Evaluation of $\alpha_{\rm QED}(q^2)$ in the space- and time-like regime

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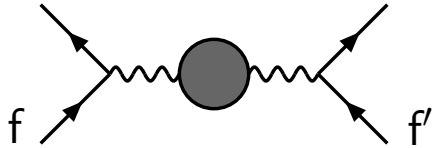
Introduction

Running QED coupling $\alpha(q^2)$: important and fundamental quantity

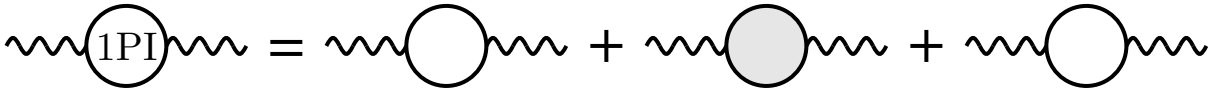
★ Appears in many places, e.g.



★ Given as a geometric series in 1PI diagram

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}, \qquad \alpha \equiv \alpha(0) \sim \frac{1}{137}$$

★ Three contributions:



leptons (5-fl) hadrons top

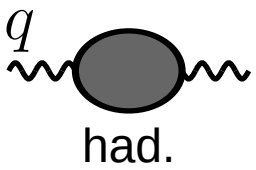
$$\Delta\alpha(q^2) = \Delta\alpha_{\rm lep}(q^2) + \Delta\alpha^{(5)}_{\rm had}(q^2) + \Delta\alpha_{\rm top}(q^2)$$

$\Delta\alpha_{\rm lep}$ and $\Delta\alpha_{\rm top}$: perturbatively calculable

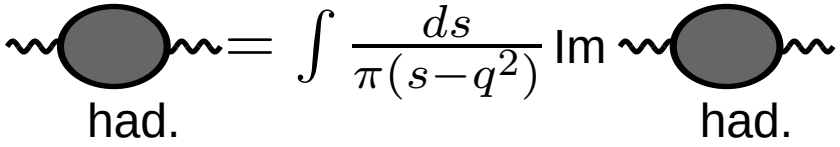
$\Delta\alpha^{(5)}_{\rm had}$: evaluation possible only by using exp. data of $e^+e^- \to$ hadrons.

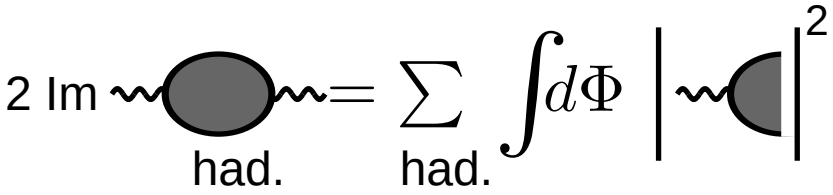
Evaluating Hadronic Contribution to $\alpha(q^2)$

The diagram to be evaluated:



pQCD not useful at low $|q^2|$. $\Longrightarrow$ Use the dispersion relation and the optical theorem.



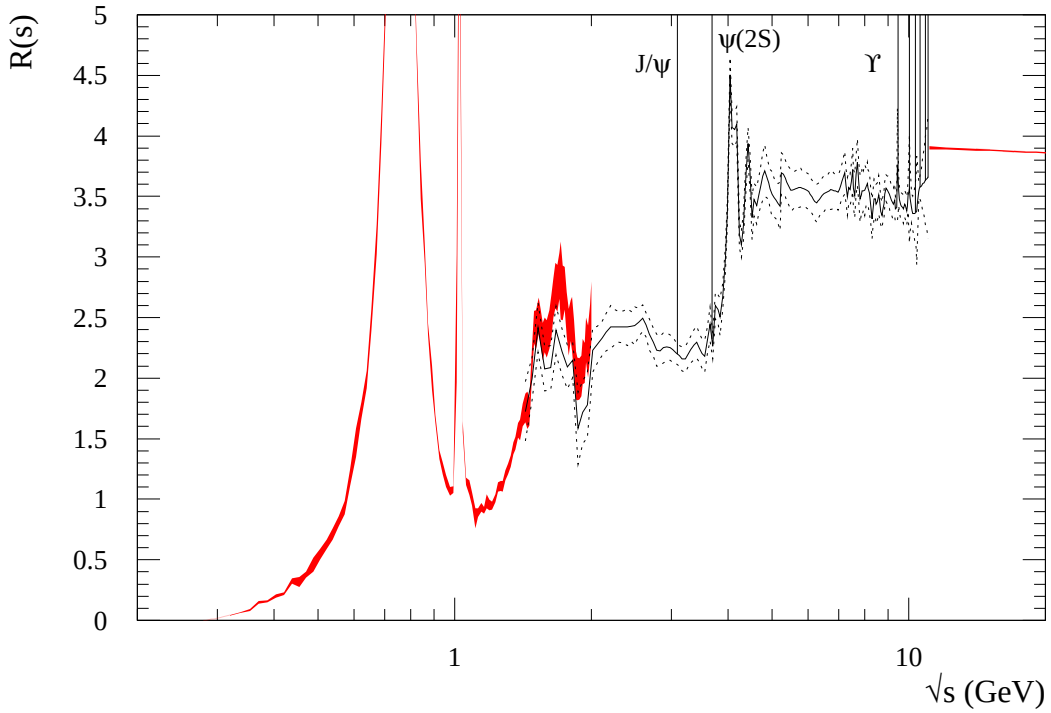


$$\Delta\alpha^{(5)}_{\rm had}(q^2) = -\frac{\alpha q^2}{3\pi}{\rm P}\int\frac{R(s')ds'}{s'(s'-q^2)}$$

- For time-like q^2, the weight function emphasizes $R(s)$ at $s \sim q^2$
  $\Longrightarrow$ For low $|q^2|$, $R(s)$ at low energies is important
- Also for space-like q^2 and time-like q^2 with $|q^2|$ as high as M_Z^2, exp. data for $R(s)$ at low energies are necessary anyway.

Input for Our Evaluation of $\Delta\alpha^{(5)}_{\rm had}(q^2)$

We use our compilation of $e^+e^- \to$ hadrons data (which we used for the evaluation of $a_\mu^{\rm had,LO}$ and $\alpha(M_Z^2)$) with updates.



$\sqrt{s} \lesssim 1.4$GeV: use exclusive measurements (red band)

$1.4\text{GeV} \lesssim \sqrt{s} \lesssim 11\text{GeV}$: use inclusively measured data (black line)

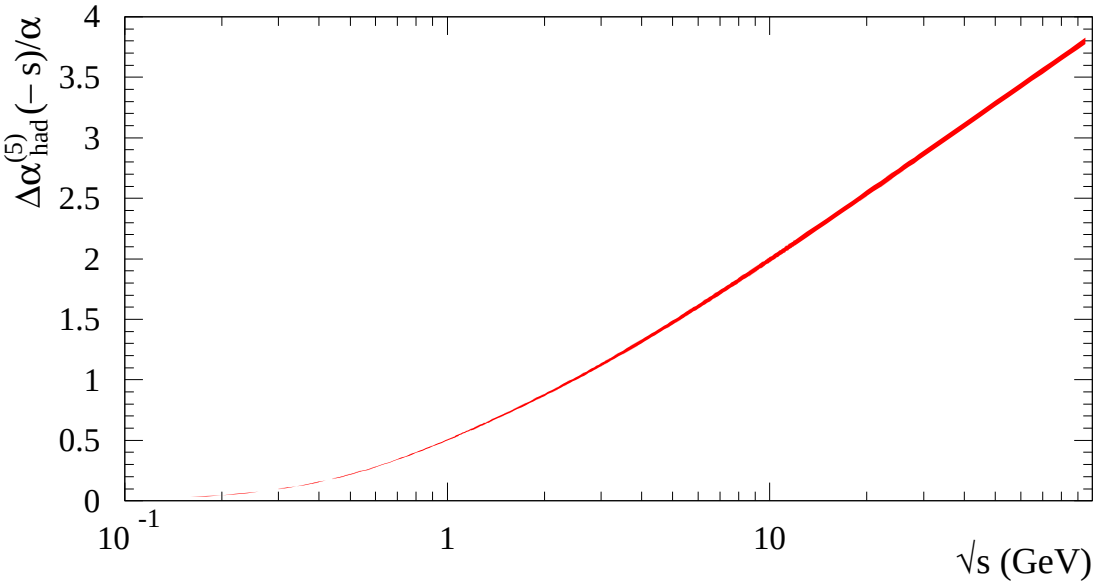
ψ, Υ: use Breit-Wigner formula

$11\text{GeV} \lesssim \sqrt{s}$: use pQCD (red band)

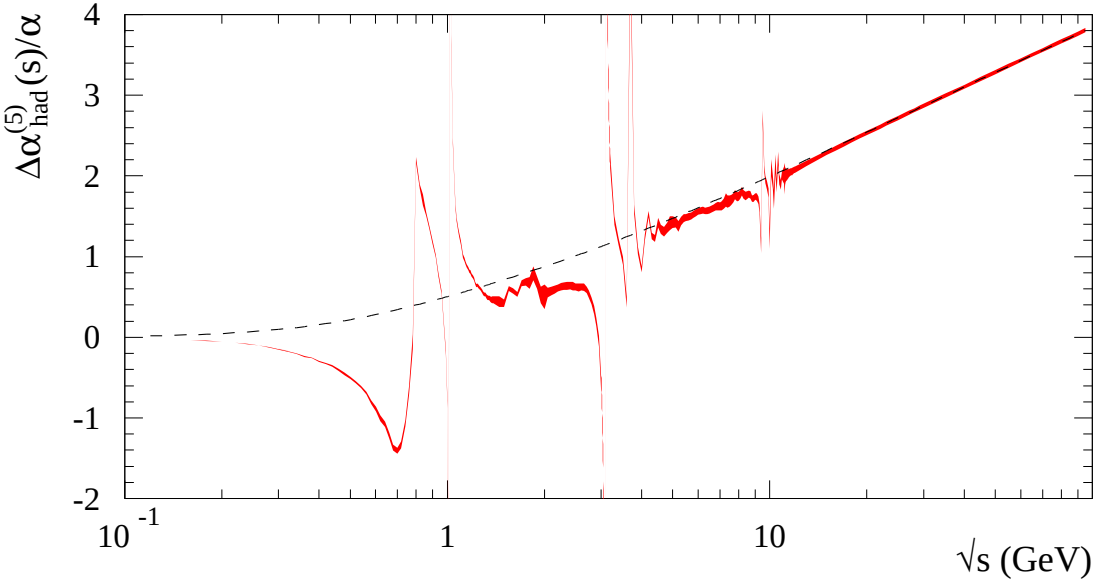
Channel	Experiments with References
$\pi^+\pi^-$	OLYA [16, 17, 18], OLYA-TOF [19], NA7 [20], OLYA and CMD [21, 22], DM1 [23], DM2 [24], BCF [25, 26], MEA [27, 28], ORSAY-ACO [29], CMD-2 [10, 11, 30]
$\pi^0\gamma$	SND [31, 32]
$\eta\gamma$	SND [32, 33], CMD-2 [34, 35, 36]
$\pi^+\pi^-\pi^0$	ND [22], DM1 [37], DM2 [38], CMD-2 [10, 13, 34, 39], SND [40, 41], CMD [42]
K^+K^-	MEA [27], OLYA [43], BCF [26], DM1 [44], DM2 [45, 46], CMD [22], CMD-2 [34], SND [47]
$K^0_SK^0_L$	DM1 [48], CMD-2 [10, 14, 49], SND [47]
$\pi^+\pi^-\pi^0\pi^0$	M3N [50], DM2 [51], OLYA [52], CMD-2 [53], SND [54], ORSAY-ACO [55], $\gamma\gamma 2$ [56], MEA [57]
$\omega(\to\pi^0\gamma)\pi^0$	ND and ARGUS [22], DM2 [51], CMD-2 [53, 58], SND [59, 60], ND [61]
$\pi^+\pi^-\pi^+\pi^-$	ND [22], M3N [50], CMD [62], DM1 [63, 64], DM2 [51], OLYA [65], $\gamma\gamma 2$ [66], CMD-2 [53, 67, 68], SND [54], ORSAY-ACO [55]
$\pi^+\pi^-\pi^+\pi^-\pi^0$	MEA [57], M3N [50], CMD [22, 62], $\gamma\gamma 2$ [56]
$\pi^+\pi^-\pi^0\pi^0\pi^0$	M3N [50]
$\omega(\to\pi^0\gamma)\pi^+\pi^-$	DM2 [38], CMD-2 [69], DM1 [70]
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	M3N [50], CMD [62], DM1 [71], DM2 [72]
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	M3N [50], CMD [62], DM2 [72], $\gamma\gamma 2$ [56], MEA [57]
$\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$	isospin-related
$\eta\pi^+\pi^-$	DM2 [73], CMD-2 [69]
$K^+K^-\pi^0$	DM2 [74, 75]
$K^0_S\pi K$	DM1 [76], DM2 [74, 75]
K^0_SX	DM1 [77]
$\pi^+\pi^-K^+K^-$	DM2 [74]
$p\bar{p}$	FENICE [78, 79], DM2 [80, 81], DM1 [82]
$n\bar{n}$	FENICE [78, 83]
incl. (< 2 GeV)	$\gamma\gamma 2$ [84], MEA [85], M3N [86], BARYON-ANTIBARYON [87]
incl. (> 2 GeV)	BES [88, 89], Crystal Ball [90, 91, 92], LENA [93], MD-1 [94], DASP [95], CLEO [96], CUSB [97], DHHM [98]

Table 1: Experiments and references for the e^+e^- data sets for the different exclusive and the inclusive channels as used in this analysis. The recent re-analysis from CMD-2 [10] supersedes their previously published data for $\pi^+\pi^-$ [11], $\pi^+\pi^-\pi^0$ [13] and $K^0_SK^0_L$ [14].

Our (Preliminary) Results for $\Delta\alpha^{(5)}_{\rm had}(q^2)$

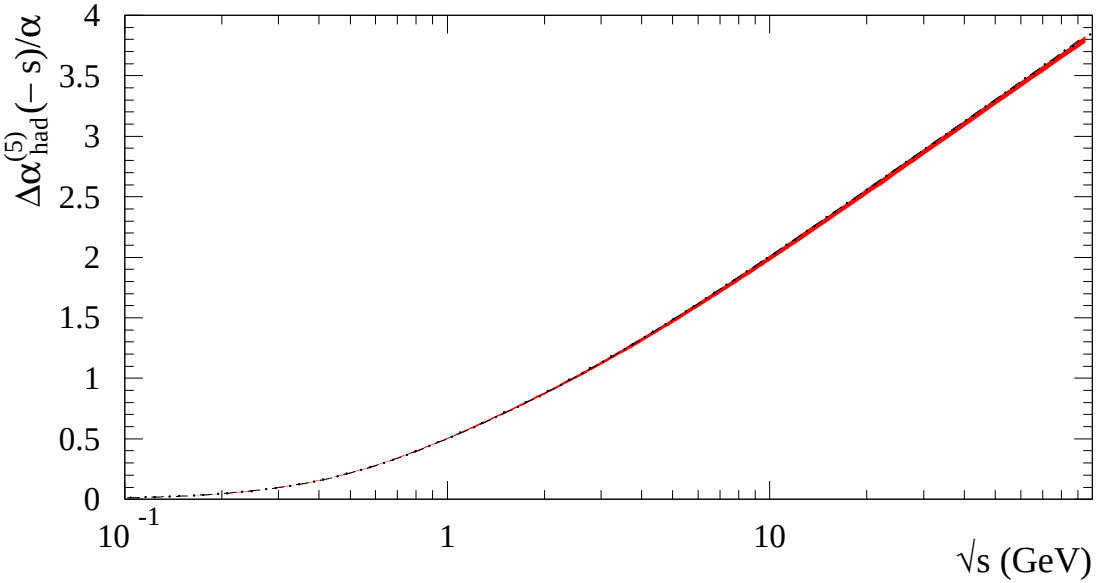


For spacelike q^2.

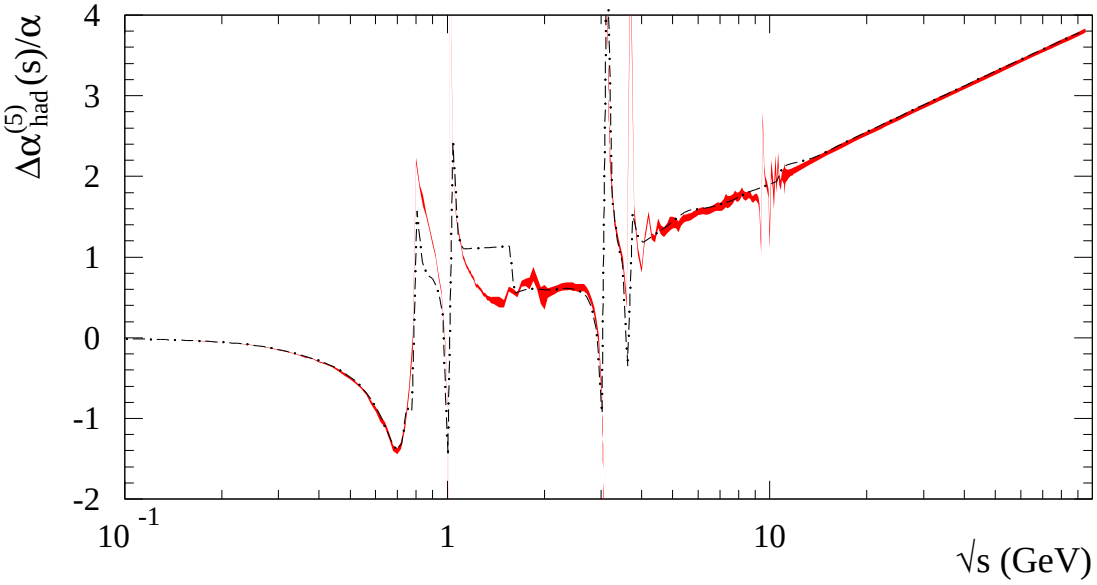


For timelike q^2.

Comparison with Other Works in Literature



For spacelike q^2.
Our results (red band) compared to Jegerlehner's results (dot-dashed line) and Burkhardt and Pietrzyk's results (dotted line)



For timelike q^2.
Our results (red band) compared to Jegerlehner's results (dot-dashed line)

Summary

We evaluated $\boldsymbol{\Delta\alpha^{(5)}_{\text{had}}(q^2)}$ in the space- and time-like regime, using our compilation of $e^+e^- \to$ hadrons data which we used to evaluate the hadronic contribution to muon $g-2$ and $\alpha(M_Z^2)$ with updates.

✓ When combined with already precisely known contributions from leptons and top, our results immediately yield $\boldsymbol{\alpha(q^2)}$.

✓ We found reasonable agreement with results in literature.

▶ We plan to provide a publicly available table of $\boldsymbol{\Delta\alpha^{(5)}_{\text{had}}(q^2)}$ and $\boldsymbol{\alpha(q^2)}$, together with their uncertainties, as functions of q^2.