~~Higgs~~ Models of EWSB and the EW Chiral Lagrangian

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- Tension: Unitarity vs. EW Constraints
- (Re)Introduce the EChL
- Matching the EChL to a ("general") ~~Higgs~~ Model (Bosonic sector)
- The S parameter from the EChL
- Observables from the EChL: Heavy SM Higgs vs. ~~Higgs~~
- Conclude and Outlook

Unitarity in ~~Higgs~~ Models

- Besides providing a source of EWSB (and fermion masses), the SM Higgs also plays a role in $V_L V_L \to V_L V_L$ scattering.
- Without something "like" a Higgs, the amplitudes for $V_L V_L$ scattering violate Unitarity @ ≈ 1.8 TeV.
- Inclusion of Higgs exchange cancels this violation. $(M_H \leq 870\, GeV)$
- In extra-d ~~Higgs~~ models, the delay of Unitarity violation is provided by the exchange of Kaluza-Klein gauge bosons (W', Z').
- Unitarity violation delayed if couplings involving both SM and KK gauge bosons obey certain sum rules . These sum rules are a general feature of all ~~Higgs~~ models. ("Model-independent")
- But, in order for this mechanism to work, $M_{W'} \approx M_{Z'} \leq 1.8$ TeV

Problems with S

- In their "barest" forms (e.g. No brane kinetic terms, bulk couplings equal, etc.), ~~Higgs~~ models produce large contributions to the S parameter due to mixing between the SM and KK gauge bosons.
- S "counts" the number of degrees of freedom participating in the EW sector.
- Typically, from the bosonic sector alone, $S \sim 1$. While, due to "custodial" symmetry present in these models, $T \sim 0$.
- The large (positive) contribution to S can be lowered by several modifications ("delocalized" fermions, brane kinetic terms, etc.).
- However, for minimal ~~Higgs~~ models, S can only be made smaller by making KK gauge bosons very heavy.

"Tension" between Unitarity constraints (light KK modes) and constraints from EW observables (heavy KK modes)

(Re)Introducing the EChL

- In any case, $\mathrm{M}_{\mathrm{V'}} \gg M_Z$ such that the first "signature" of ~~Higgs~~ models may lie in precision EW observables.
- Becomes useful to use techniques of EFT's.
- One EFT which provides an extremely economical description of the EWSB sector is the Electroweak Chiral Lagrangian (EChL). Originally, the EChL was used to describe strongly interacting theories (e.g. Heavy Higgs, Technicolor and composite Higgs).
- Based on an expansion in (p^2/Λ^2):

$$L_{EChL} = L^{(2)} + L^{(4)} + \cdots$$

- Physical d.o.f.'s: $W^{\pm}$, Z and associated GB's.
- Incorporates local ($SU(2)_L \times U(1)_Y$) and (appoximate) global ($SU(2)_L \times SU(2)_R$) symmetries of the SM

Building the EChL

- The basic building blocks ($W^{\pm}$, Z and GB's) are introduced via the matrices:

$$U = \exp\left(\frac{i\, w^a \tau^a}{v}\right) \qquad W_\mu = W_\mu^i \tau^i$$

$$B_{\mu\nu} = \frac{1}{2}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right) \qquad W_{\mu\nu} = \frac{1}{2}\left(\partial_\mu W_\nu - \partial_\nu W_\mu - \frac{ig}{2}[W_\mu, W_\nu]\right)$$

- And the covariant derivative: $D_\mu U = \partial_\mu U + igW_\mu U - ig' B_\mu U \tau^3$

- At lowest order, there is only one term which respects the symmetries:

$$L^{(2)} = \frac{v^2}{4} Tr\, D_\mu U\, D^\mu U^\dagger$$

- Note: the full Lagrangian consists of $L^{(2)}$ plus usual gauge kinetic, gauge-fixing and Fadeev-Popov terms.

- In Unitary gauge ($U \to 1$), $L^{(2)}$ just reproduces $W^{\pm}$, Z mass terms (v ≈ 246 GeV).

O(p^4) Terms in the EChL

- At next-to-leading order, the EChL contains 5 terms which respect the $SU(2)_L \times SU(2)_R$ symmetry and involve 5 free parameters:

$$L^{(4)} = L_1 (Tr(D_\mu U D^\mu U^\dagger))^2 + L_2 Tr(D_\mu U D_\nu U^\dagger) Tr(D^\mu U D^\nu U^\dagger)$$
$$-igL_{9,L} Tr(W^{\mu\nu} D_\mu U D_\nu U^\dagger) - ig' L_{9,R} Tr(B^{\mu\nu} D_\mu U D_\nu U^\dagger)$$
$$+ g g' L_{10} Tr(U^\dagger B^{\mu\nu} U W_{\mu\nu})$$

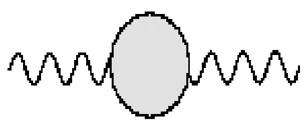
- Of the 5 free parameters: L_{10} enters into oblique corrections ($S = -16\pi L_{10}$)
  L_9's and L_{10} contribute to $\gamma WW/ZWW$ vertices
  L_1 and L_2 play a role in $V_L V_L$ scattering

- Different scenarios of EWSB simply lead to different values for the EChL parameters. Thus, measuring the L_i allows to differentiate the possibilities.

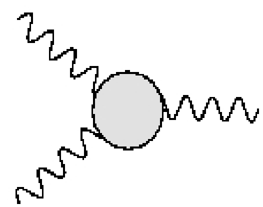
Calculating the EChL Parameters

- Formally, the EChL parameters can be calculated by "coupling" the heavy fields to the EChL and, subsequently, integrating out these fields.

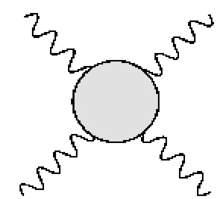
- Less formally, they can also be determined by matching Green's functions calculated in the two theories (EChL vs. "Full" model)



L_{10}



$\{L_{9,L}, L_{9,R}, L_{10}\}$



$\{L_1, L_2\}$

- For L_1 and L_2, it is enough to match the tree-level $V_L V_L$ scattering amplitudes in the two theories.

- To extract L_{10}, we need to match the full one-loop amplitudes for the two-point functions calculated in the two theories.

A Side-note: $W_L Z_L \to W_L Z_L$

- The scattering amplitude for this process has two potentially harmful terms which grow with energy as E^4 and E^2.

- The E^4 term can be canceled if the couplings between the SM and KK gauge bosons obey the sum rule:

$$g_{ZZWW} = g_{ZWW}^2 + \sum_i (g_{ZWW^i})^2$$

- The E^2 term can also be canceled if the couplings obey an additional sum rule:

$$2(g_{ZZWW} - g_{ZWW}^2)(M_W^2 + M_Z^2) + g_{ZWW}^2 \frac{M_Z^4}{M_W^2} = \sum_i (g_{ZWW^i})^2 \left[3(M_i^{\pm})^2 - \frac{(M_Z^2 - M_W^2)^2}{(M_i^{\pm})^2} \right]$$

- Note: these sum rules are "model-independent"

- In the following, we will assume that the sum rules are "saturated" for the first KK resonance, such that:

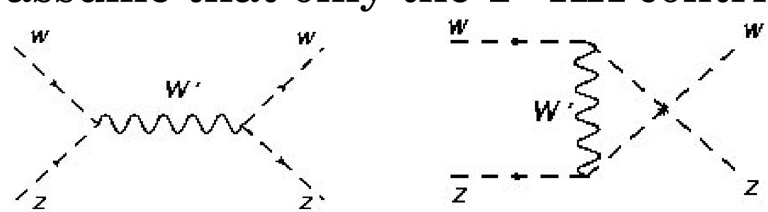
$$g_{ZWW'} \leqslant \frac{g_{ZWW} M_Z^2}{\sqrt{3} M_{W'} M_W}$$

EChL Parameters: L_1 and L_2

- To extract L_1 and L_2 , we need only match the tree-level amplitudes. In particular, consider $wz \to wz$. From the EchL, we have:

$$A^{wz}_{EChL} = \frac{t}{v^2} + \frac{4}{v^4}\left[2 L_1 t^2 + L_2 (s^2 + u^2)\right]$$

- On the full theory side, we have s- and u-channel contributions from the KK tower of W′s. If we assume that only the 1st KK contribution is important:



- Taking $M^2_{W'} \gg s, u$

$$A^{wz}_{W'} \simeq g^2_{ZWW'}\left[\frac{3t}{M^2_{W'}} + \frac{1}{M^4_{W'}}(-2t^2 + (s^2 + u^2)) + O\left(\frac{s^3}{M^6_{W'}}\right)\right]$$

- Matching the coefficients of t^2 and s^2:

$$L_1 = -L_2 = \frac{-v^4}{4}\left(\frac{g^2_{ZWW'}}{M^4_{W'}}\right) \simeq \frac{-v^2 M^2_W M^2_Z}{6 M^6_{W'}}$$

The L_{10} Parameter

- In order to extract L_{10}, we need to match the one-loop self-energies:

$$\Pi_L^{ab} + \Pi_{CT}^{ab} = \hat{\Pi}_L^{ab} + \hat{\Pi}_{CT}^{ab} + \Pi_{EChL}^{\hat{ab}} \qquad \text{(where } ab = WW, ZZ, \gamma\gamma \text{ and } \gamma Z\text{)}$$

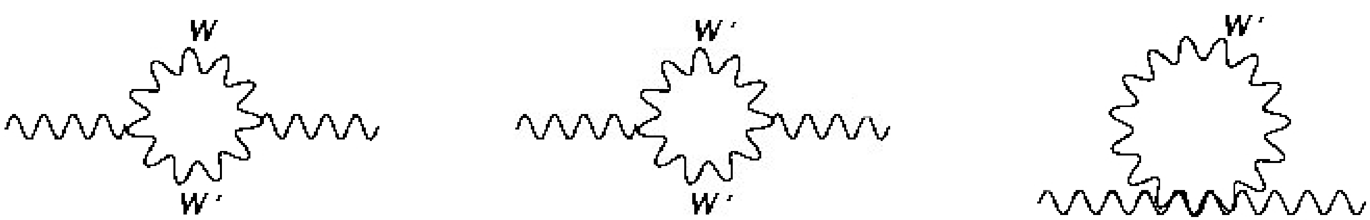
- To perform the matching, it's useful to separate the amplitudes into three distinct tensor structures:

$$\Pi_i^{ab} = C_1^{ab} g^{\mu\nu} + C_2^{ab}(q^2 g^{\mu\nu} - q^\mu q^\nu) + C_3^{ab} q^\mu q^\nu$$

- In this way, L_{10} is easily extracted. In the mass eigenstate basis, we find:

$$L_{10} = \frac{1}{g\,g'}\left[(\cos^2\theta_w - \sin^2\theta_w) C_2^{\gamma Z} + \sin\theta_w \cos\theta_w (C_2^{\gamma\gamma} - C_2^{ZZ})\right]$$

- And, in general, the amplitudes are calculated from:



L_{10} and the S Parameter

- The amplitudes are calculated in the Unitary gauge using dimensional regularization. To estimate the size of the contributions from the W', we make the identification:

$$\frac{1}{\epsilon}(4\pi)^{\epsilon}\Gamma(1+\epsilon)\rightarrow\log\left(\frac{\Lambda^2}{\mu^2}\right)$$

- We can write the self-energies in terms of 3 "fundamental" amplitudes. In particular,

$$\frac{1}{g^2_{\gamma WW'}}\Pi^{\gamma\gamma}=\frac{1}{g_{\gamma WW'}g_{ZWW'}}\Pi^{\gamma Z}=A_0^{\mu\nu}+2A_2^{\mu\nu}=0$$

- Thus, the only contribution to L_{10} comes from C_2^{ZZ}

$$L_{10}=\frac{-s_w c_w}{g\,g'}\left[\frac{g^2_{ZWW'}}{16\pi^2}\frac{17}{12}\frac{M^2_{W'}}{M^2_W}\right]\log\left(\frac{\Lambda^2}{\mu^2}\right)\approx\frac{-c_w^2}{16\pi^2}\frac{17}{36}\log\left(\frac{\Lambda^2}{M^2_{W'}}\right)$$

- Finally, the contribution to the S parameter: $\Delta S=-16\pi L_{10}$

Limits on S without a Higgs

- To compare ΔS with experiment, we first must "scale down" our result from $M_{W'}$ to M_Z:

$$\Delta S \to \frac{c_w^2}{\pi}\frac{17}{36}\log\left(\frac{\Lambda^2}{M_{W'}^2}\right)+\frac{1}{12\pi}\log\left(\frac{M_{W'}^2}{M_Z^2}\right)$$

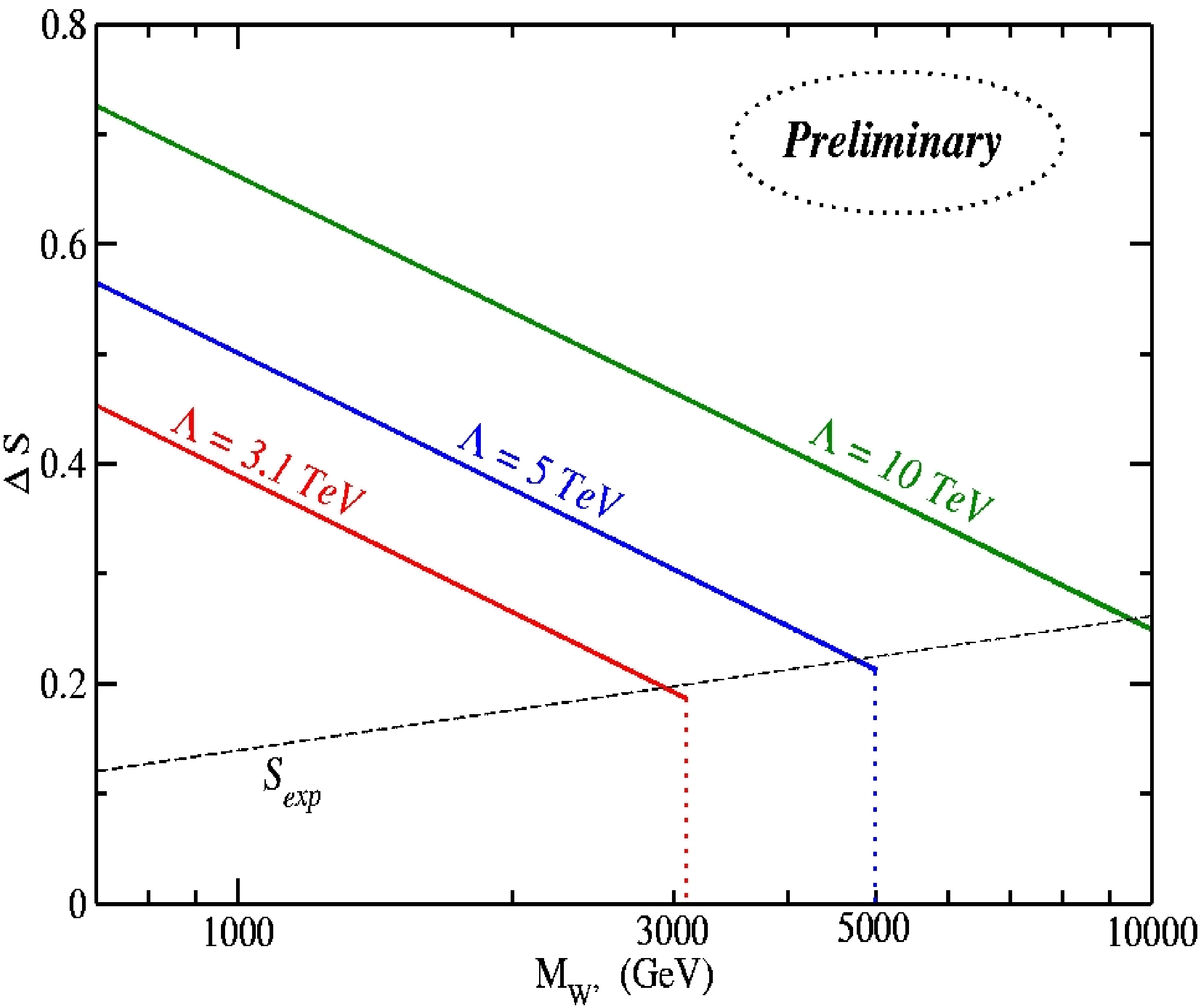
- Next, since most global analyses are performed assuming the presence of a SM Higgs, we have to subtract the contribution of the Higgs to S:

$$S_{Higgs}=\frac{-1}{6\pi}\left[\frac{5}{72}-\log\left(\frac{M_H}{M_Z}\right)\right]$$

- Then, we can compare with experimental limits on S by applying:

$$S_{\exp}=S_{ref}(M_H^{ref})+\frac{5}{72\pi}+\frac{1}{6\pi}\log\left(\frac{M_{W'}}{M_H^{ref}}\right)$$

S and the Scale of Unitarity Violation



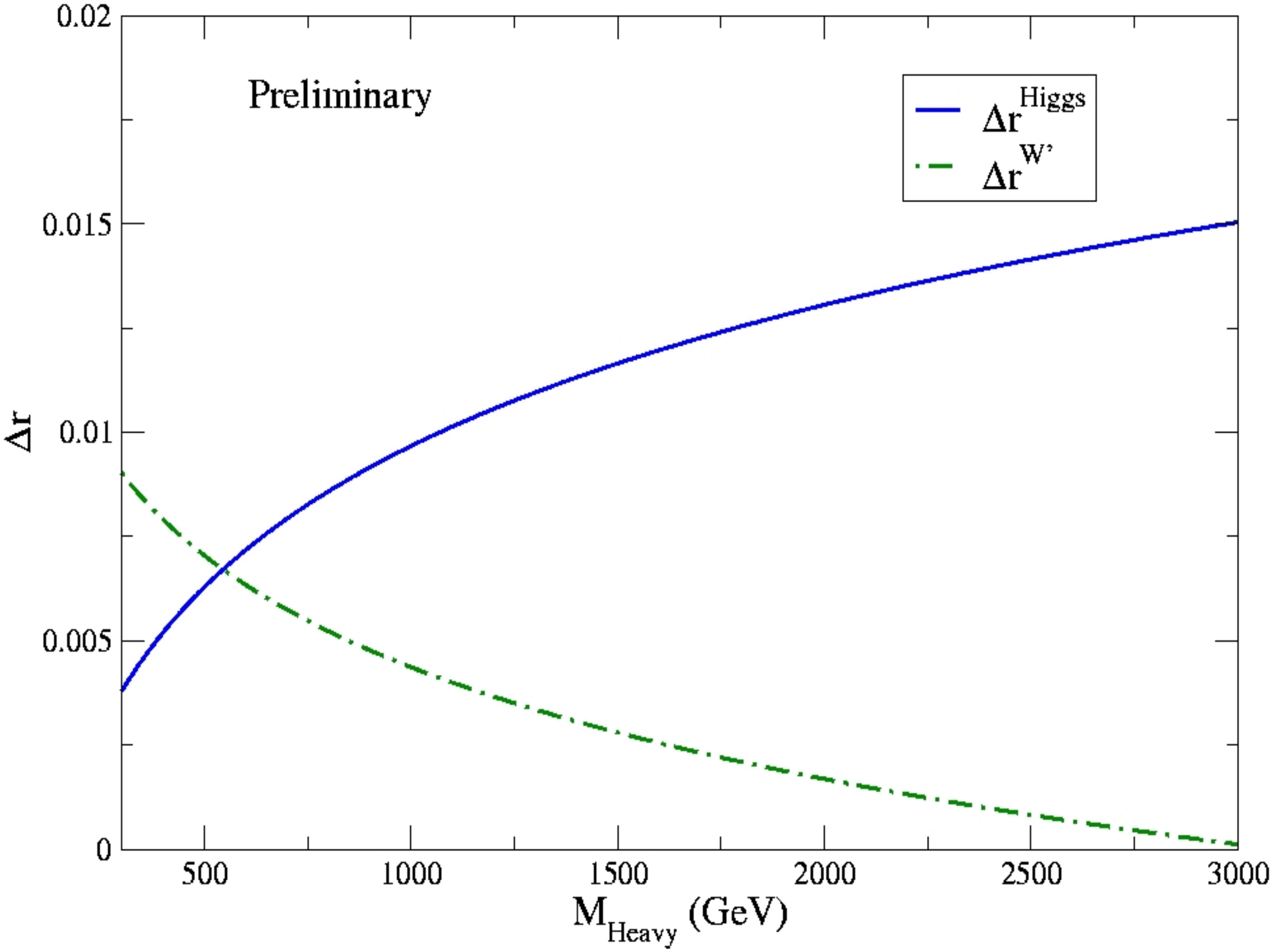
- For M_H^{ref} = 600 GeV and $S \geq 0$:

  $S_{ref} \leq 0.09$ (PDG2004).

- In each case, in order to satisfy bounds on S, $M_{W'} \approx \Lambda$.

- Tension between unitarity and constraints on S is, in a sense, somewhat "model independent."

Heavy Higgs vs. ~~Higgs~~ in the EChL



- Complete EChL analysis of heavy Higgs performed by Herrero and Ruiz-Morales (NPB418, 431 (1994)).

- Radiative corrections to the $W^{\pm}$ mass:

$$M_W^2 = \frac{\pi \alpha}{\sqrt{2} G_\mu s_w^2} (1 + \Delta r)$$

- For ~~Higgs~~: $\Delta r \approx \frac{-8 \pi \alpha}{s_w^2} L_{10}$

- "Inverse" behavior of heavy Higgs vs. ~~Higgs~~

Conclusions and Outlook

- Extra-dimensional ~~Higgs~~ models (in their 'barest' forms) exhibit a "tension" between Unitarity constraints (light KK's) and constraints coming from EW observables (heavy KK's).

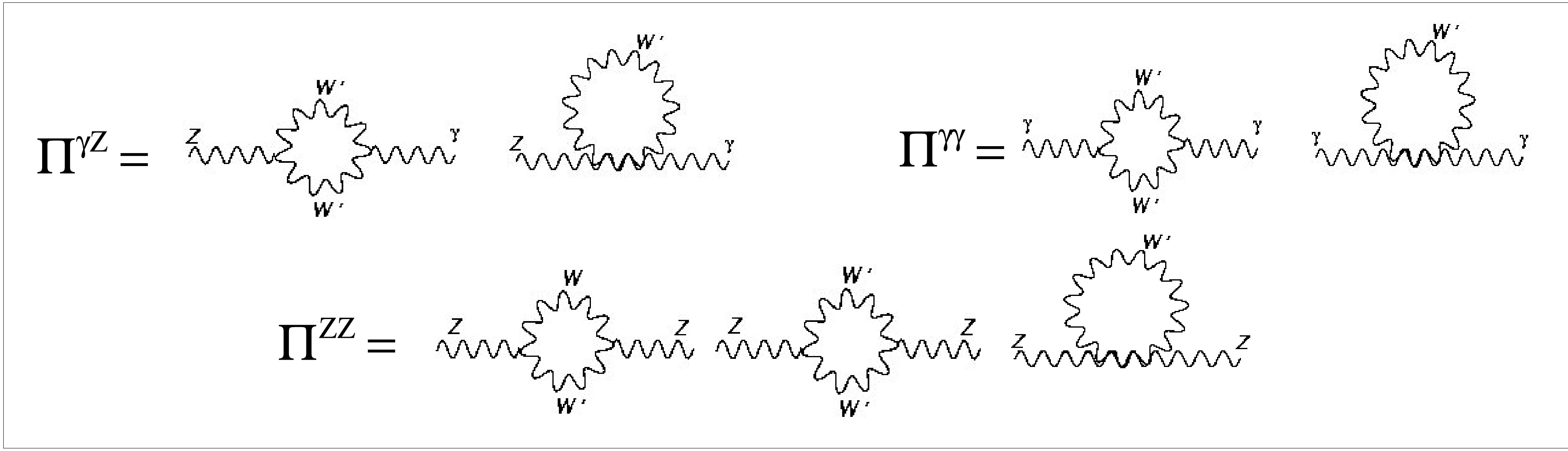
- The EChL provides an extremely economical framework in which to study these models.

- The EChL only involves light physical degrees of freedom, while incorporating both local and global symmetries present in the minimal SM.

- We have calculated the S parameter from a general ~~Higgs~~ model, using only the "model independent" sum rules. This calculation provides a good example of the tension present in these models.

- Finally, to determine the full EChL parameter space, one needs to match the $\gamma WW/ZWW$ vertices (@ one loop) in the two theories. Also, need to include the fermionic sector considering the important role this sector plays in the most realistic of these models (See Chivukula et al., PRD72 075012 (2005)).

Backup Slide: One-loop Amplitudes

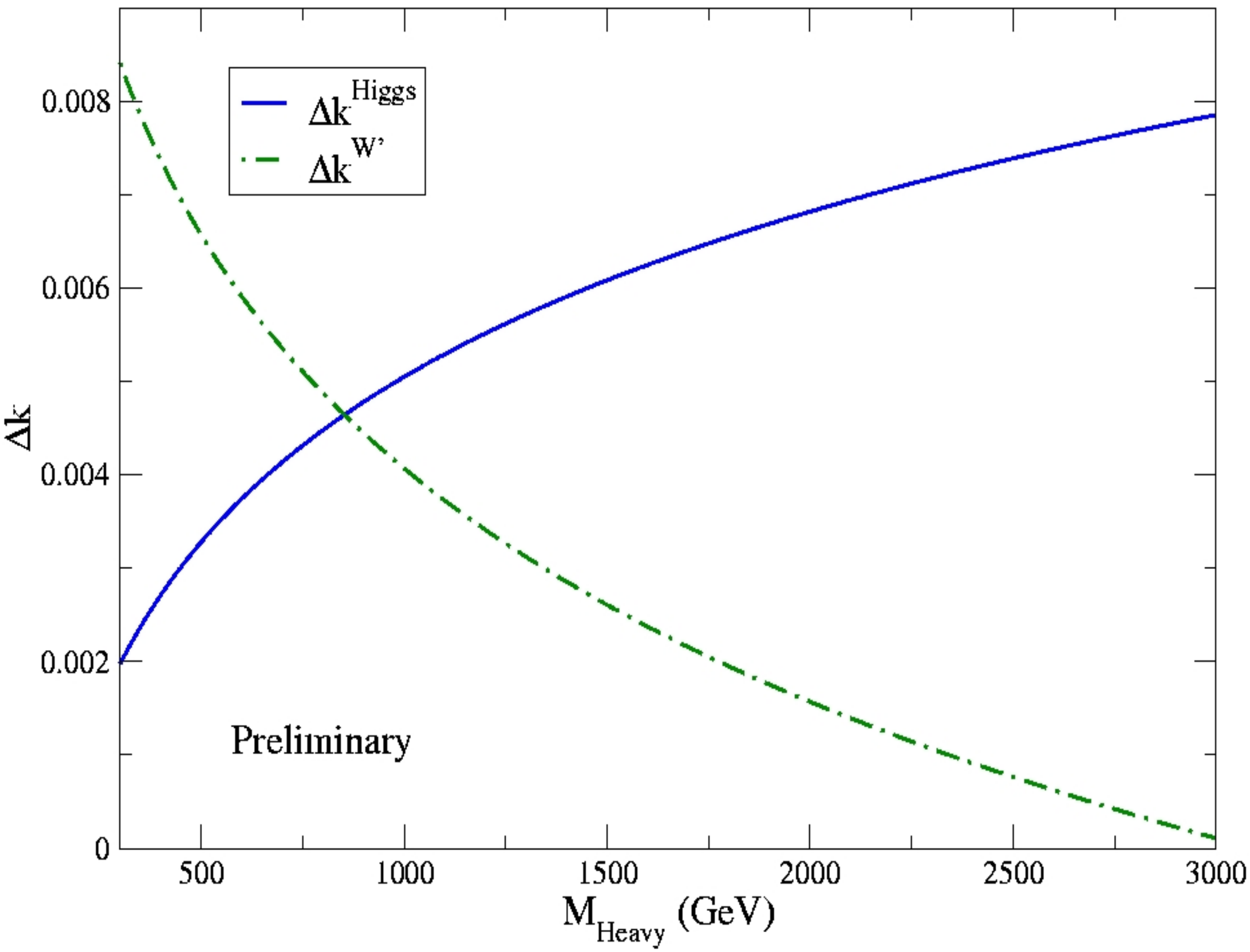


- Using DimReg in unitary gauge and identifying: $\frac{1}{\epsilon}(4\pi)^{\epsilon}\Gamma(1+\epsilon)\rightarrow\log\left(\frac{\Lambda^2}{\mu^2}\right)$

$$A_0^{\mu\nu}(q^2;M)=\frac{1}{16\pi^2}\left[\left(\frac{9M^2}{2}\right)g^{\mu\nu}+7q^{\mu}q^{\nu}\right]\log\left(\frac{\Lambda^2}{\mu^2}\right)\qquad A_2^{\mu\nu}(q^2;M)=\frac{1}{16\pi^2}\left[\left(\frac{-9M^2}{4}\right)g^{\mu\nu}\right]\log\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$A_1^{\mu\nu}(q^2;M_1,M_2)=\frac{1}{16\pi^2}M_2^2\left[g^{\mu\nu}\left(3-\frac{3M_2^2}{4M_1^2}+\frac{17}{12}\frac{q^2}{M_1^2}\right)+\frac{7}{6}\frac{q^{\mu}q^{\nu}}{M_1^2}\right]\log\left(\frac{\Lambda^2}{\mu^2}\right)$$

Backup Slide: Heavy Higgs vs. ~~Higgs~~



- Radiative corrections to A_{FB}^{μ} in $e^+e^- \rightarrow \mu^+\mu^-$

$$A_{FB}^{\mu} = 3\left(\frac{g_V g_A}{g_V^2 + g_A^2}\right)^2$$

$$g_A = \frac{1}{2}$$

$$g_V = -\frac{1}{2} + 2\sin^2(1+\Delta k)\, s_w^2$$

- For ~~Higgs~~:

$$\Delta k \approx -\frac{4\pi\alpha}{s_w^2}\frac{1}{c_w^2 - s_w^2} L_{10}$$