# Violations of the Narrow Width Approximation in SUSY

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## Outline

- Simplest case
- Sanity Checks
- More complicated examples
- Process Observable @ LHC
- Summary

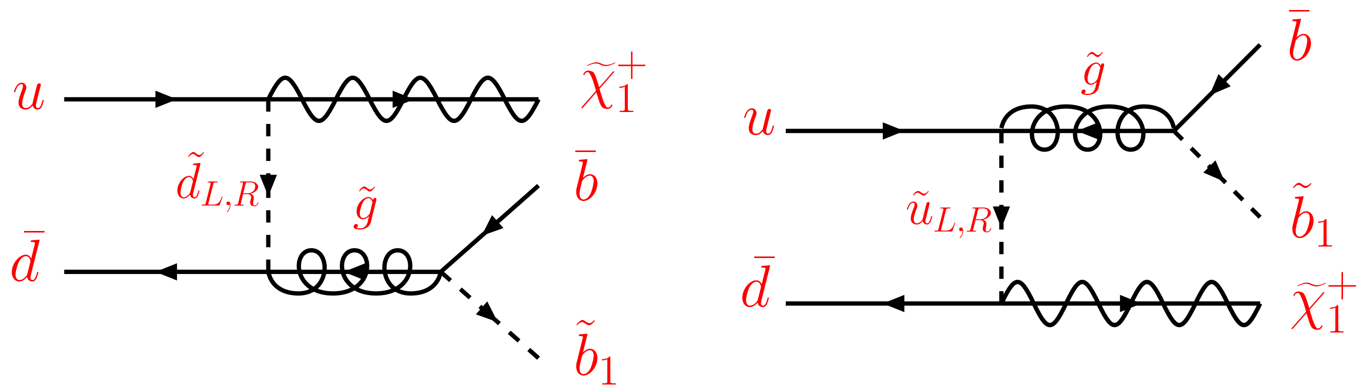
# Introduction

We investigate the validity of the Narrow Width Approximation

- Usually treat intermediate states on shell, then decay
- Makes several assumptions about the process
- Violations normally due to non-resonant diagrams
- We show the SUSY mass spectrum $\longrightarrow$ new violations
- Effects are usually jumbled with other complications (PDFs, interference, etc.)
- More diagrams $\longrightarrow$ worse complications

# Simplest: One resonance, Fixed Energy Beams

Few diagrams, with charged currents & mixed gauge couplings
$u\bar{d} \to \widetilde{\chi}_1^+ \bar{b}\tilde{b}_1$ is:




- No other resonant or non-resonant diagrams
- See NWA violations even *without* non-resonant diagrams
- $\chi_1^+$ can be long-lived; ignore decay cascade

Examine validity of NWA without complications

# The Narrow Width Approximation

$n$-body phase space factorizes:

$$
\begin{aligned}
d\Phi^n &= (2\pi)^4\delta^4\Big(k_1 + k_2 - \sum_i p_i\Big)\prod_{i=1}^{n}\frac{d^3p_i}{(2\pi)^3 2E_i} \\
&= (2\pi)^4\delta^4\Big(k_1 + k_2 - q - \sum_i p_i\Big)\prod_{i=3}^{n}\frac{d^3p_i}{(2\pi)^3 2E_i} \times \frac{d^3q}{(2\pi)^3 2E_q^0} \times \frac{dq^2}{2\pi} \times d\Phi^2(q; p_1, p_2)
\end{aligned}
$$

Ideal for resonances, via $\int dq^2$ and a 2-body decay.
Assume the Breit-Wigner propagator is separable, then:

$$
\begin{aligned}
\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} \frac{dq^2}{2\pi}\left(\frac{1}{q^2 - m^2 + im\Gamma}\right)^2 &= \int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}} \frac{dq^2}{2\pi}\,\frac{1}{(q^2 - m^2)^2 + (m\Gamma)^2} \\
&= \int_{q^2_{\mathrm{min}}-m^2}^{q^2_{\mathrm{max}}-m^2} dx\,\frac{1}{x^2 + (m\Gamma)^2} = \int_{-m^2}^{s-m^2} dx\,\frac{1}{x^2 + (m\Gamma)^2} \\
&\approx \int_{0}^{\infty} dx\,\frac{1}{x^2 + (m\Gamma)^2} = 2\int_{-\infty}^{\infty} dx\,\frac{1}{x^2 + (m\Gamma)^2} = \frac{\pi}{m\Gamma}\,.
\end{aligned}
$$

Assumes massless final states!

# Comparisons

We look at $u\bar{d} \longrightarrow \widetilde{\chi}_1^+ \bar{b}\tilde{b}_1$ over a variety of $\sqrt{s}$ Compare:

- $\sigma_{tot}$ = cross section w/o decays $[fb]$
- $\sigma_{ONS}$ = cross section w/ decays calculated on-shell
- $\sigma_{OFS}$ = cross section w/ decays calculated off-shell
- $\sigma_{ONS}/\sigma_{ONS}$

$\longrightarrow$ If NWA is valid, $\sigma_{ONS}/\sigma_{ONS} = 1$ for all $\sqrt{s}$

# Simplest case with fixed beams

| $\sqrt{s}$ [TeV] | $\sigma_{\rm tot}$ [fb] | $\sigma_{\rm ONS}$ | $\sigma_{\rm OFS}$ | $\sigma_{\rm OFS}/\sigma_{\rm ONS}$ |
| --- | --- | --- | --- | --- |
| 1.0 | 482 | 50.5 | 49.7 | 0.98 |
| 2.0 | 574 | 60.2 | 64.4 | 1.07 |
| 4.0 | 203 | 21.3 | 24.1 | 1.13 |
| 10 | 36.7 | 3.85 | 4.65 | 1.21 |
| 20 | 9.37 | 0.983 | 1.24 | 1.26 |
| 40 | 2.36 | 0.248 | 0.329 | 1.33 |
| 100 | 0.378 | 0.0397 | 0.0558 | 1.41 |
| 200 | 0.0946 | 0.00993 | 0.0146 | 1.47 |
| 400 | 0.0237 | 0.00248 | 0.00376 | 1.52 |
| 1000 | 0.00379 | 0.000397 | 0.000618 | 1.56 |

$\sigma_{\rm tot}$ & $\sigma_{\rm ONS}$ fall off as 1/s as expected

$\sigma_{\rm OFS}/\sigma_{\rm ONS}$ grows as $Li_2(s)$, but with $m_{\tilde{b}_1}$ coefficient: $\sim m_{\tilde{b}_1} Li_2(s)$

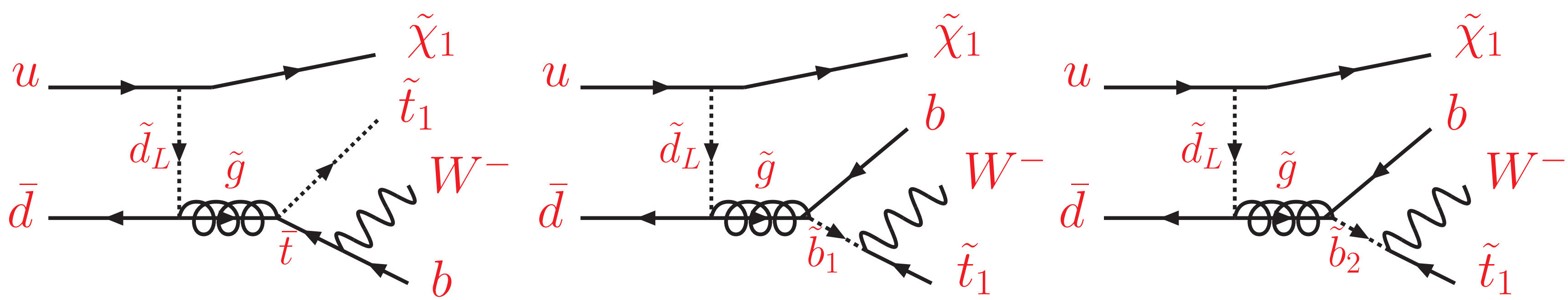
This result is found analytically as well as numerically.

## Sanity Checks

1. $\Gamma_{\tilde{g}} = 0$ in M.E., multiply by $\frac{(q^2-m^2)^2}{(q^2-m^2)^2+(m\Gamma)^2}$ outside $\longrightarrow$ same result.

2. What if $\tilde{b}$ decays?

Slightly more complicated, due to additional diagrams:




- $Li_2(s)$ growth still present, although with scale shift.

(more on the $\tilde{b} \longrightarrow W\tilde{t}$ decay later...)

3. What if $W$ decays? $\longrightarrow$ no change

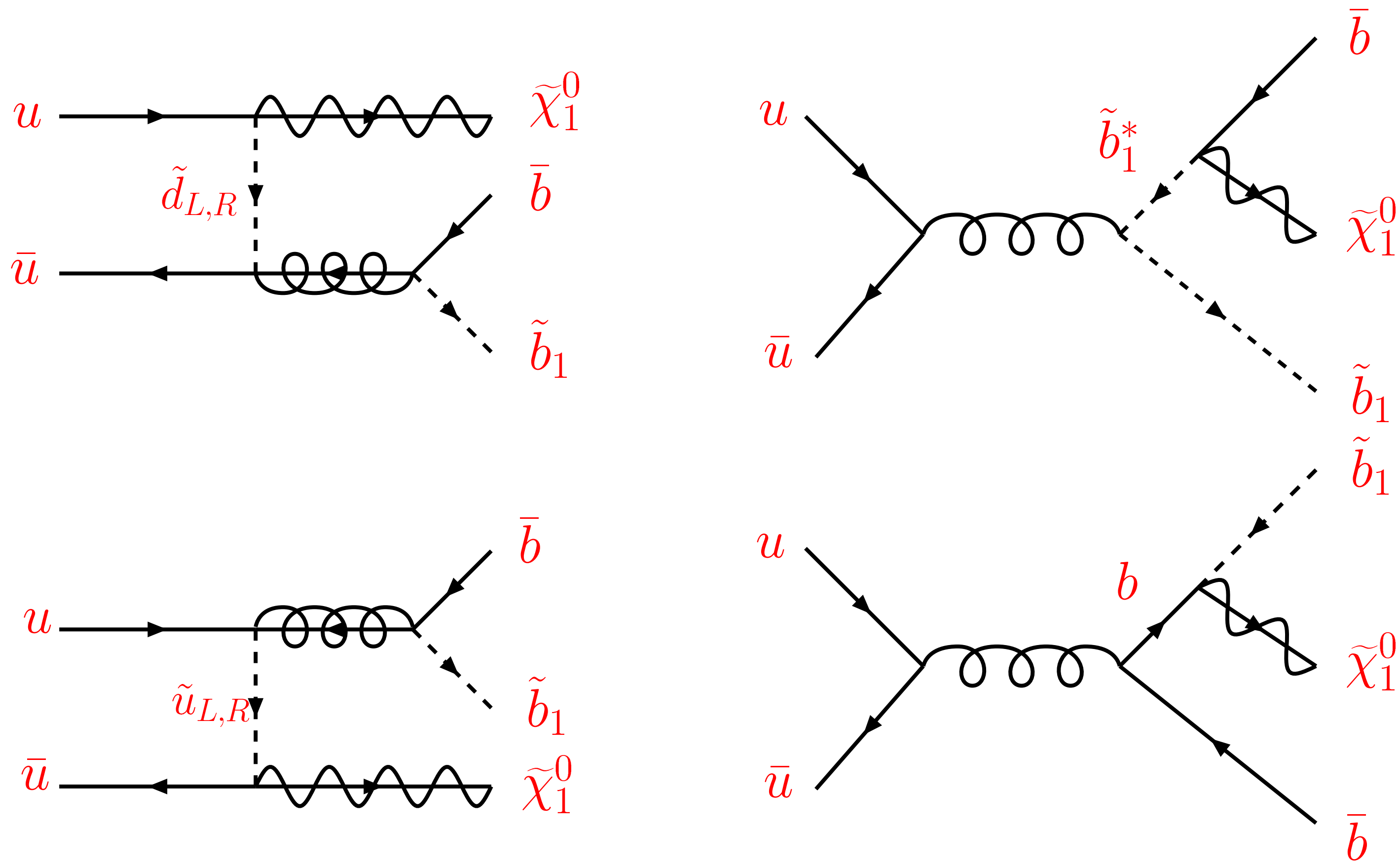
# More complicated processes

Some salient results:

- Enhancements present for variety of initial/final states
- Need interactions with fermions or vector bosons
- Need significant $\left(\frac{m_{daughter}}{m_{parent}}\right)$
- Cascade decays sometimes complicate: effect is unpredictable
- Interference effects can become important
- PDFs tend to smear and hide but not remove
- Kinematics mostly unaffected, but not always
- Basically: Everything you might expect to cause complications, does

# Potentially Observable Process at the LHC

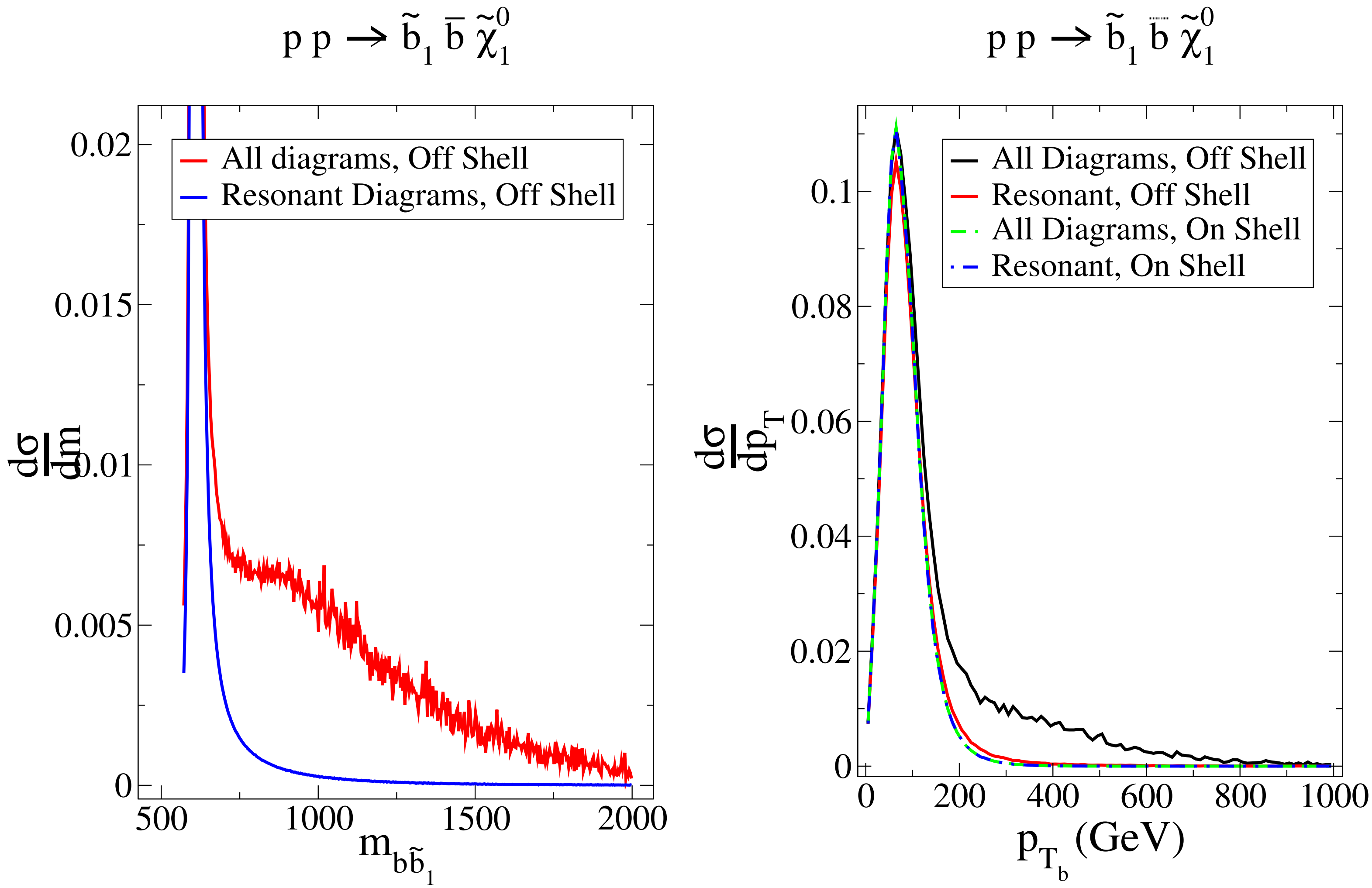
Consider $pp \rightarrow \tilde{\chi}_1^0 \tilde{g} \rightarrow \tilde{\chi}_1^0 \bar{b} \tilde{b}_1$:




$\longrightarrow$ Two more diagrams appear; one is $\tilde{b}_1$-resonant

$\longrightarrow$ Produce effects beyond those of resonant diagrams taken off-shell

## $pp \rightarrow \tilde{g}\tilde{\chi}_1^0 \rightarrow \bar{b}\tilde{b}_1\tilde{\chi}_1^0$ off-shell   +50% correction to rate!




spreads... reason: no interference, but new $\tilde{b}_1$ pole

(separable by jet/lepton edges? probably , but need to check)

# Conclusions

- interesting theoretical feature for off-shell SUSY processes:
  $\sigma_{\rm OFS}$ can grow as $Li_2(s)$ relative to $\sigma_{\rm ONS}$
  1. always occurs for gluino
  2. occurs for $\tilde{Q} \to V\tilde{Q}$
  3. ... ?
- off-shell SUSY @ LHC has large non-res. corrections
  $\longrightarrow$ most processes have same kinematics as on-shell,
  but not always true
  (slightly smeared-out, but same pole & invariants)
- Each decay in cascades potentially has off shell effects: this must be checked for each process
- ... but effect mitigated at (V)LHC by PDF's
- rather insidious implications for Linear Colliders