Five-dimensional Trinification

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- C. D. Carone and J. M. Conroy,
"Higgsless GUT breaking and trinification,"
Phys. Rev. D **70**, 075013 (2004) [arXiv:hep-ph/0407116].

And a brief advertisement for:

- C. D. Carone,
"Tri-N-ification,"
Phys. Rev. D **71**, 075013 (2005) [arXiv:hep-ph/0503069].

Extra Dimensions provide new mechanisms for gauge symmetry breaking.

Example: $G = SU(2)$ on $Z_2 \times Z_2$ orbifold

$$A^\mu(x,-y) \to P\,A^\mu(x,y)\,P^{-1}$$
$$A^5(x,-y) \to -PA^5(x,y)P^{-1}$$
$$A^\mu(x,\pi R - y) \to P'\,A^\mu(x,\pi R + y)\,P'^{-1}$$
$$A^5(x,\pi R - y) \to -P'\,A^\mu(x,\pi R + y)\,P'^{-1}$$

where $P = diag(1,-1)$ and $P' = diag(1,1)$.

Orbifold parity P and SU(2) generators T^a do not commute.

zero-modes $\leftrightarrow$ unbroken U(1)

Same approach can be used to break

SU(5) $\to$ SU(3) $\times$ SU(2) $\times$ U(1)

Note: rank of the gauge group is preserved.

Larger GUT groups require additional symmetry breaking.

Previous example can be described via boundary conditions on the interval $y \in [0, \pi R]$

$$A_\mu^{1,2}(x,0) = \partial_5 A_\mu^3(x,0) = \partial_5 A_\mu^a(x,\pi R) = 0$$
$$\partial_5 A_5^{1,2}(x,0) = A_5^3(x,0) = A_5^a(x,\pi R) = 0$$

Other boundary conditions are possible.

Consider a 5D SU(N) gauge field

$$S = -\frac{1}{2}\int d^4x \int_0^{\pi R} dy \, \mathrm{Tr}\ F^{MN} F_{MN}$$

which includes

$$S \supset -\frac{1}{2}\int d^4x \int_0^{\pi R} dy \, (\partial_\mu A_\nu^a \partial^\mu A^{a\nu} - \partial_5 A^{a\mu} \partial_5 A_\mu^a)$$

Consider the variation δA_ν^a:

$$\delta S = -\int d^4x \int_0^{\pi R} dy \, (\partial_\mu A_\nu^a \partial^\mu \delta A^{a\nu} - \partial_5 A_\nu^a \partial_5 \delta A^{a\nu})$$

Integrate by parts, retain surface terms at $y = 0$ and $y = \pi R$:

$$\begin{aligned} \delta S &= \int d^4x \int_0^{\pi R} dy \, (\partial_\mu \partial^\mu A_\nu^a - (\partial_5)^2 A_\nu^a) \delta A^{a\nu} \\ &+ \int d^4x \, (\partial_5 A^{a\nu}) \delta A_\nu^a \, |_0^{\pi R} \end{aligned}$$

Add a brane-localized mass $-V\mathrm{Tr}A^\mu A_\mu$ at πR

$$\begin{aligned}\delta S &= \int d^4x \int_0^{\pi R} dy\, (\partial_\mu \partial^\mu A^a_\nu - (\partial_5)^2 A^a_\nu)\delta A^{a\nu} \\ &+ \int d^4x\, (\partial_5 A^{a\nu})\delta A^a_\nu \,|_0^{\pi R} - V A^{a\nu}\delta A^a_\nu \,|^{\pi R}\end{aligned}$$

This gives the bulk EOM + BC's

$$\begin{aligned}(\Box - \partial_5^2)A^{a\mu} &= 0 \\ (\partial_5 A^{a\nu})\delta A^a_\nu &= 0 \qquad (y=0) \\ (\partial_5 A^{a\nu} - V A^{a\nu})\delta A^a_\nu &= 0 \qquad (y=\pi R)\end{aligned}$$

Decompose

$$A^\mu(x^\nu, y) = A^{(k)\mu}(x) f^{(k)}(y)$$

so that

$$\begin{aligned}\Box A^{(k)\mu}(x) &= -M_k^2 A^{(k)\mu}(x) \\ -\partial_5^2 f^{(k)}(y) &= M_k^2 f^{(k)}(y)\end{aligned}$$

and choose

$$\partial_5 f\,|_0 = 0 \longrightarrow f^{(k)}(y) = N_k \cos(M_k y)$$

The remaining condition $(\partial_5 - V) f \mid_{\pi R} = 0$ yields the transcendental equation

$$M_k \tan(M_k \pi R) = -V$$

Spectrum for large V ($M_c \equiv 1/R$):

$$M_k \approx \frac{M_c}{2}(2n+1)(1+\frac{M_c}{\pi V}+\cdots)$$

$n = 0, 1, 2, \ldots.$

Comments:

- Can be realized via vevs v_h in a boundary Higgs sector, $-V \sim \mathcal{O}(g_5^2 v_h^2)$.
- Rank of the gauge group can be reduced.
- Large v_h limit decouples boundary Higgs sector, BUT

$$M_1 \rightarrow \frac{M_c}{2}$$

This corresponds to "Higgsless" symmetry breaking.

Application to Trinification

Gauge group: $G_T =$SU(3)$_C\times$SU(3)$_L\times$SU(3)$_R$

matter generation:

$$\psi(\mathbf{27}) = \psi(\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) + \psi(\bar{\mathbf{3}},\mathbf{1},\mathbf{3}) + \psi(\mathbf{3},\bar{\mathbf{3}},\mathbf{1})$$

$$\equiv \psi_C + \psi_L + \psi_R$$

$$\psi_c = \begin{pmatrix} E^{0c} & E & e \\ -E^c & E^0 & \nu \\ e^c & N^c & N \end{pmatrix}, \quad \psi_L = \begin{pmatrix} u^c_{\bar{r}} & u^c_{\bar{g}} & u^c_{\bar{b}} \\ d^c_{\bar{r}} & d^c_{\bar{g}} & d^c_{\bar{b}} \\ B^c_{\bar{r}} & B^c_{\bar{g}} & B^c_{\bar{b}} \end{pmatrix}$$

$$\psi_R = \begin{pmatrix} u_r & d_r & B_r \\ u_g & d_g & B_g \\ u_b & d_b & B_b \end{pmatrix}$$

Z_3 symmetry permutes the labels (C,L,R)

$\rightarrow g_C = g_L = g_R$ at M_{GUT}

Example:

$$Z_3[\psi_R\psi_L\phi_C] = \psi_R\psi_L\phi_C + \psi_C\psi_R\phi_L + \psi_L\psi_C\phi_R$$

Conventional Symmetry Breaking:

$$\phi(\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v_1 \end{pmatrix}, \quad \chi(\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & v_2 & 0 \end{pmatrix}$$

$+$ complicated potential. Via BC's:

$$\partial_5 A^\mu_C(x,0) = \partial_5 A^\mu_C(x,\pi R) = 0 \ .$$

$$\partial_5 A^a_L(x,0) = \partial_5 A^a_L(x,\pi R) = 0 \text{ for } a = 1\ldots 3$$

$$\left.\begin{array}{c} \partial_5 A^a_L(x,0) = 0 \\ \partial_5 A^a_L(x,\pi R) = V_L A^a_L(x,\pi R) \end{array}\right\} \text{ for } a = 4\ldots 7$$

$$\left.\begin{array}{c} \partial_5 A^a_R(x,0) = 0 \\ \partial_5 A^a_R(x,\pi R) = V_R A^a_R(x,\pi R) \end{array}\right\} \text{ for } a = 1,2,4\ldots 7$$

3 remaining U(1)s: A^8_L, A^3_R, A^8_R. We want:

$$A^\mu_Y = -\frac{1}{\sqrt{5}}(A^8_L + \sqrt{3}A^3_R + A^8_R)^\mu$$

to obtain the GUT-scale prediction

$$\sin^2\theta_W = 3/8$$

Brane mass matrix, $M = 0$ eigenvector:

$$(-1/\sqrt{5}, -\sqrt{3}/\sqrt{5}, -1/\sqrt{5})$$

Most general solution: $\mathcal{A} \equiv (A_L^8, A_R^3, A_R^8)^\mu$

$\partial_5 \mathcal{A}(x, 0) = 0,$ and $\partial_5 \mathcal{A}(x, \pi R) = \mathcal{M}\mathcal{A}(x, \pi R)$

with

$$\mathcal{M} = \begin{pmatrix} V_1 & -\frac{1}{2\sqrt{3}}(V_1 + V_3) & -\frac{1}{2}(V_1 - V_3) \\ -\frac{1}{2\sqrt{3}}(V_1 + V_3) & \frac{1}{6}(V_2 + V_3) & \frac{1}{2\sqrt{3}}(V_1 - V_2) \\ -\frac{1}{2}(V_1 - V_3) & \frac{1}{2\sqrt{3}}(V_1 - V_2) & \frac{1}{2}(V_2 - V_3) \end{pmatrix}$$

Comments:

- $\phi(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ and $\chi(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ boundary Higgs correspond to special choices for V_1, V_2, V_3.
- Matter / EW Higgs $(\mathbf{27} + \overline{\mathbf{27}})$ on πR brane
- Exotic components can be decoupled if:

1. Include $\mathbf{27}s$ with $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ vevs, and $\mathbf{108}$ with $(\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$ vev at πR boundary.
2. Allow higher-dimension ops with cutoff Λ
3. Take the Higgsless limit holding V_i/Λ fixed.

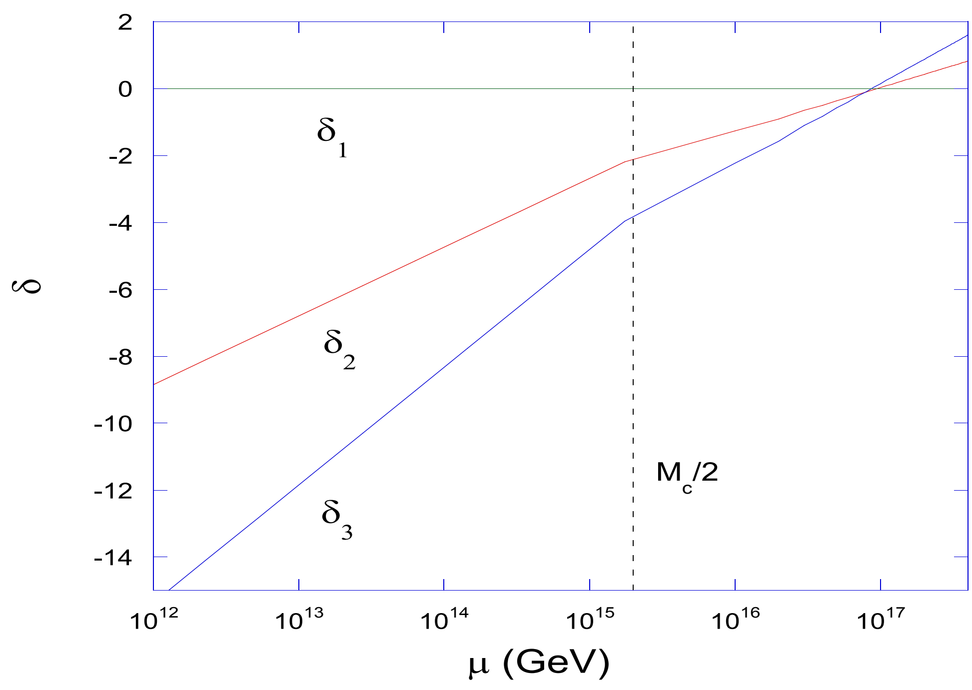
Gauge Unification

Consider $\delta_i(\mu) = \alpha_i^{-1}(\mu) - \alpha_1^{-1}(\mu)$. Above $M_c/2$,

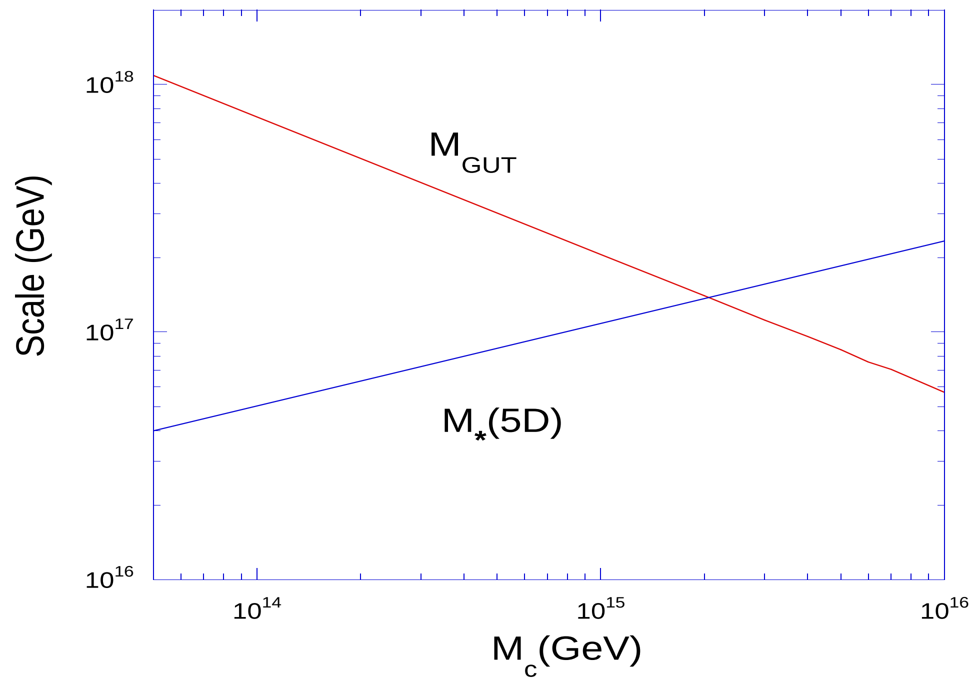
$$\delta_i(\mu) = \delta_i(M_c/2) - \frac{1}{2\pi} R_i(\mu)$$

$$\begin{aligned} R_2(\mu) &= -\frac{28}{5}\log(\frac{\mu}{M_c/2}) - 4 \sum_{0<nM_c<\mu} \log(\frac{\mu}{nM_c}) \\ &+ 4 \sum_{0<(n+1/2)M_c<\mu} \log(\frac{\mu}{[n+1/2]M_c}), \end{aligned}$$

$$\begin{aligned} R_3(\mu) &= -\frac{48}{5}\log(\frac{\mu}{M_c/2}) - 6 \sum_{0<nM_c<\mu} \log(\frac{\mu}{nM_c}) \\ &+ 6 \sum_{0<(n+1/2)M_c<\mu} \log(\frac{\mu}{[n+1/2]M_c}) \end{aligned}$$



Unification is delayed by a tower of threshold corrections



Unification and 5D Planck scale may coincide.

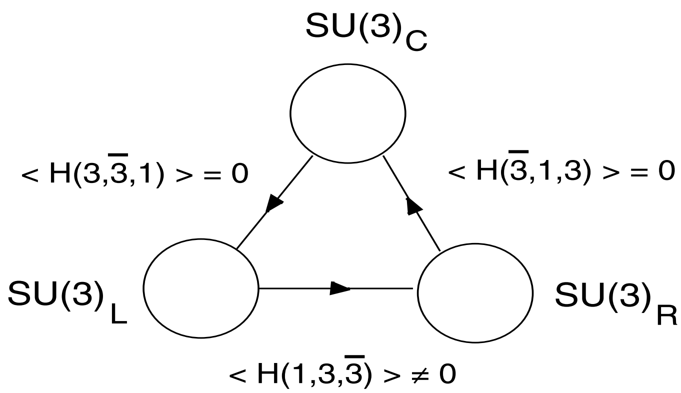
$$M_{*(4)}^{2} \sim (\pi R)^{n} \, M_{*(4+n)}^{n+2}$$

A Conclusion and an Advertisement

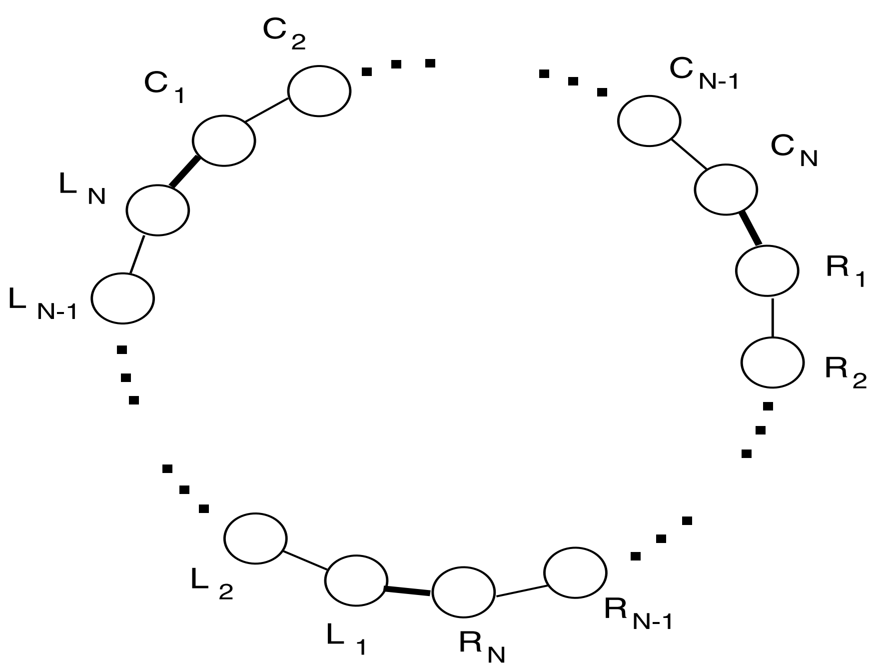
"Higgsless" symmetry-breaking can be useful in constructing GUT's with rank > 4 and decoupling unwanted physics from the symmetry breaking sector (proton decay).

The physics just described can be recovered in 4D theories via deconstruction.

Ordinary trinification:



Tri-N-ification: (dark links are special)



See: C. D. Carone, Phys. Rev. D **71**, 075013 (2005) [arXiv:hep-ph/0503069]