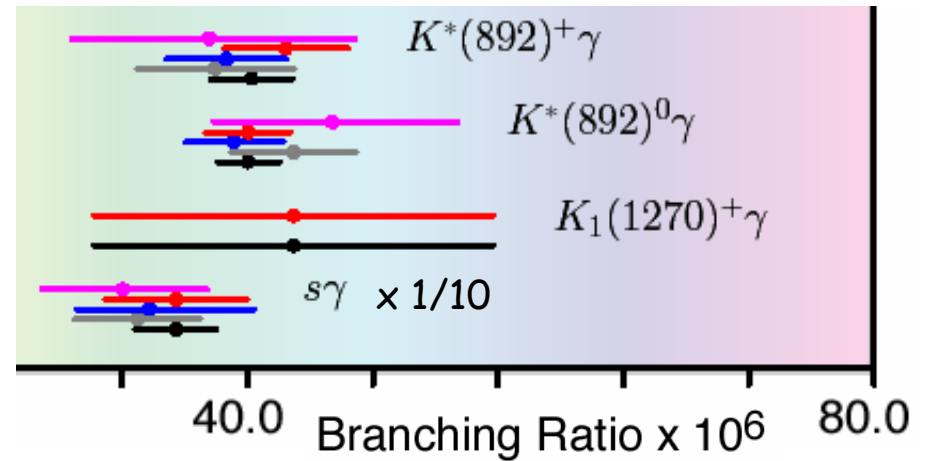
$$b \rightarrow d \gamma$$

$$b \rightarrow s \gamma$$

Status (HFAG Winter 2006)



$$\sigma \approx 30\%$$



$$\sigma \approx 10\%$$

$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.16 \pm 0.02$$

$$\sigma \approx 13\%$$



Δm_s (CDF)



Copied from Guillermo Gomez-Ceballos's talk, FPCP 2006

Frequency domain: **amplitude scan**

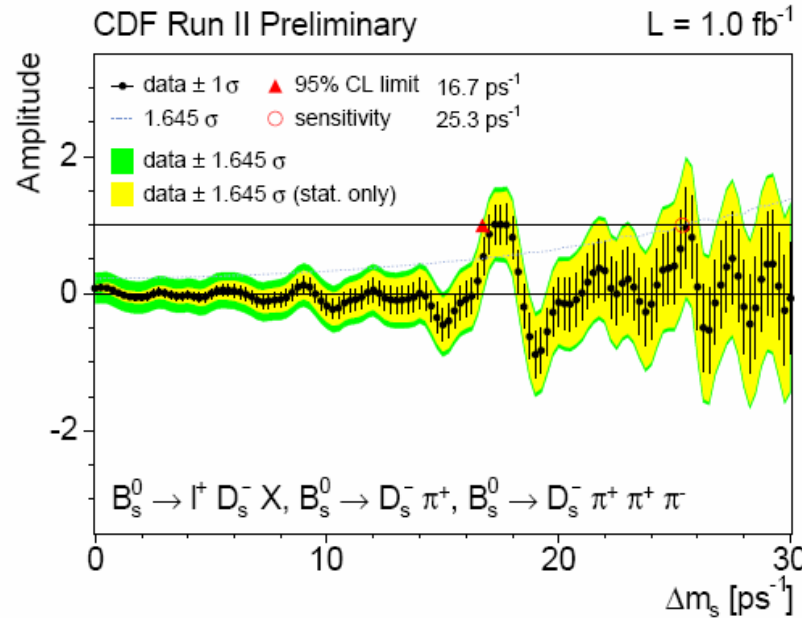
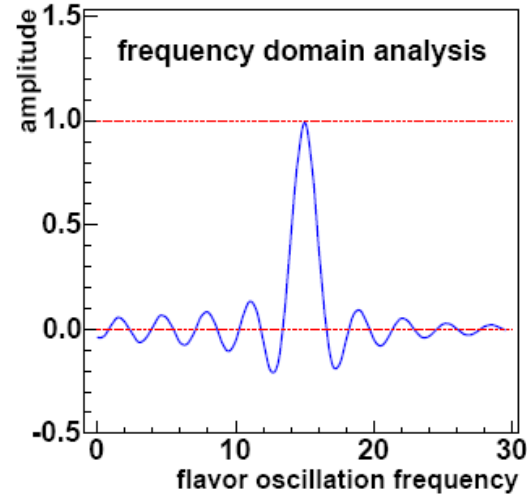
+ introduce amplitude:

$$P(t) \sim (1 \pm \mathcal{A}D \cos \Delta m_s t)$$

+ fit for \mathcal{A} at different Δm_s

⇒ obtain frequency spectrum

+ true $\Delta m_s \Rightarrow \mathcal{A} = 1$, else $\mathcal{A} = 0$



\mathcal{A} compatible with 1 for $\Delta m_s \sim 17.25$ ps⁻¹!



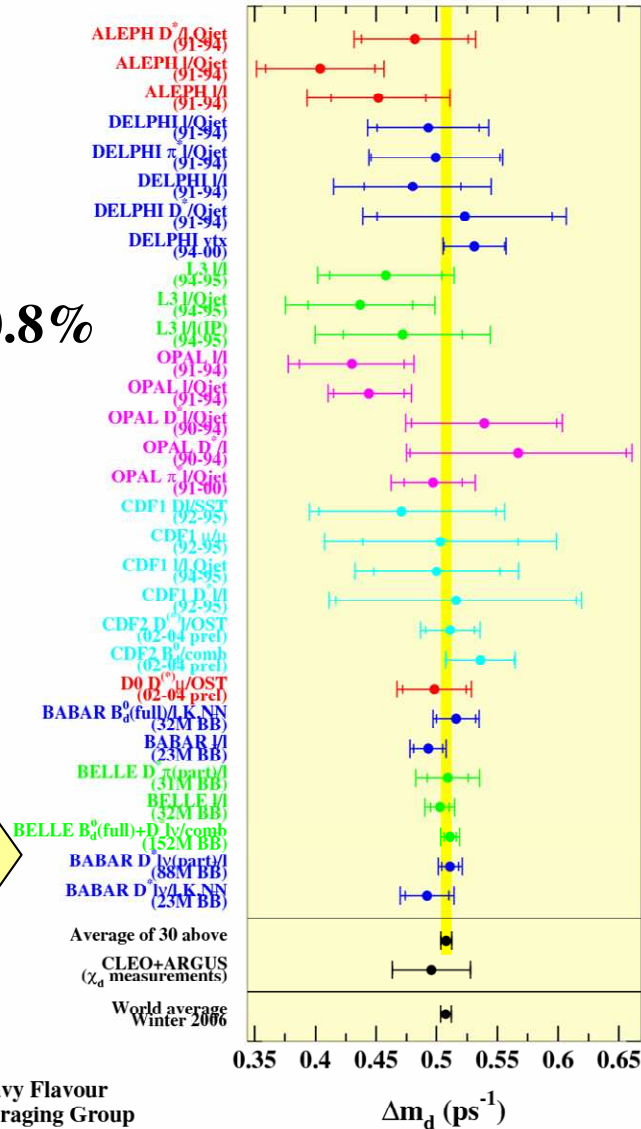
Δm_d and Δm_s Status



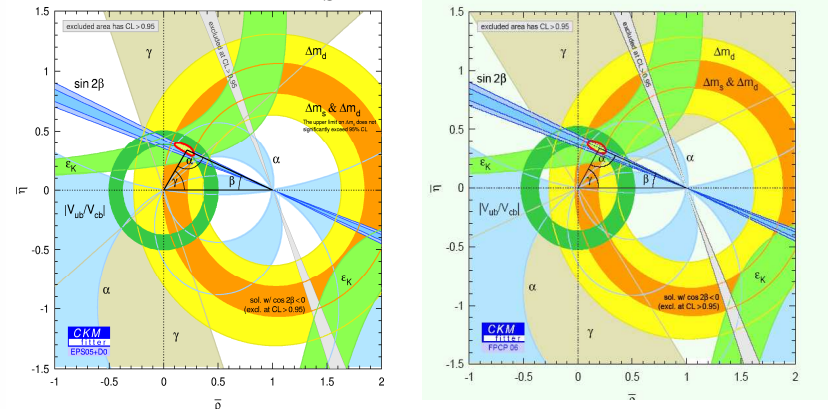
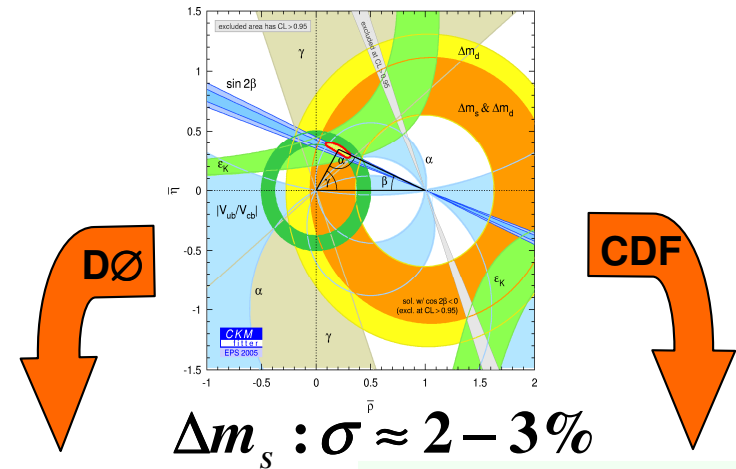
$\Delta m_d : \sigma \approx 0.8\%$

Δm_d from BaBar shown in this talk

Heavy Flavour Averaging Group



- 0.482 ± 0.044 ± 0.024 ps^{-1}
- 0.404 ± 0.045 ± 0.027 ps^{-1}
- 0.452 ± 0.039 ± 0.044 ps^{-1}
- 0.493 ± 0.042 ± 0.027 ps^{-1}
- 0.499 ± 0.053 ± 0.015 ps^{-1}
- 0.480 ± 0.040 ± 0.051 ps^{-1}
- 0.523 ± 0.072 ± 0.043 ps^{-1}
- 0.531 ± 0.025 ± 0.007 ps^{-1}
- 0.458 ± 0.046 ± 0.032 ps^{-1}
- 0.437 ± 0.043 ± 0.044 ps^{-1}
- 0.472 ± 0.049 ± 0.053 ps^{-1}
- 0.430 ± 0.043 ± 0.028 ps^{-1}
- 0.444 ± 0.029 ± 0.020 ps^{-1}
- 0.539 ± 0.060 ± 0.024 ps^{-1}
- 0.567 ± 0.089 ± 0.029 ps^{-1}
- 0.497 ± 0.024 ± 0.025 ps^{-1}
- 0.471 ± 0.078 ± 0.068 ± 0.034 ps^{-1}
- 0.503 ± 0.064 ± 0.071 ps^{-1}
- 0.500 ± 0.052 ± 0.043 ps^{-1}
- 0.516 ± 0.099 ± 0.029 ± 0.035 ps^{-1}
- 0.511 ± 0.020 ± 0.014 ps^{-1}
- 0.536 ± 0.028 ± 0.006 ps^{-1}
- 0.498 ± 0.026 ± 0.016 ps^{-1}
- 0.516 ± 0.016 ± 0.010 ps^{-1}
- 0.493 ± 0.012 ± 0.009 ps^{-1}
- 0.509 ± 0.017 ± 0.020 ps^{-1}
- 0.503 ± 0.008 ± 0.010 ps^{-1}
- 0.511 ± 0.005 ± 0.006 ps^{-1}
- 0.511 ± 0.007 ± 0.007 ps^{-1}
- 0.492 ± 0.018 ± 0.013 ps^{-1}
- 0.508 ± 0.004 ps^{-1}
- 0.495 ± 0.032 ps^{-1}
- 0.507 ± 0.004 ps^{-1}



$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.208^{+0.008}_{-0.007}$$

$\sigma \approx 4\%$