Interacting Dark Energy and the Cosmic Coincidence Problem

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Cosmological Evolution

Two components

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = 0\ ,$$
$$\dot{\rho}_m + 3H(1 + w_m)\rho_m = 0$$

Define

$$\Omega_\Lambda = \frac{8\pi\rho_\Lambda}{3M_p^2 H^2}\ , \qquad \Omega_m = \frac{8\pi\rho_m}{3M_p^2 H^2}\ , \qquad \Omega_\Lambda + \Omega_m = 1$$

Evolution equation becomes

$$\boxed{\frac{d\Omega_\Lambda}{dx} = -3\Omega_\Lambda(1 - \Omega_\Lambda)\left[w_\Lambda - w_m\right]}$$

where $x = \ln(a/a_0)$.

Familiar fixed points at Ω_Λ=0,1.

Interacting Dark Energy

Conservation of the energy-momentum tensor

$$\dot{\rho}_\Lambda + 3H(1+w_\Lambda)\rho_\Lambda = -Q \ ,$$
$$\dot{\rho}_m + 3H(1+w_m)\rho_m = Q$$

Define effective equations of state

$$\dot{\rho}_\Lambda + 3H(1+w_\Lambda^{\rm eff})\rho_\Lambda = 0 \ ,$$
$$\dot{\rho}_m + 3H(1+w_m^{\rm eff})\rho_m = 0$$

Defining the interaction rate Γ

$$\Gamma = \frac{Q}{\rho_\Lambda}$$

$$w_\Lambda^{\rm eff} = w_\Lambda + \frac{\Gamma}{3H} \ , \qquad w_m^{\rm eff} = w_m - \frac{\rho_\Lambda}{\rho_m}\frac{\Gamma}{3H}$$

Evolution equation

Define ratio r $r = \frac{\rho_m}{\rho_\Lambda}\,, \qquad r = \frac{1-\Omega_\Lambda}{\Omega_\Lambda}\,, \qquad \dot{r} = -\frac{\dot{\Omega}_\Lambda}{\Omega_\Lambda^2}$

$$\dot{r} = 3Hr\left[w_\Lambda - w_m + \frac{1+r}{r}\frac{\Gamma}{3H}\right] = 3Hr\left[w_\Lambda^{\text{eff}} - w_m^{\text{eff}}\right]$$

Evolution equation for the dark energy

$$\frac{d\Omega_\Lambda}{dx} = -3\Omega_\Lambda(1-\Omega_\Lambda)\left[w_\Lambda^{\text{eff}} - w_m^{\text{eff}}\right]$$

where $x = \ln(a/a_0)$.

Depending on the form of the interaction, additional fixed points can arise (when the effective equations of state coincide).

Physically, the decay of dark energy into matter can produce a stable equilibrium.

Horava & Minic, PRL 85, 1610 (2000)
Thomas, PRL 89, 091391 (2002)

Holographic assumption

Specification of the dark energy density is equivalent to the specification of a length scale.

$$\rho_\Lambda = \frac{3c^2 M_p^2}{8\pi L_\Lambda^2}$$

The evolution equations imply a relationship between rates

$$\Gamma = 3H(-1 - w_\Lambda) + 2\frac{\dot{L}_\Lambda}{L_\Lambda}$$

Two physical assumption for rates determines the third rate. Also

$$w_\Lambda^{\mathrm{eff}} = -1 + \frac{2}{3H}\frac{\dot{L}_\Lambda}{L_\Lambda}$$

Which obviously reduces to the usual result for a cosmological constant.

Deceleration Parameter

Acceleration is determined

$$\frac{\dot{L}_\Lambda}{L_\Lambda} = -\frac{\dot{H}}{H} - \frac{1}{2}\frac{\dot{\Omega}_\Lambda}{\Omega_\Lambda}$$

Asmptotic value of the effective equation of state in terms of the deceleration parameter

$$w_\Lambda^{\text{eff}} = -\frac{1}{3} + \frac{2}{3}q - \frac{1}{3H}\frac{\dot{\Omega}_\Lambda}{\Omega_\Lambda}$$

$$w_\Lambda^{\text{eff}} \rightarrow -\frac{1}{3} + \frac{2}{3}q$$

Examples I

Hubble horizon

$$L_\Lambda = R_{\mathrm{HH}} = \frac{1}{H}$$

$$\rho_\Lambda = \frac{3c^2 M_p^2 H^2}{8\pi}$$

Effective equation of state:

$$w_\Lambda^{\mathrm{eff}} = -\frac{1}{3} + \frac{2}{3} q$$

Identically satisfied for all epochs.

Horava & Minic, PRL 85, 1610 (2000)
Thomas, PRL 89, 091391 (2002)

Examples II

Past horizon:

$$R_{\mathrm{PH}} = a\int_0^t \frac{dt}{a} = a\int_0^a \frac{da}{Ha^2}$$

Future horizon:

$$R_{\mathrm{FH}} = a\int_t^\infty \frac{dt}{a} = a\int_a^\infty \frac{da}{Ha^2}$$

M. Li, Phys.Lett.B603,1(2004)

$$\begin{aligned}\frac{1}{H}\frac{\dot{L}_\Lambda}{L_\Lambda} &= -\frac{\dot{H}}{H^2} \quad (\mathrm{HH})\,,\\ &= 1+\frac{\sqrt{\Omega_\Lambda}}{c} \quad (\mathrm{PH})\,,\\ &= 1-\frac{\sqrt{\Omega_\Lambda}}{c} \quad (\mathrm{FH})\end{aligned}$$

Holographic Length = Horizon

The PH and FH cases have an effective equations of state that is a function of Ω_Λ

$$
\begin{aligned}
w_\Lambda^{\text{eff}} &= -1 - \frac{2}{3}\frac{\dot{H}}{H^2} \quad (\text{HH}) , \\
&= -\frac{1}{3} + \frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c} \quad (\text{PH}) , \\
&= -\frac{1}{3} - \frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c} \quad (\text{FH}) ,
\end{aligned}
$$

Functions of Ω_Λ

and $w_\Lambda^{\text{eff}} = -1$ for a cosmological constant.

Interactions

Assume the interaction is a function of Ω_Λ

$$\frac{\Gamma}{3H} = b^2 g(\Omega_\Lambda)$$

Kim, Lee, Myung
Phys.Lett.B632,605(2006)

Dimensionless constant $b^2 > 0$

$$w_m^{\text{eff}} = w_m - \frac{\Gamma}{3H}\frac{\Omega_\Lambda}{(1-\Omega_\Lambda)}$$

Take $w_m = 0$ and e.g. $g(\Omega_\Lambda) = 1/\Omega_\Lambda$:

$$w_m^{\text{eff}} = -\frac{b^2}{(1-\Omega_\Lambda)}$$

Interactions

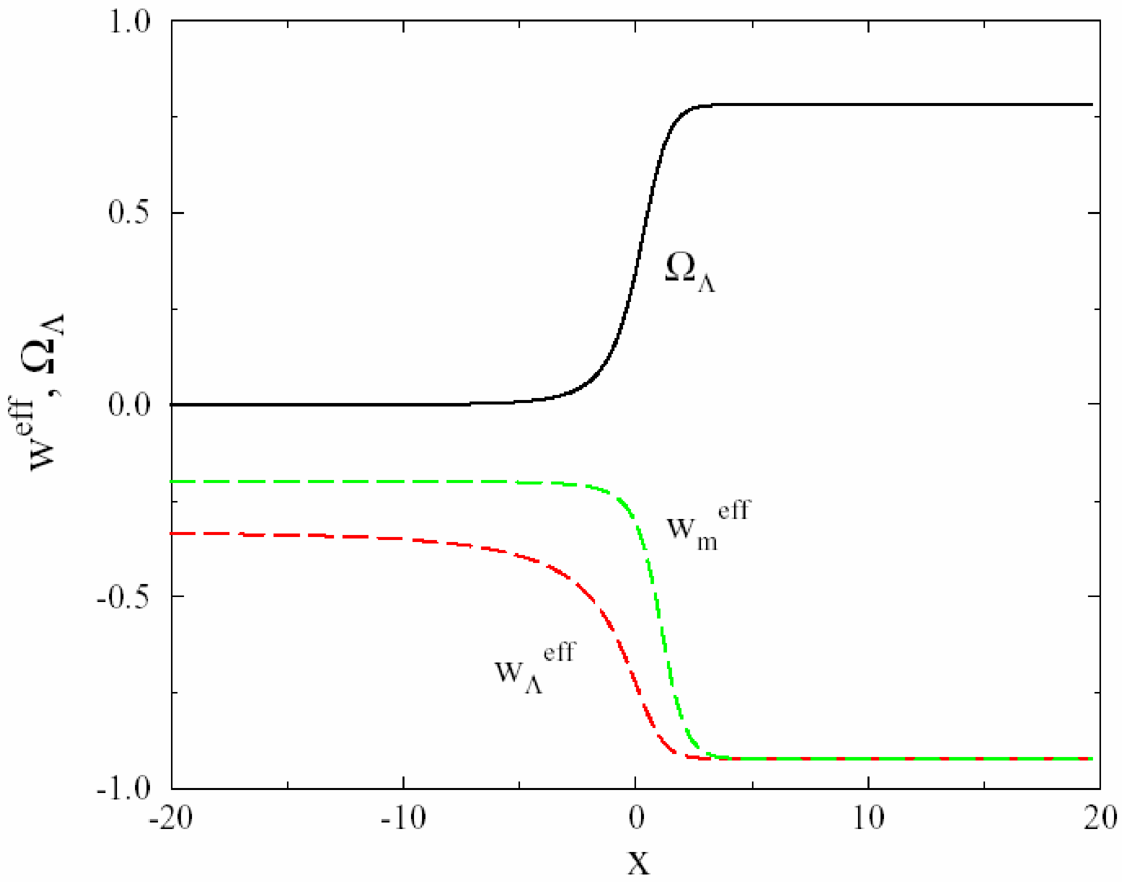
$$\frac{d\Omega_\Lambda}{dx} = -3\Omega_\Lambda(1-\Omega_\Lambda)\left[w_\Lambda^{\rm eff} - w_m^{\rm eff}\right] \ .$$

$$\frac{d\Omega_\Lambda}{dx} = 3\Omega_\Lambda(1-\Omega_\Lambda)\left[1 - \frac{2}{3H}\frac{\dot{L}_\Lambda}{L_\Lambda} - \frac{\Gamma}{3H}\frac{\Omega_\Lambda}{(1-\Omega_\Lambda)}\right]$$

Qualitative behavior of solutions follows in a straightforward manner from the physical assumptions

Evolution example for interacting dark energy

Future horizon (FH) and $\frac{\Gamma}{3H} = \frac{b^2}{\Omega_\Lambda}$, $b^2 = 0.2$



Other Interactions

Consider a more general set of interactions, e.g.

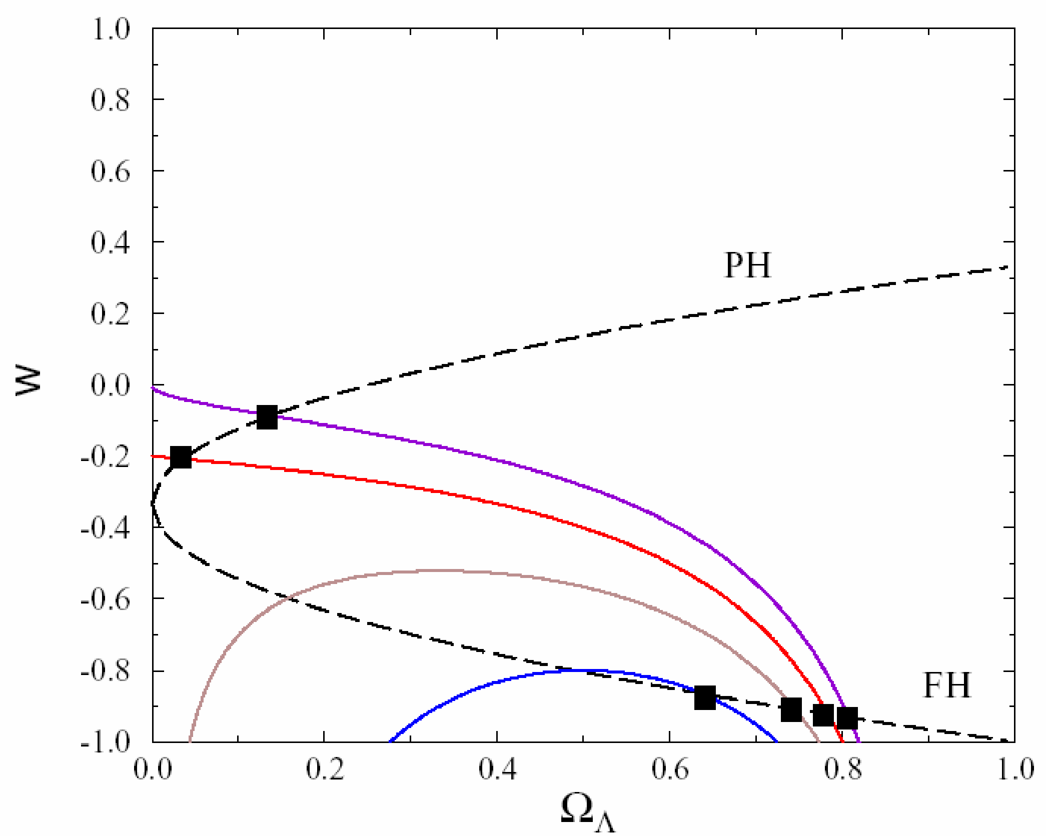
$$\frac{\Gamma}{3H} = \frac{b^2}{\Omega_\Lambda^n}$$

$$\frac{d\Omega_\Lambda}{dx} = 3\Omega_\Lambda(1-\Omega_\Lambda)\left[1 - \frac{2}{3H}\frac{\dot{L}_\Lambda}{L_\Lambda} - \frac{\Gamma}{3H}\frac{\Omega_\Lambda}{(1-\Omega_\Lambda)}\right]$$

$$w_\Lambda^{\mathrm{eff}} = -\frac{1}{3} - \frac{2}{3}\frac{\sqrt{\Omega_\Lambda}}{c} \quad \text{(FH)}\ ,$$

General behavior for various physical assumptions

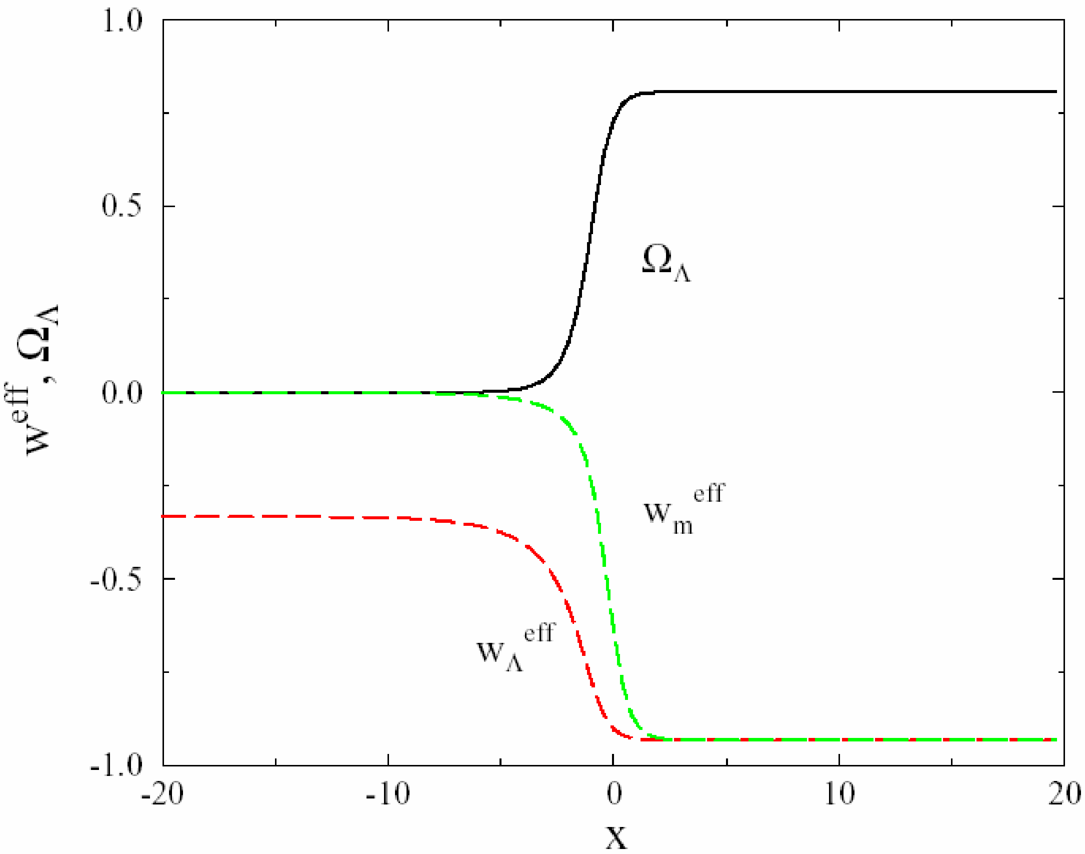
Effective equations of state



$n = \frac{1}{2}, 1, \frac{3}{2}, 2$ from top to bottom

Interacting Dark Energy

Future horizon (FH) and $\frac{\Gamma}{3H} = \frac{b^2}{{\Omega_\Lambda}^{1/2}}$



Conclusions

- Holographic models give a generalized description of the size of the dark energy component.
- Decaying dark energy can lead to a stable equilibrium solution and hence address the coincidence problem.
- Interaction rate is related to the Hubble parameter H. Microscopic mechanism is unclear.
- Detailed tests versus the present and future observational data (Supernova, CMB) can distinguish between various scenarios (in progress).