Choosing the Factorization Scale in Perturbative QCD

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Based on work in progress

with F. Maltoni and S. Willenbrock

The parton model

A hadron is a bundle of partons ($q, \bar{q}, g$), each species described by a number density $f(x, \mu)$:

- x is the fraction of the hadron's momentum carried by the parton.
- μ is the factorization scale.

Hadronic cross sections are convolutions of PDFs with hard-scattering cross sections:

$$\sigma = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1, \mu) f_j(x_2, \mu) \hat{\sigma}_{ij}(x_1 x_2, \mu).$$

The exact cross section σ is independent of μ, but finite-order pQCD calculations depend on μ.

Some thoughts about μ

Is the choice of μ arbitrary?

- Not if we want reliable answers from fixed-order pQCD!

In a process characterized by a single scale Q, the "obvious" choice is $\mu = Q$.

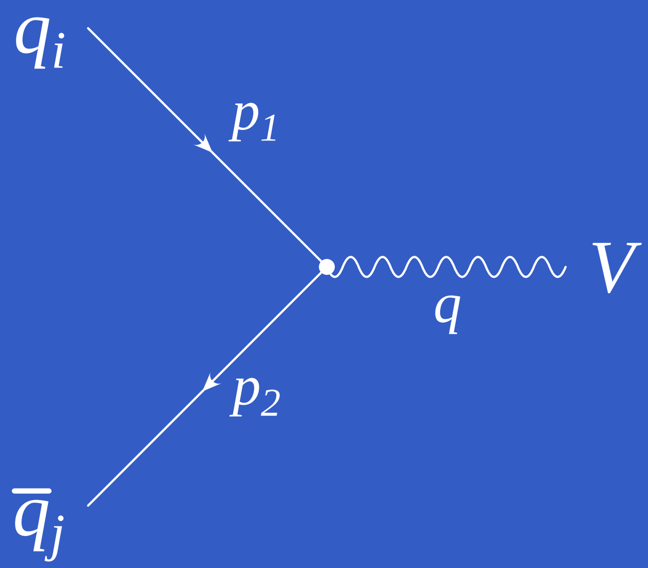
- No good argument for choosing $\mu = Q$ *exactly*: Q only sets the order of magnitude.
- Sometimes even the "obvious" choice is not obvious. *e.g.* $t\bar{t}$ production: $\mu = m_t$ or $\mu = 2m_t$?

Can we do better?

Some thoughts about μ

- μ is often thought of as characterizing the resolution with which the hadron is probed . . .
- . . . or separating long-distance physics (hadronic structure) from short-distance physics (hard scattering).
- Let's try to choose μ such that collinear physics is contained in the pdfs, and non-collinear physics in the hard-scattering cross section.
  - J. Collins (1990)
  - T. Plehn (2003)
  - E. Boos & T. Plehn (2004)
  - F. Maltoni, Z. Sullivan, & S. Willenbrock (2003)

Drell-Yan (leading order)



$$V = \gamma^*, W^{\pm}, Z$$

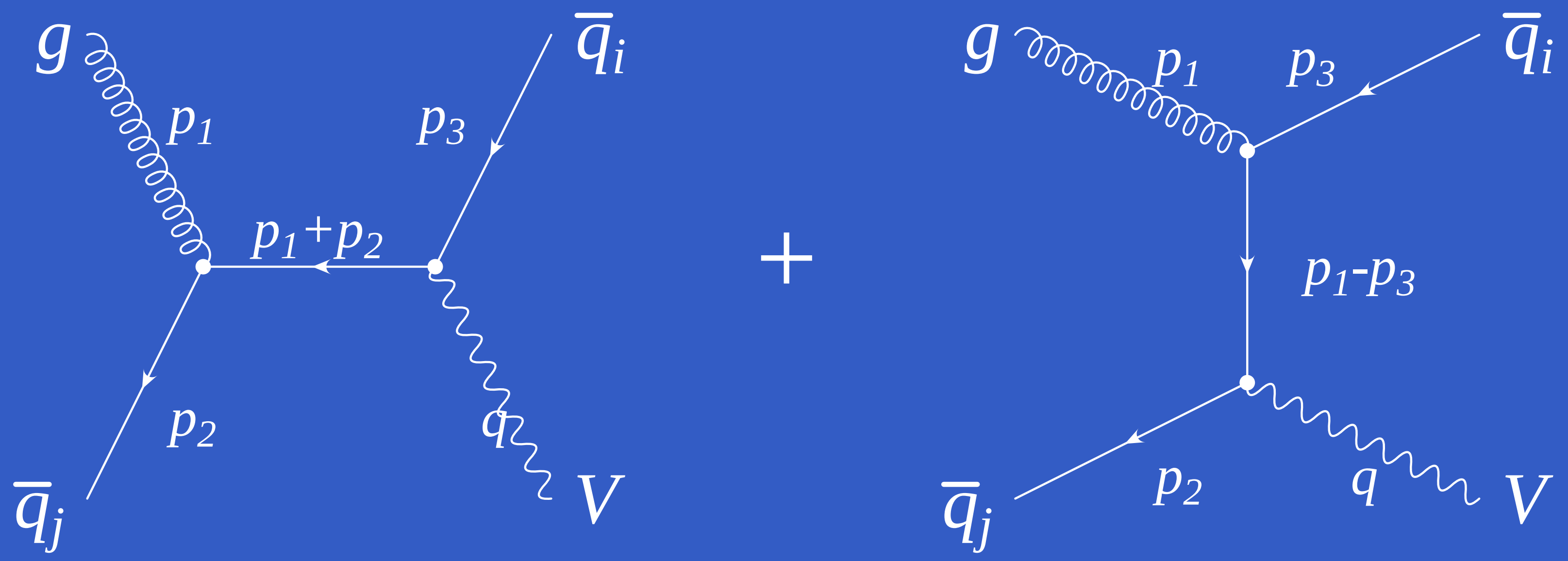
$$q^2 = Q^2$$

In dimensional regularization ($D = 4 - 2\epsilon$):

$$\sigma_0 = \frac{4\pi^2\alpha}{3S}(1-\epsilon)\mu_D^{2\epsilon}\sum_{i,j} C_{ij}\int_{-\eta_0}^{\eta_0} d\eta$$
$$\times \left[q_{0i}(\hat{x}_1)\bar{q}_{0j}(\hat{x}_2) + \bar{q}_{0j}(\hat{x}_2)q_{0i}(\hat{x}_1)\right]$$

where $\hat{x}_{1,2} \equiv (Q/\sqrt{S})e^{\pm\eta}$.

Initial gluon



Collinear singularity at $t \equiv (p_1 - p_3)^2 = 0$ due to the splitting $g \to \bar{q}_i q_i$.

We want to absorb the collinear physics into the PDF $q_i(x)$.

Initial gluon: Differential cross section

$$\frac{d\sigma_{\text{init}}}{dt} = \frac{\pi\alpha\alpha_s}{3S}\frac{(4\pi\mu_D^4)^\epsilon}{\Gamma(1-\epsilon)}\sum_{i,j} C_{ij}\int_{-\eta_0}^{\eta_0} d\eta \int_{Q^2-t}^{Se^{-2|\eta|}} ds$$
$$\times \left[g(x_1)\bar{q}_j(x_2) + \bar{q}_j(x_1)g(x_2)\right]$$
$$\times \frac{1}{s^2}\left(\frac{s}{tu}\right)^\epsilon \left[(1-\epsilon)\left(\frac{s}{-t} + \frac{-t}{s}\right) - \frac{2uQ^2}{st} + 2\epsilon\right]$$

where $x_{1,2} \equiv \sqrt{s/S}e^{\pm\eta}$ and $u \equiv Q^2 - s - t$.

Let’s worry only about the “collinear” terms.

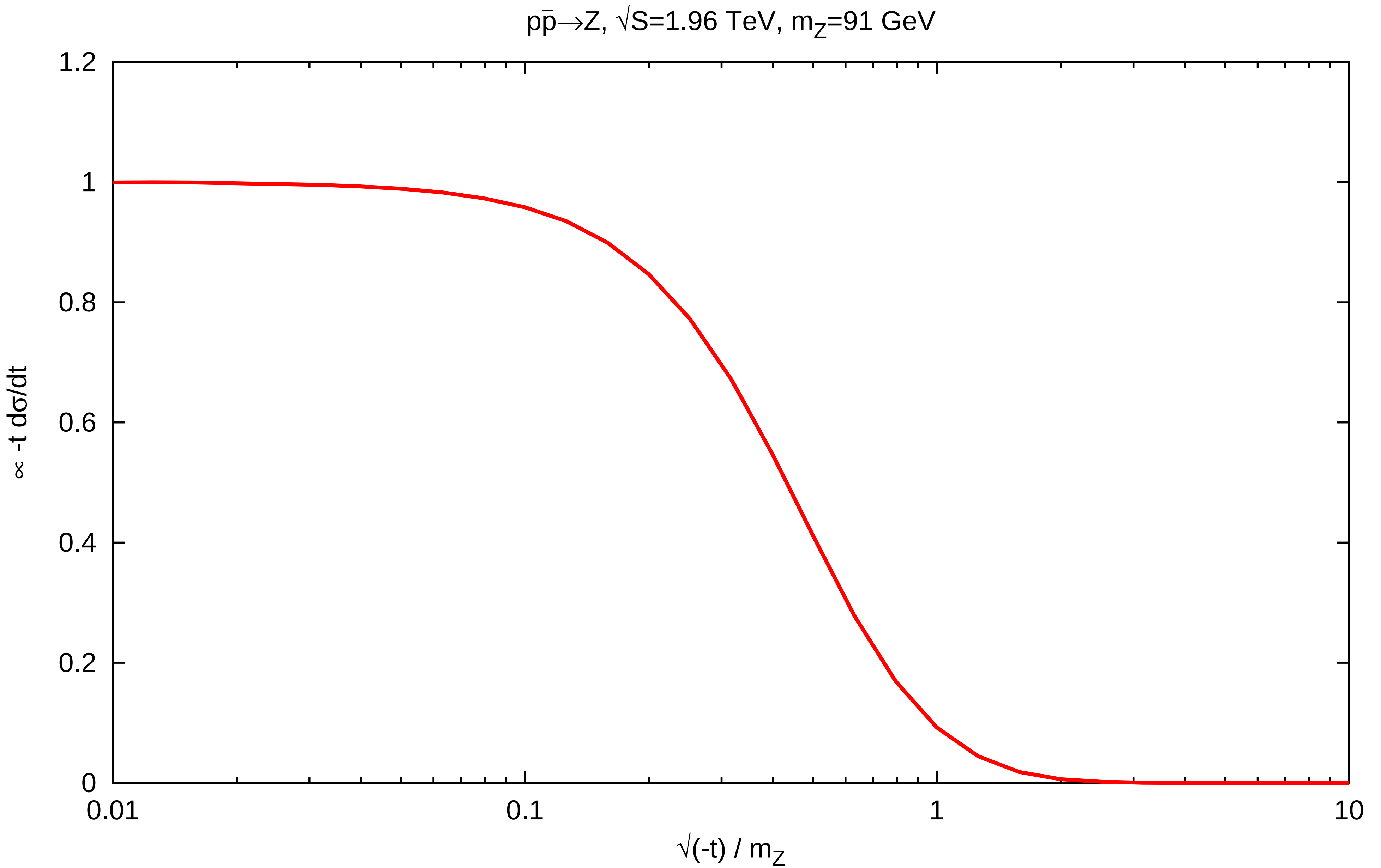
Initial gluon: Differential cross section

$$\frac{d\sigma_{\text{init}}}{dt} \sim \frac{\pi\alpha\alpha_s}{3S}\frac{(4\pi\mu_D^4)^\epsilon}{\Gamma(1-\epsilon)}\sum_{i,j} C_{ij}\int_{-\eta_0}^{\eta_0} d\eta \int_{Q^2-t}^{Se^{-2|\eta|}} ds$$

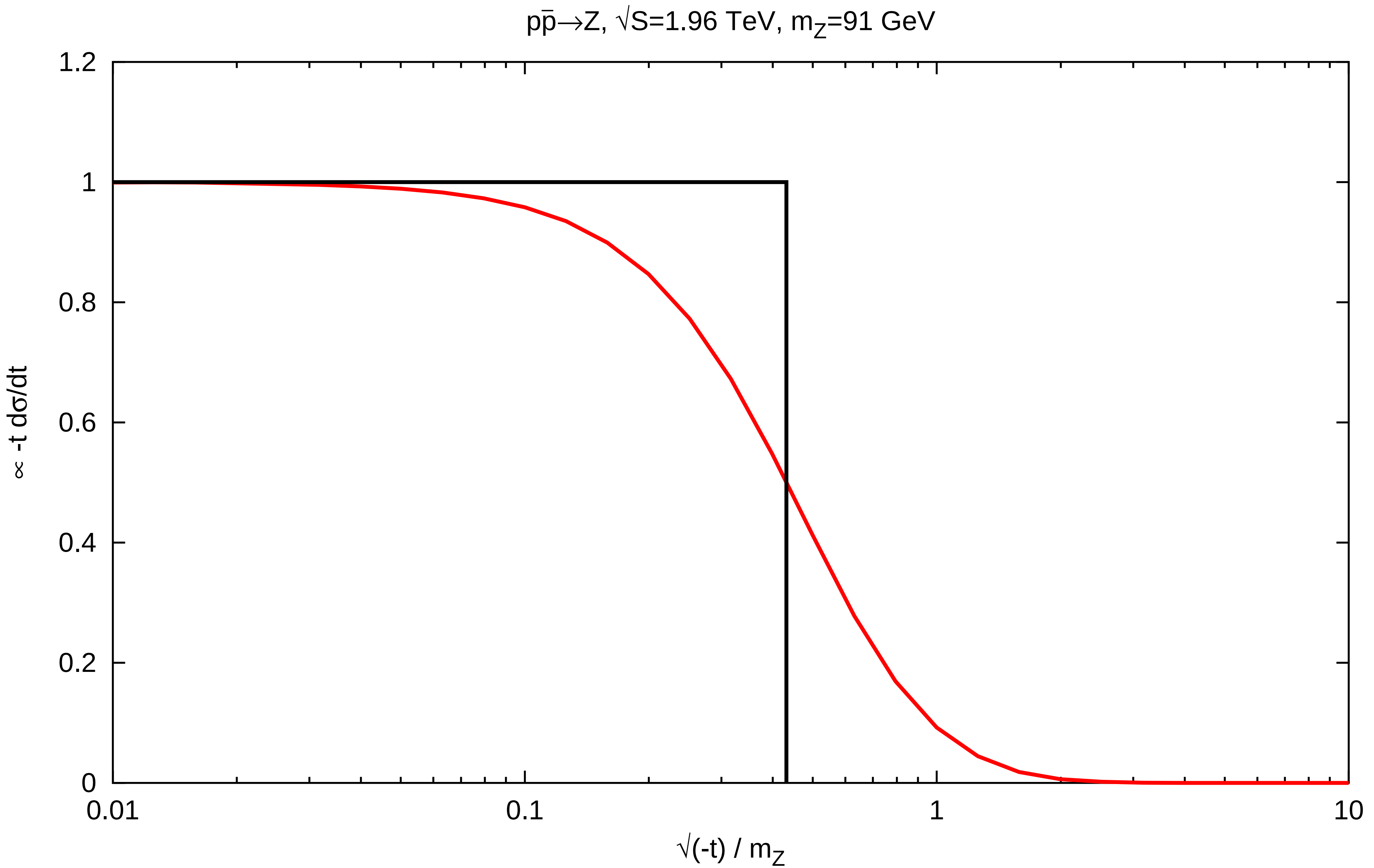
$$\times \left[g(x_1)\bar{q}_j(x_2) + \bar{q}_j(x_1)g(x_2)\right]$$

$$\times \frac{1}{s^2}\left(\frac{s}{-t(s-Q^2)}\right)^\epsilon \left[(1-\epsilon)\left(\frac{s}{-t}\right) + \frac{2(s-Q^2)Q^2}{st}\right].$$

Initial gluon: Collinear plateau



Initial gluon: Collinear plateau



Initial gluon: Collinear residue

Integrate the residue of $d\sigma_{\text{init}}/dt$ up to a scale μ:

$$\bar{\sigma}_{\text{init}} \equiv \left[\lim_{t\to 0} t^{1+\epsilon}\frac{d\sigma_{\text{init}}}{dt}\right]\int_{-\mu^2}^{0}\frac{dt}{t^{1+\epsilon}}$$

Initial gluon: Collinear residue

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$$\bar{\sigma}_{\text{init}} \equiv \left[\lim_{t\to 0} t^{1+\epsilon} \frac{d\sigma_{\text{init}}}{dt}\right] \int_{-\mu^2}^{0} \frac{dt}{t^{1+\epsilon}}$$

$$\bar{\sigma}_{\text{init}} = -\frac{2\pi\alpha\alpha_s}{3S}(1-\epsilon)\mu_D^{2\epsilon} \sum_{i,j} C_{ij} \int_{-\eta_0}^{\eta_0} d\eta \int_{Q^2}^{Se^{-2|\eta|}} \frac{ds}{s}$$

$$\times \left[g(x_1)\bar{q}_j(x_2) + \bar{q}_j(x_1)g(x_2)\right]$$

$$\times \left\{ \left[\frac{1}{\epsilon} - \gamma + \ln \frac{4\pi\mu_D^2 s}{\mu^2(s-Q^2)}\right] P_{qg}\left(\frac{Q^2}{s}\right) - \frac{Q^2}{s}\left(1-\frac{Q^2}{s}\right)\right\},$$

where $P_{qg}(z) \equiv \frac{1}{2}\left[z^2 + (1-z)^2\right]$.

Initial gluon: Correction to the PDF

If we define

$$\delta q_i(x) = -\frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\, g\left(\frac{x}{z}\right) \times \left\{\left[\frac{1}{\epsilon} - \gamma + \ln\left(\frac{4\pi\mu_D^2}{\mu^2(1-z)}\right)\right] P_{qg}(z) - z(1-z)\right\},$$

then

$$\bar{\sigma}_{\text{init}} = \frac{4\pi^2\alpha}{3S}(1-\epsilon)\mu_D^{2\epsilon}\int_{-\eta_0}^{\eta_0} d\eta\,\left[\delta q_i(\hat{x}_1)\bar{q}_j(\hat{x}_2)+\bar{q}_j(\hat{x}_1)\delta q_i(\hat{x}_2)\right].$$

We can include this in the LO cross section by replacing

$$q_{0i}(x) \longrightarrow q_i(x) \equiv q_{0i}(x) + \delta q_i(x).$$

Initial gluon: Conversion to $\overline{\text{MS}}$

This prescription defines a factorization scheme.
But most PDF sets use the $\overline{\text{MS}}$ scheme, where

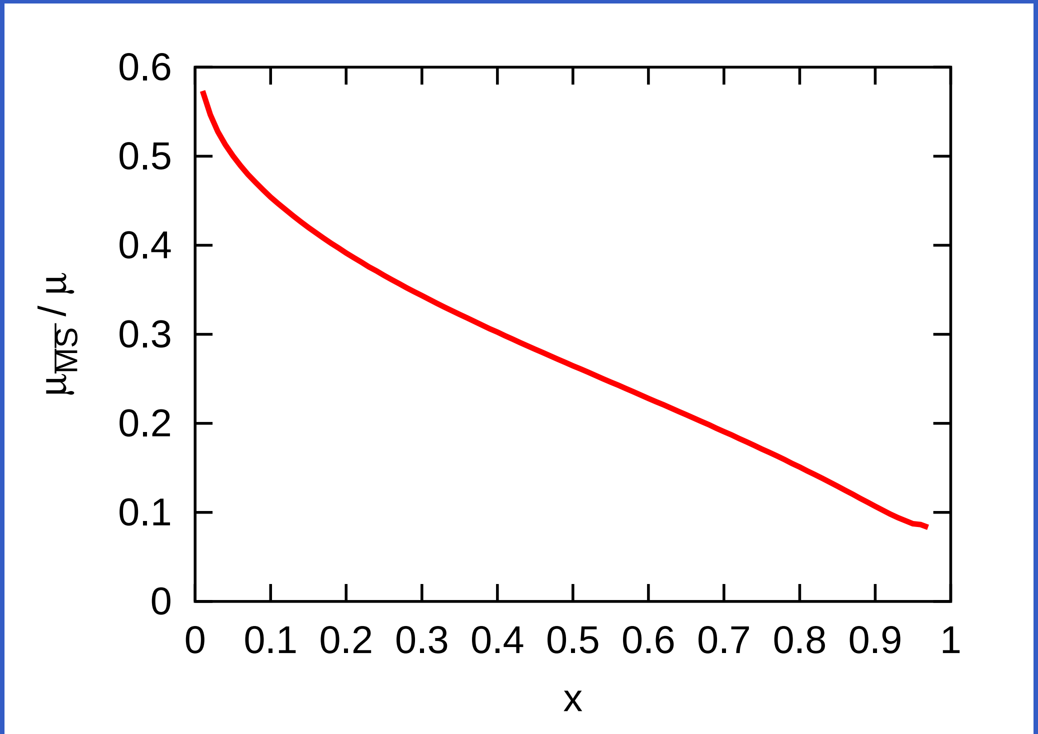
$$\delta q_i^{\overline{\text{MS}}}(x) = -\frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\, g\left(\frac{x}{z}\right)\left[\frac{1}{\epsilon} - \gamma + \ln\left(\frac{4\pi\mu_D^2}{\mu_{\overline{\text{MS}}}^2}\right)\right] P_{qg}(z).$$

If we equate the corrections in the two schemes,
$\delta q_i(x) = \delta q_i^{\overline{\text{MS}}}(x)$, then

$$\mu_{\overline{\text{MS}}}^2 = \mu^2 \exp\left\{\frac{\int_x^1 \frac{dz}{z}\, g\left(\frac{x}{z}\right)\left[\ln(1-z)P_{qg}(z) + z(1-z)\right]}{\int_x^1 \frac{dz}{z}\, g\left(\frac{x}{z}\right) P_{qg}(z)}\right\}.$$

But this solution depends on x!

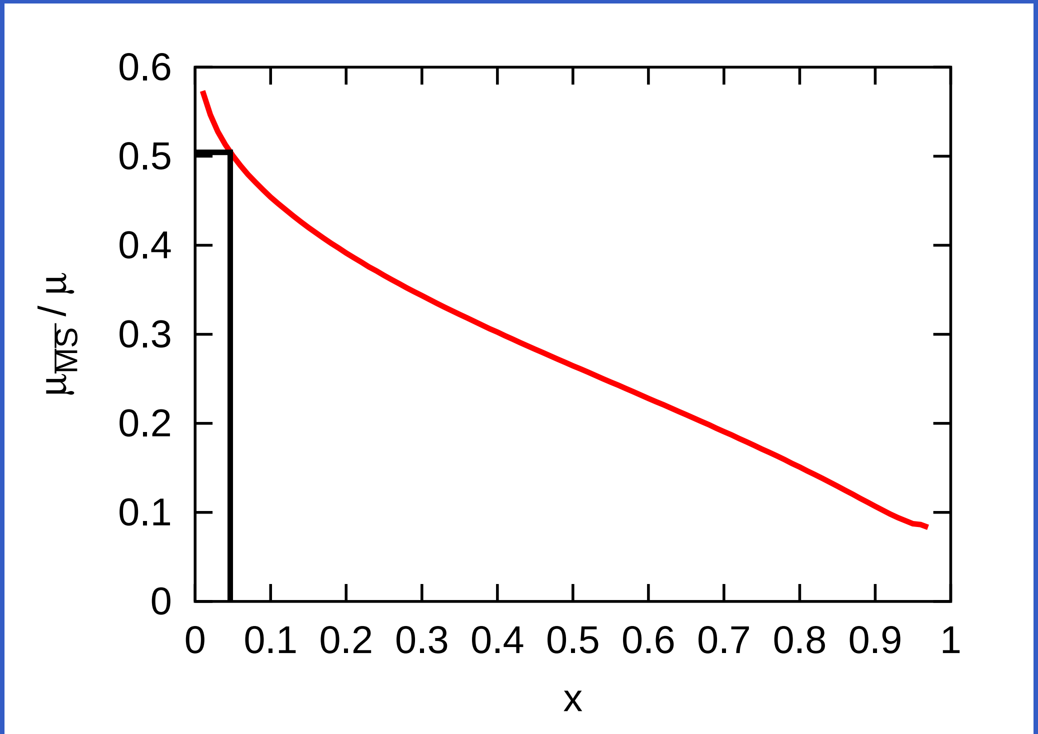
Initial gluon: Conversion to $\overline{\text{MS}}$



$$\mu^2_{\overline{\text{MS}}} = \mu^2 \exp\left\{ \frac{\int_x^1 \frac{dz}{z}\, g\left(\frac{x}{z}\right)\, \left[\ln(1-z)P_{qg}(z) + z(1-z)\right]}{\int_x^1 \frac{dz}{z}\, g\left(\frac{x}{z}\right)\, P_{qg}(z)} \right\}.$$

We must choose one value of x: $x = Q/\sqrt{S}$.

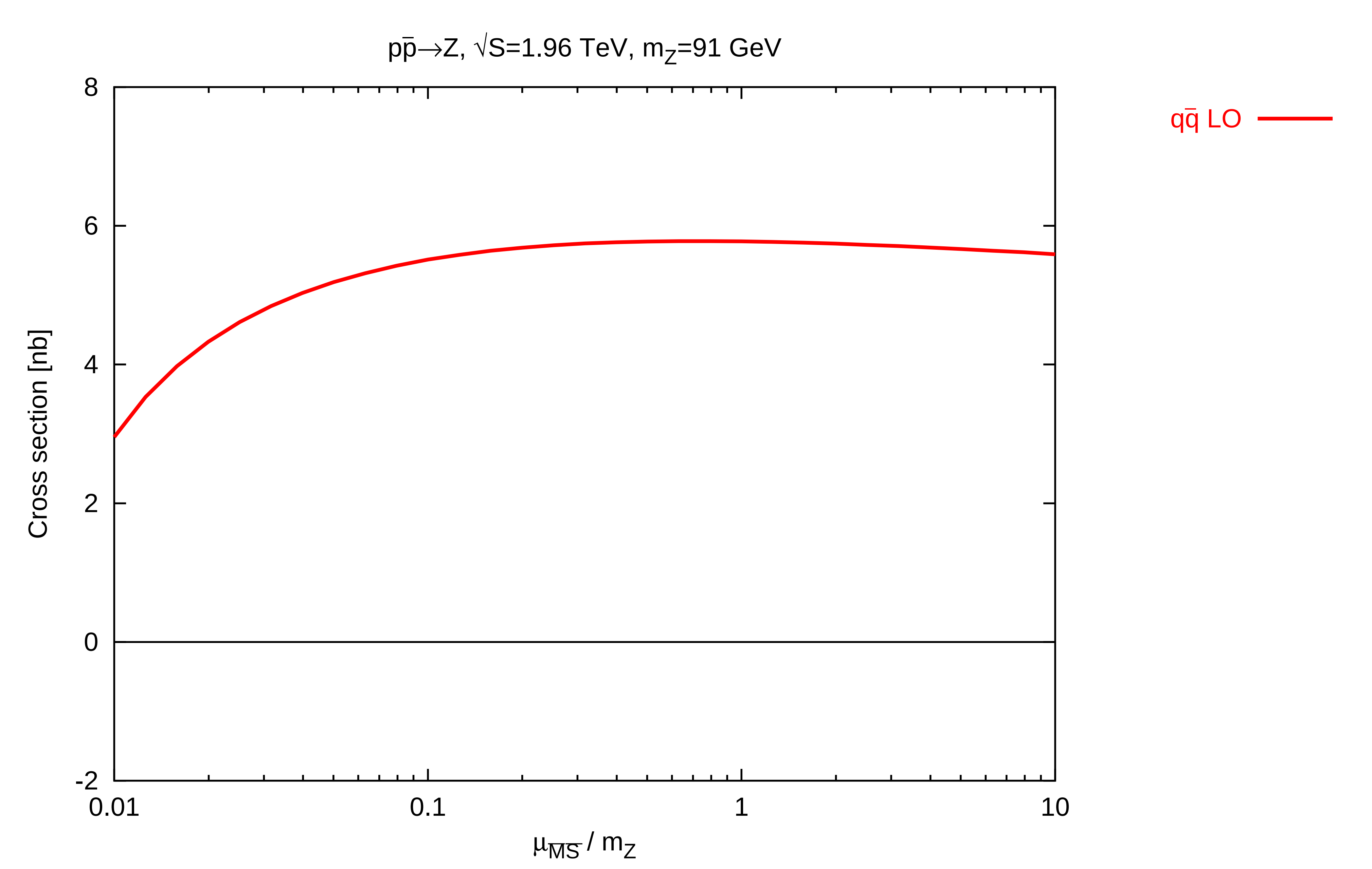
Initial gluon: Conversion to $\overline{\text{MS}}$



$$\mu_{\overline{\text{MS}}}^2 = \mu^2 \exp\left\{ \frac{\int_x^1 \frac{dz}{z}\, g\left(\frac{x}{z}\right) \left[\ln(1-z) P_{qg}(z) + z(1-z) \right]}{\int_x^1 \frac{dz}{z}\, g\left(\frac{x}{z}\right) P_{qg}(z)} \right\}.$$

We must choose one value of x: $x = Q/\sqrt{S}$.

$m_Z = 91$ GeV at the Tevatron



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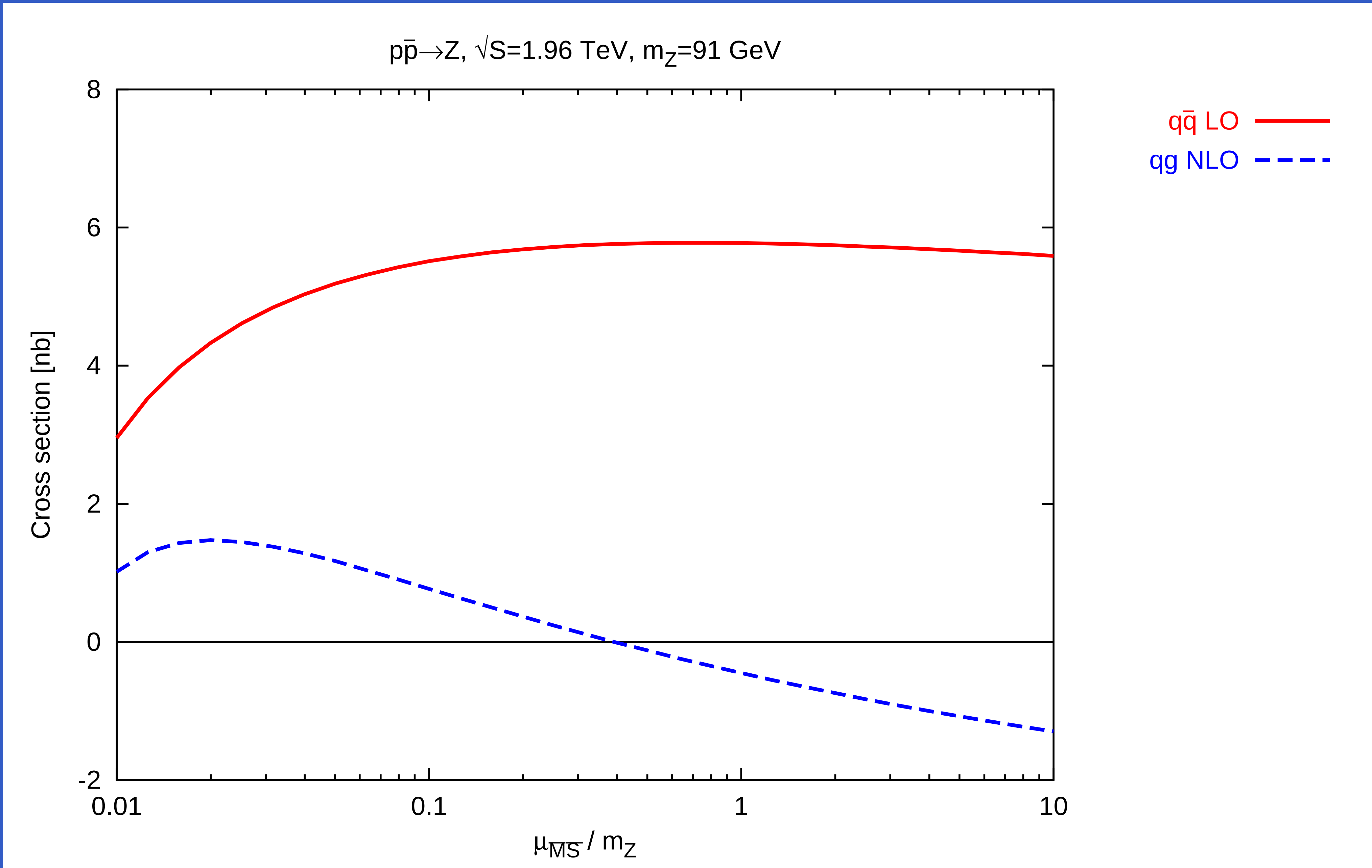
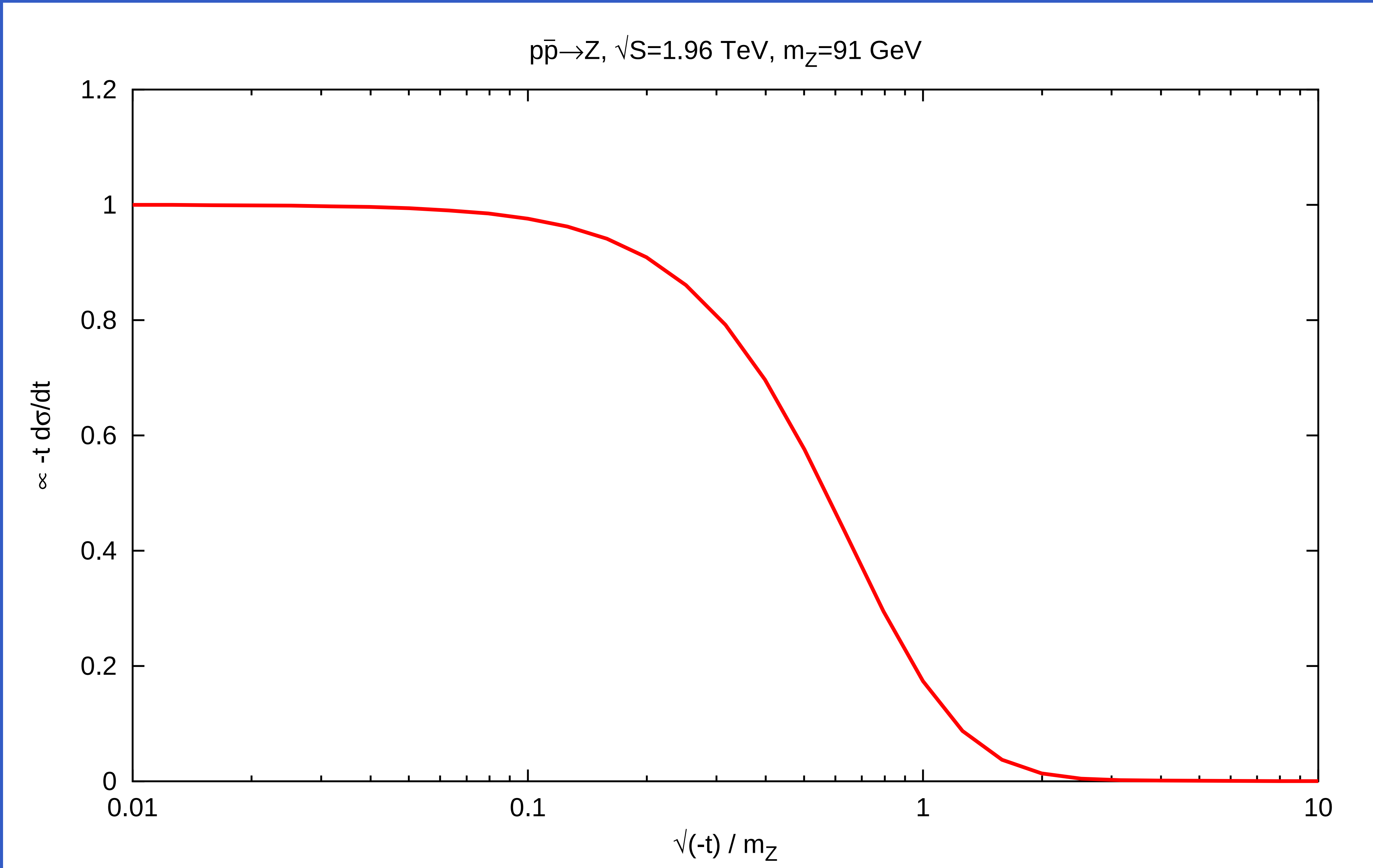
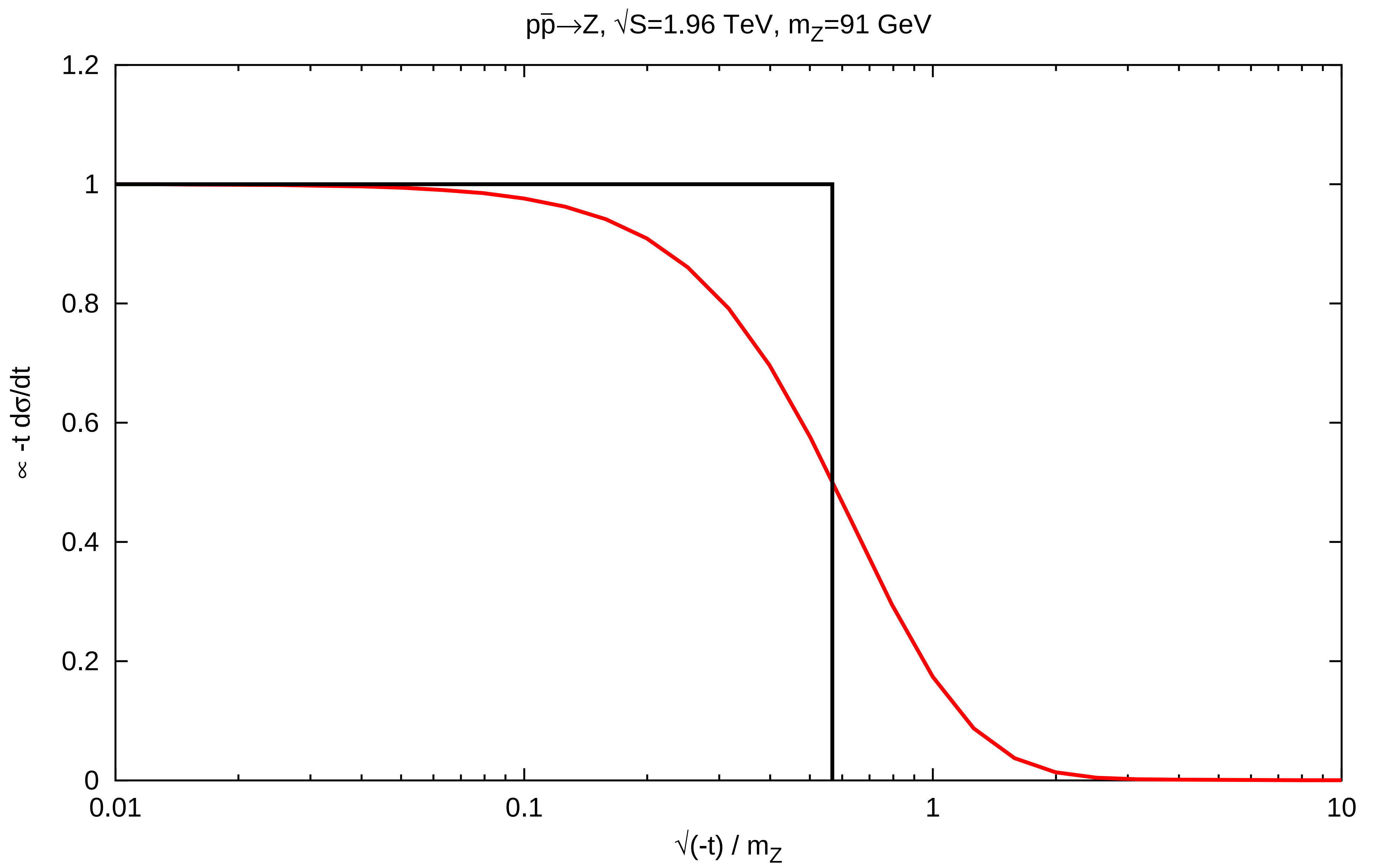


Real and virtual gluons: Collinear plateau



Real and virtual gluons: Collinear plateau



Real and virtual gluons: Correction to the PDF

$$\delta q_i(x) = -\frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\, q_i\left(\frac{x}{z}\right)$$

$$\times\left[\left(\frac{1}{\epsilon} - \gamma + \ln\frac{4\pi\mu_D^2}{\mu^2}\right) P_{qq}(z) - \Delta_{qq}(z)\right]$$

where

$$P_{qq}(z) \equiv \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)_+$$

and

$$\Delta_{qq}(z) \equiv \frac{4}{3}(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + \left(\frac{4\pi^2}{3} - 4\right)\delta(1-z) + \frac{4}{3}(1-z)$$

Real and virtual gluons: Conversion to $\overline{\text{MS}}$

$$\delta q_i^{\overline{\text{MS}}}(x) = -\frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\, q_i\left(\frac{x}{z}\right)\left(\frac{1}{\epsilon} - \gamma + \ln\frac{4\pi\mu_D^2}{\mu_{\overline{\text{MS}}}^2}\right) P_{qq}(z)$$

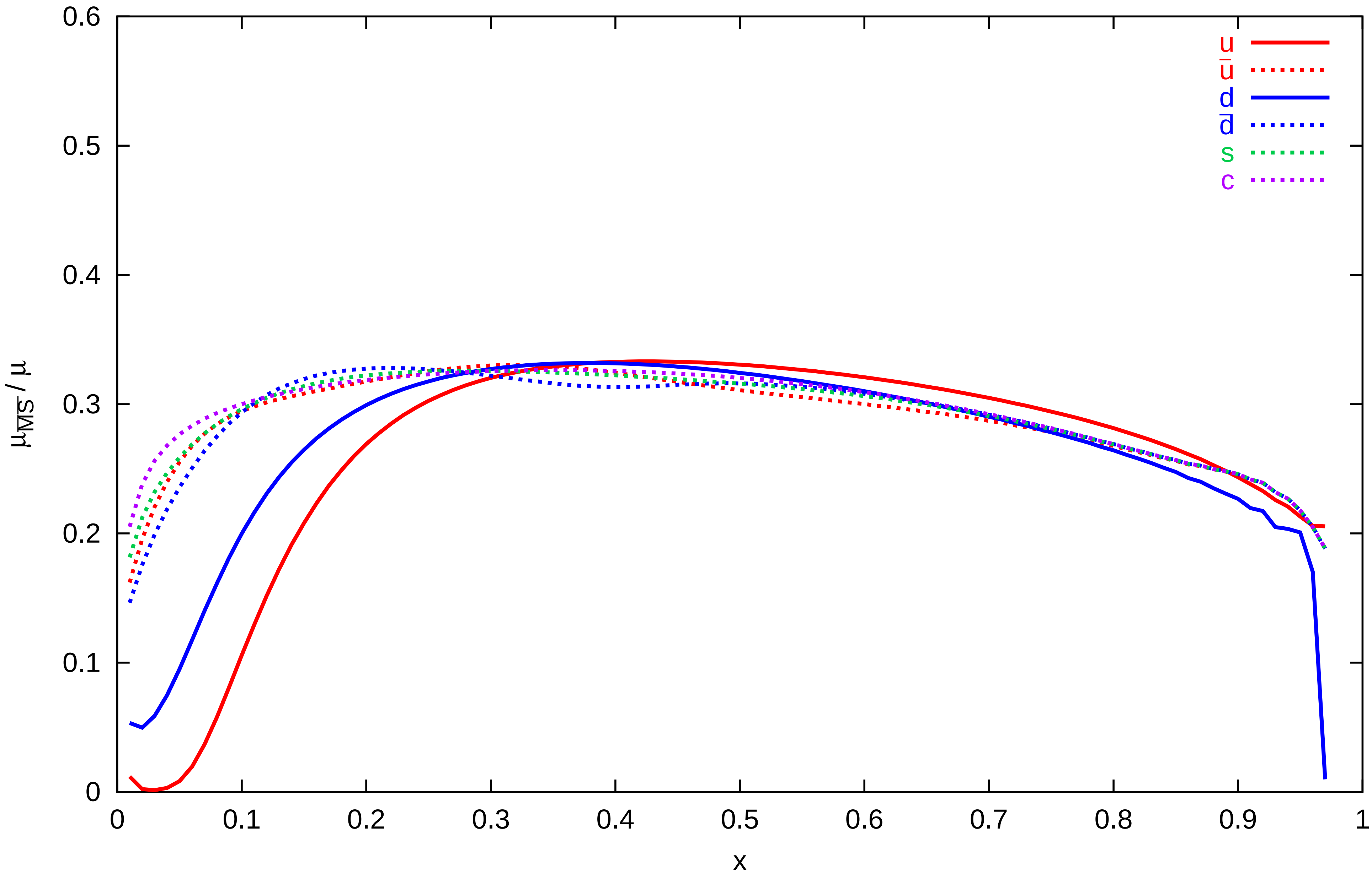
To convert to $\overline{\text{MS}}$, we choose

$$\mu_{\overline{\text{MS}}}^2 = \mu^2 \exp\left\{\frac{\int_x^1 \frac{dz}{z}\, q_i\left(\frac{x}{z}\right)\Delta_{qq}(z)}{\int_{x_0}^1 \frac{dz}{z}\, q_i\left(\frac{x}{z}\right) P_{qq}(z)}\right\}.$$

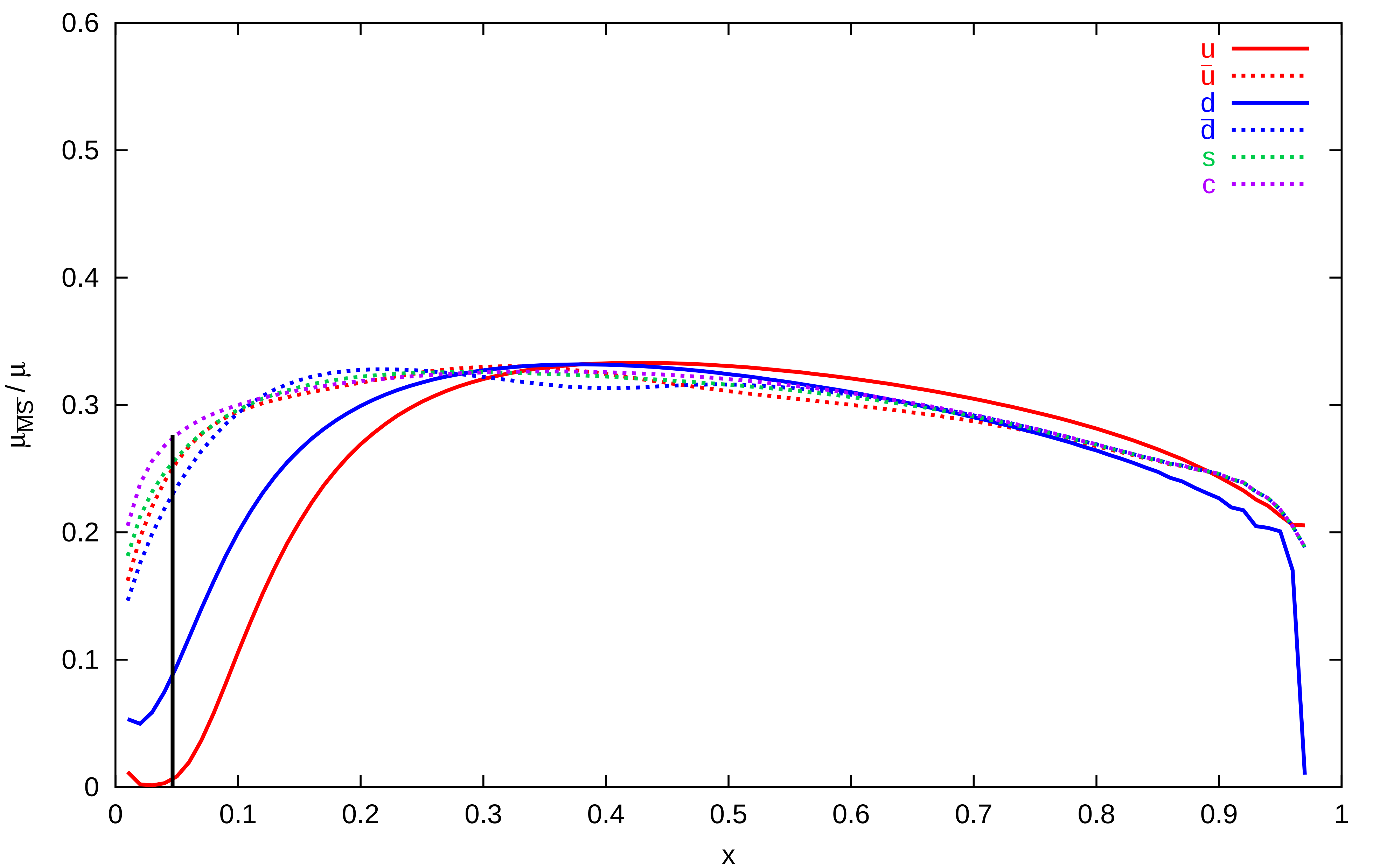
But this solution depends on x …

…and on the quark flavor!

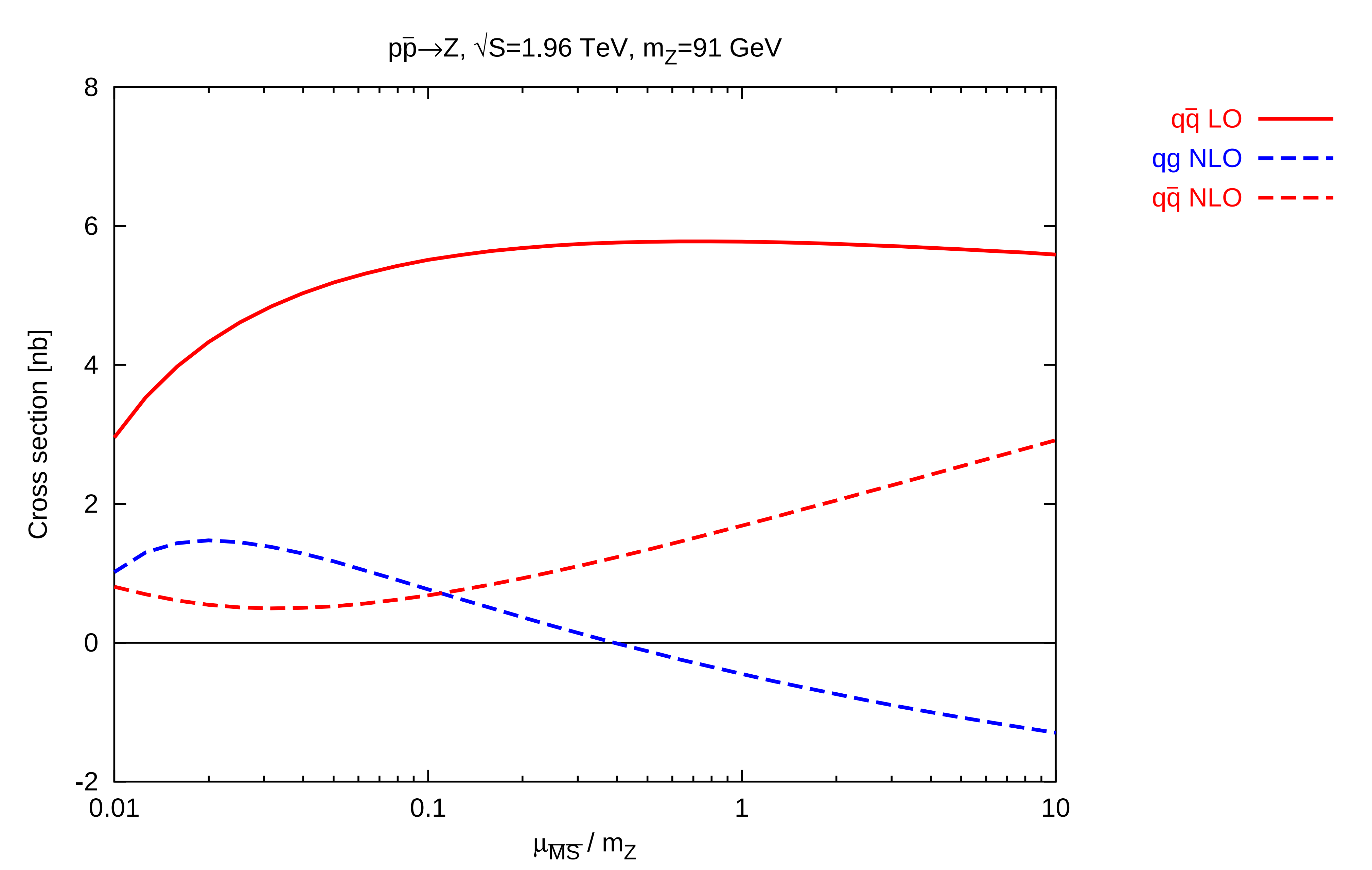
Real and virtual gluons: Conversion to $\overline{\text{MS}}$



Real and virtual gluons: Conversion to $\overline{MS}$



$m_Z = 91$ GeV at the Tevatron



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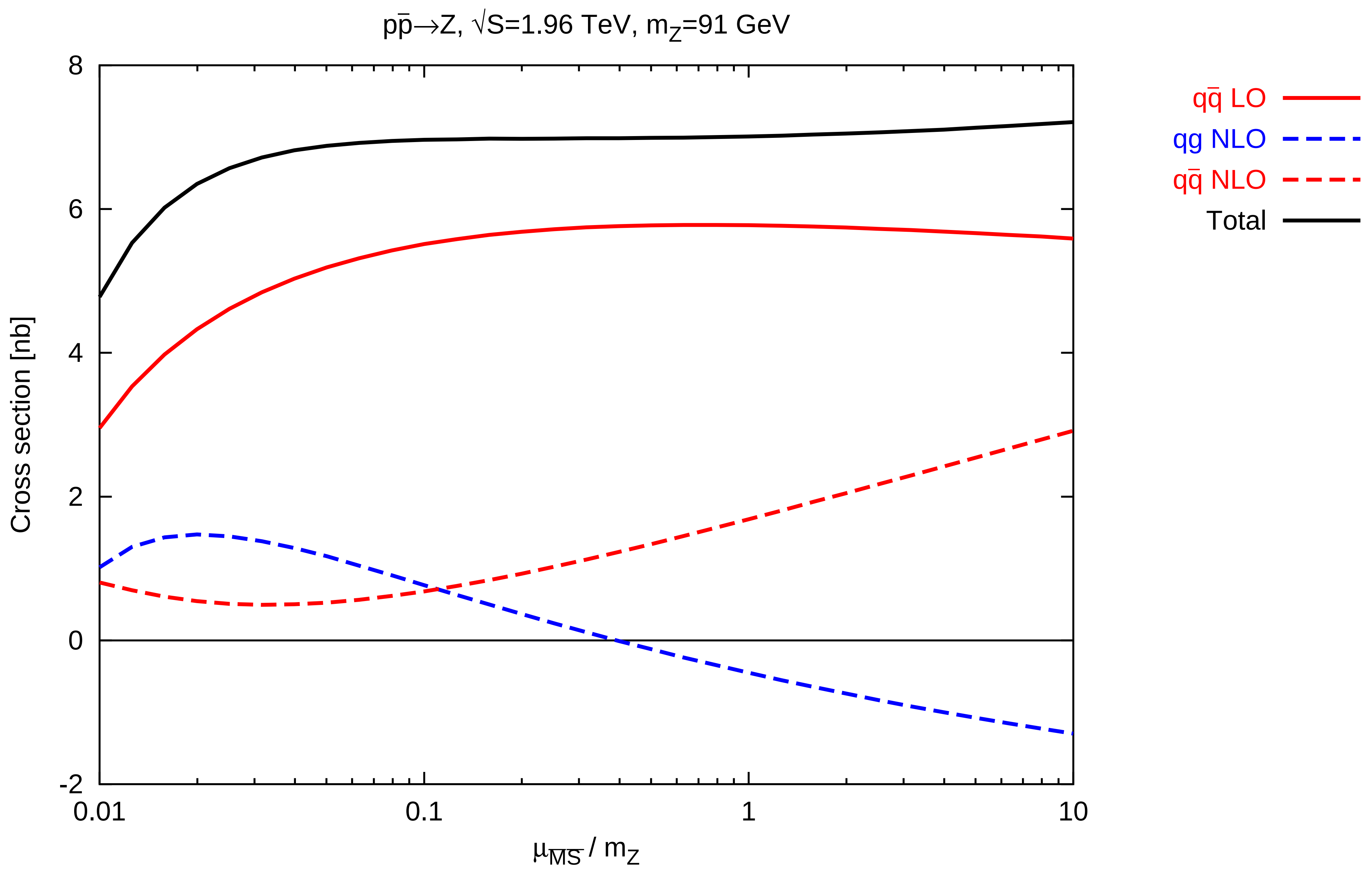
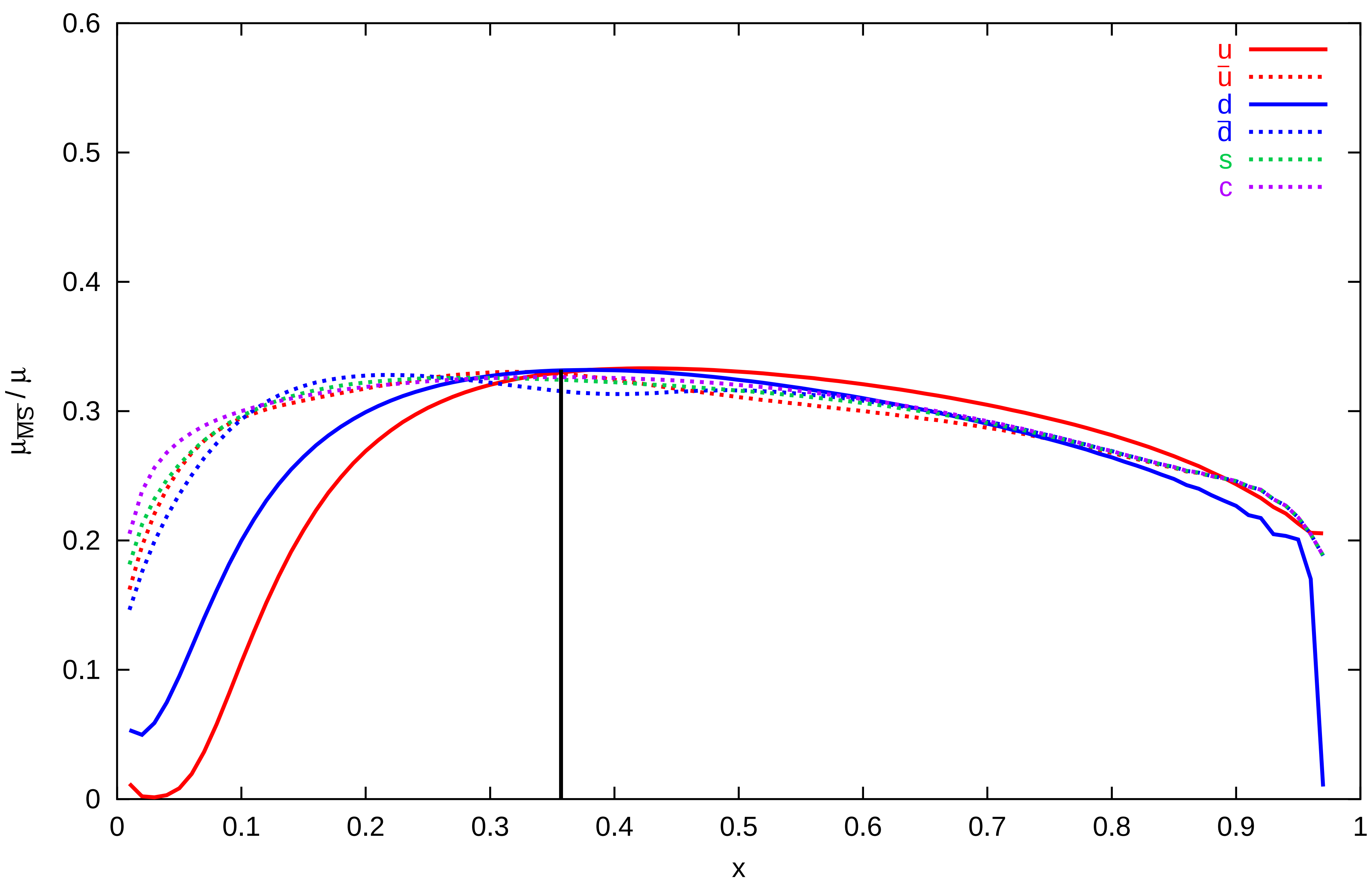
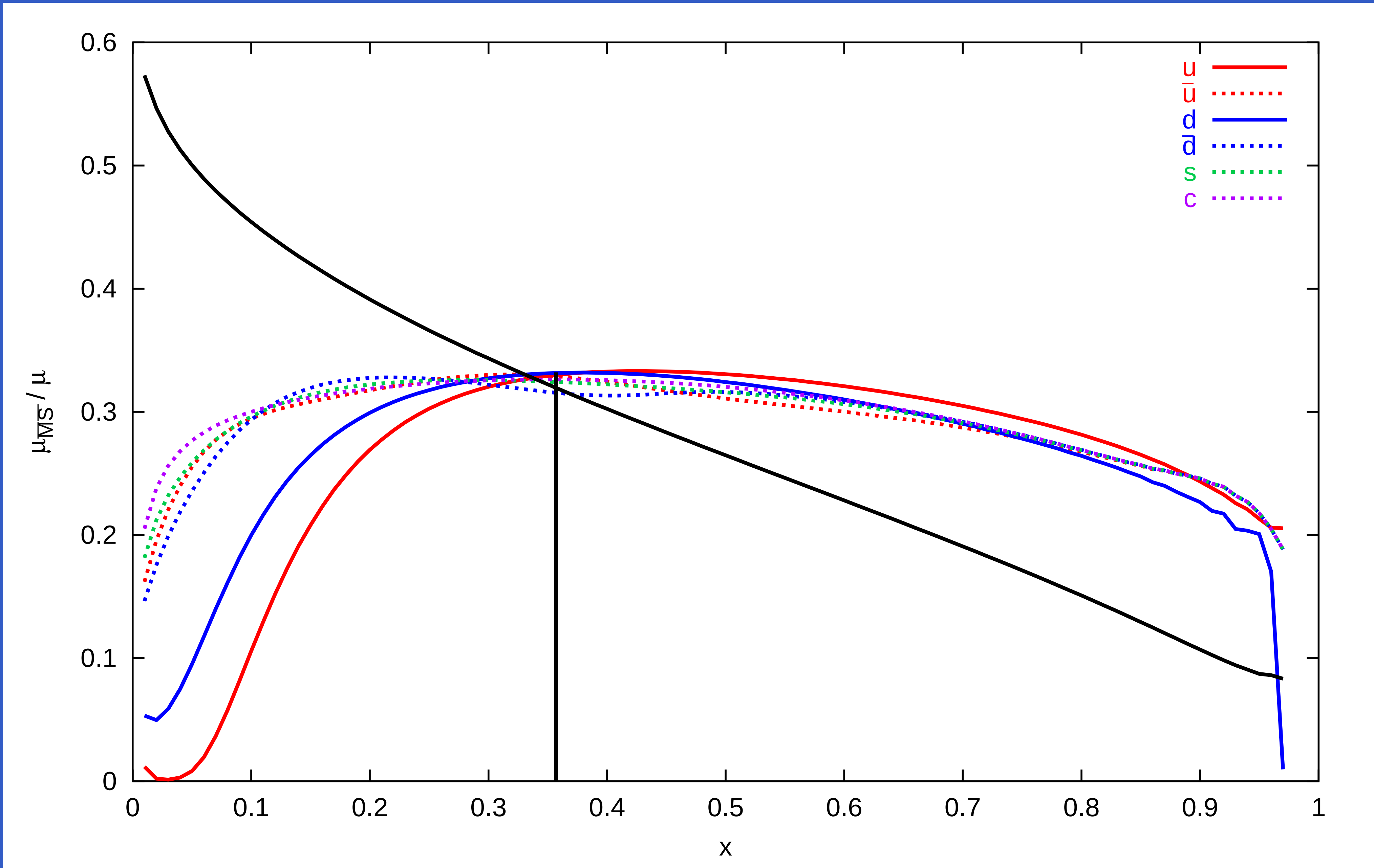


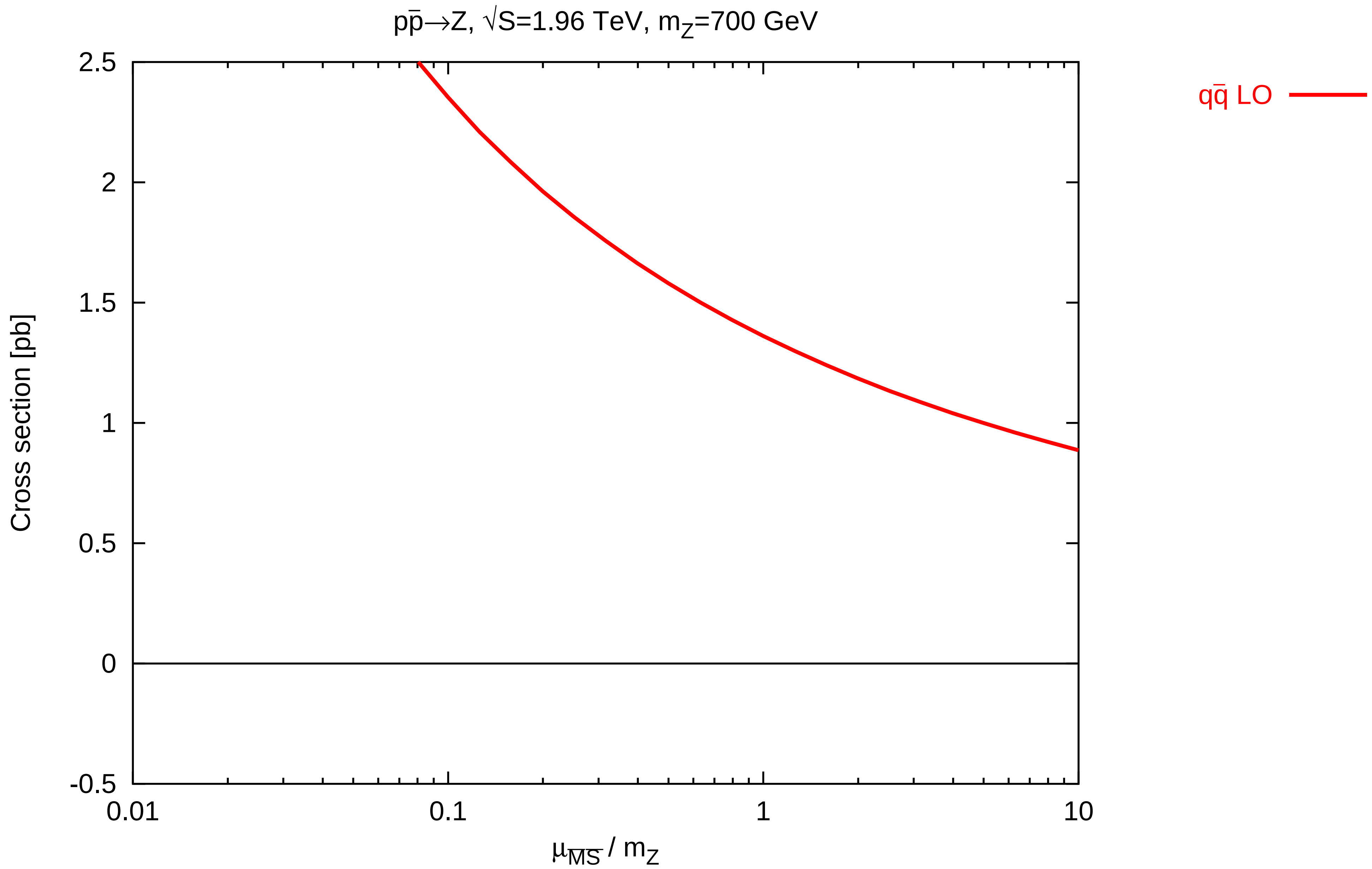
Real and virtual gluons: Conversion to $\overline{\rm MS}$



Real and virtual gluons: Conversion to $\overline{\text{MS}}$



$m_Z = 700$ GeV at the Tevatron



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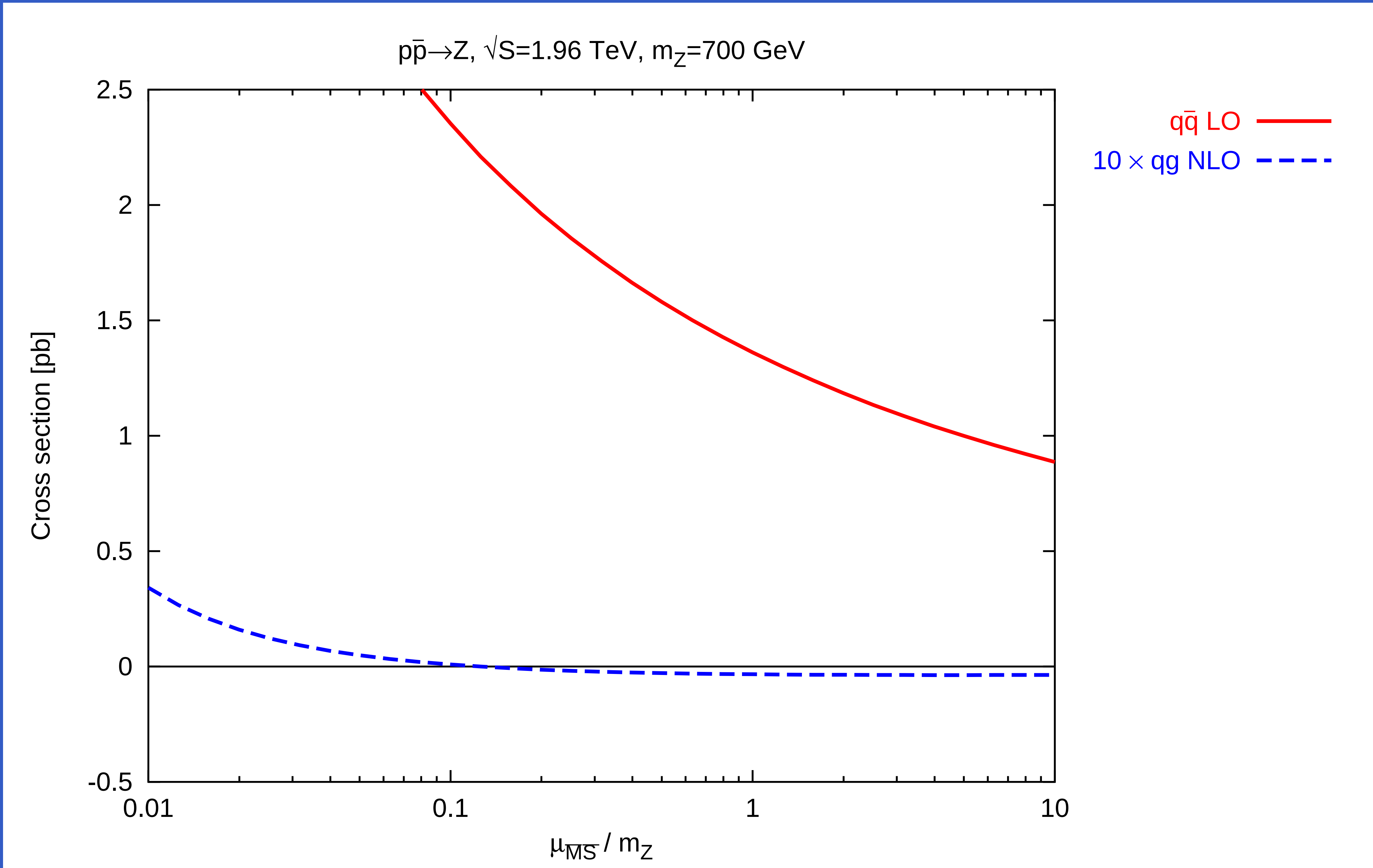

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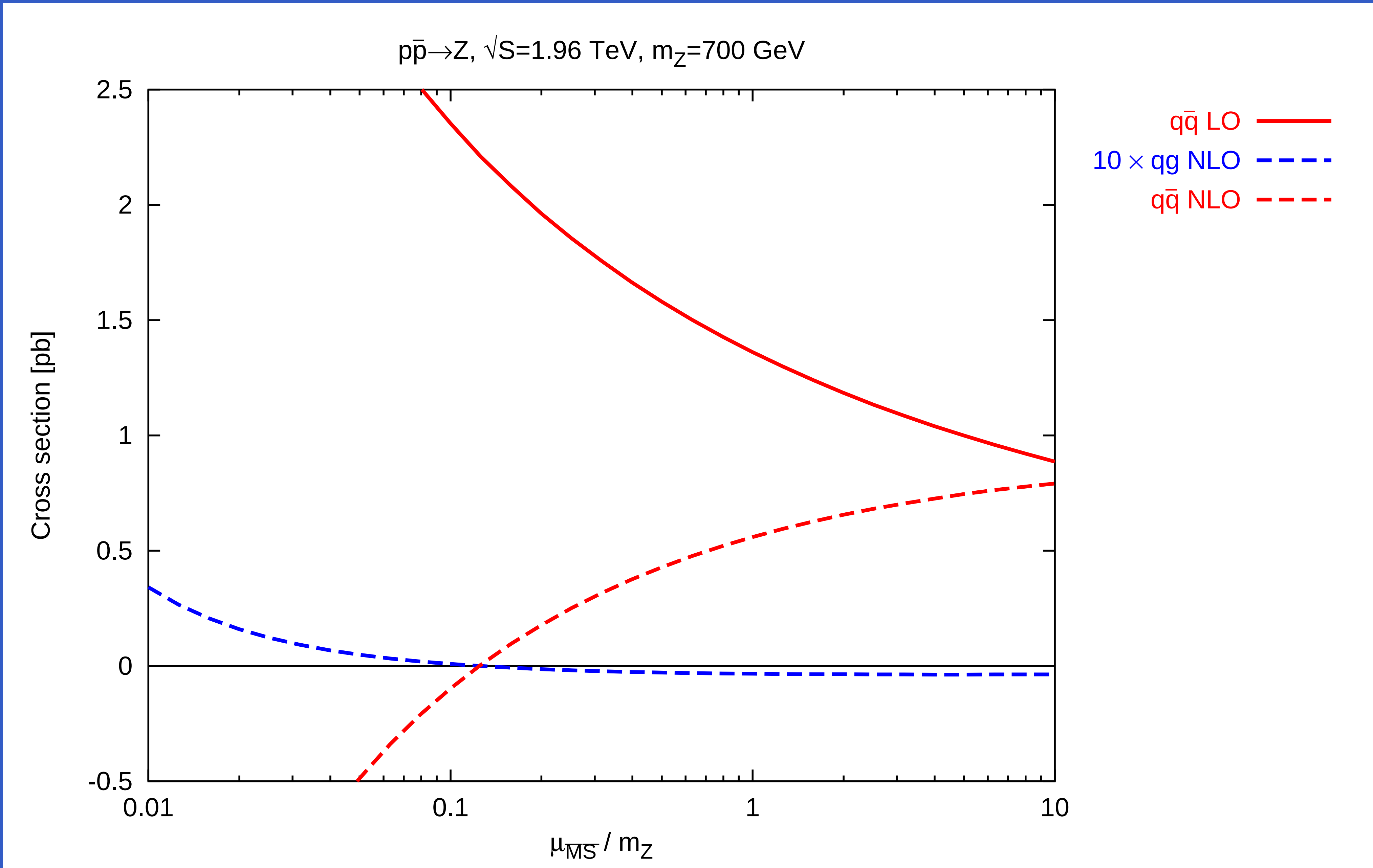

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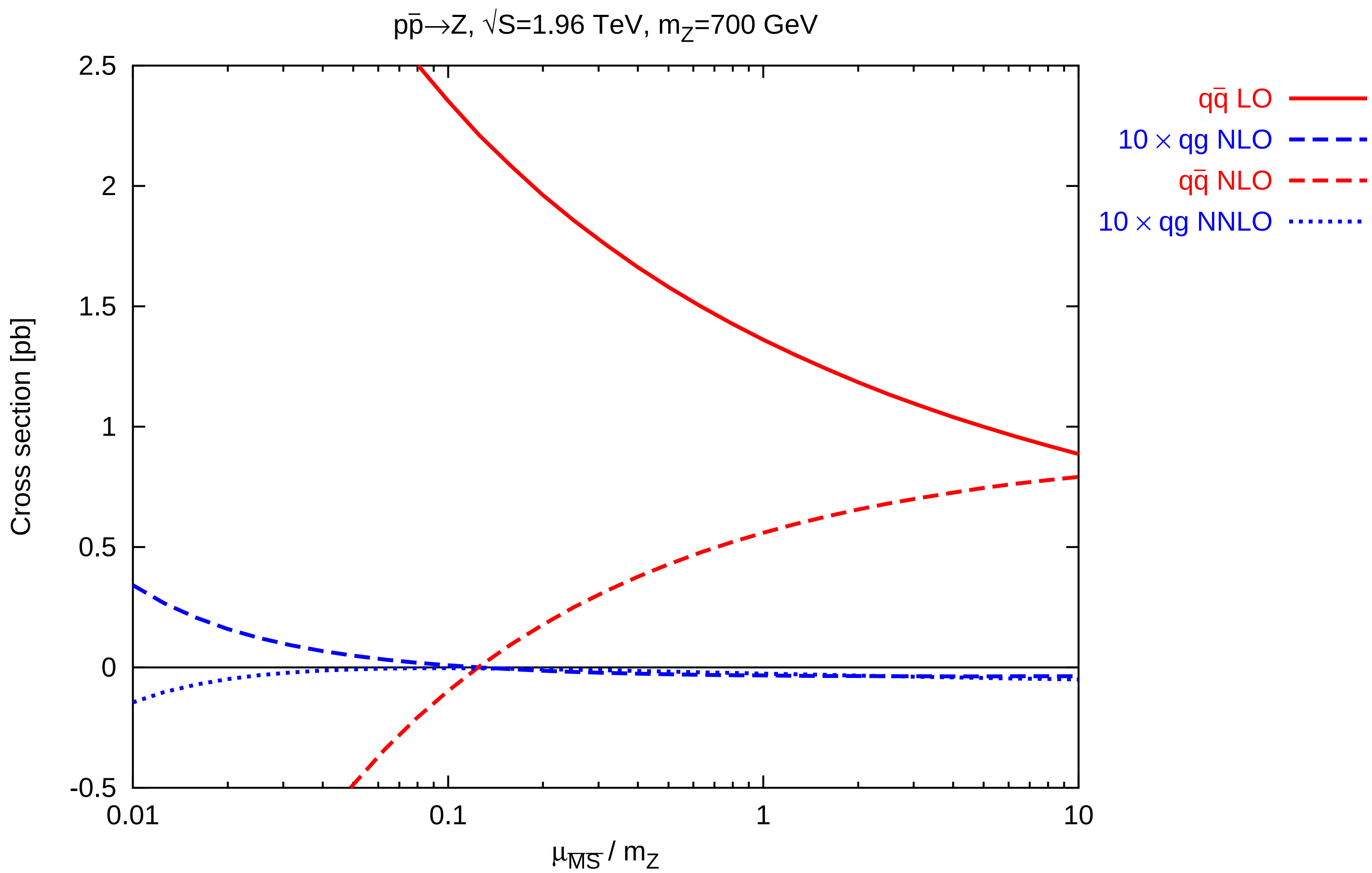

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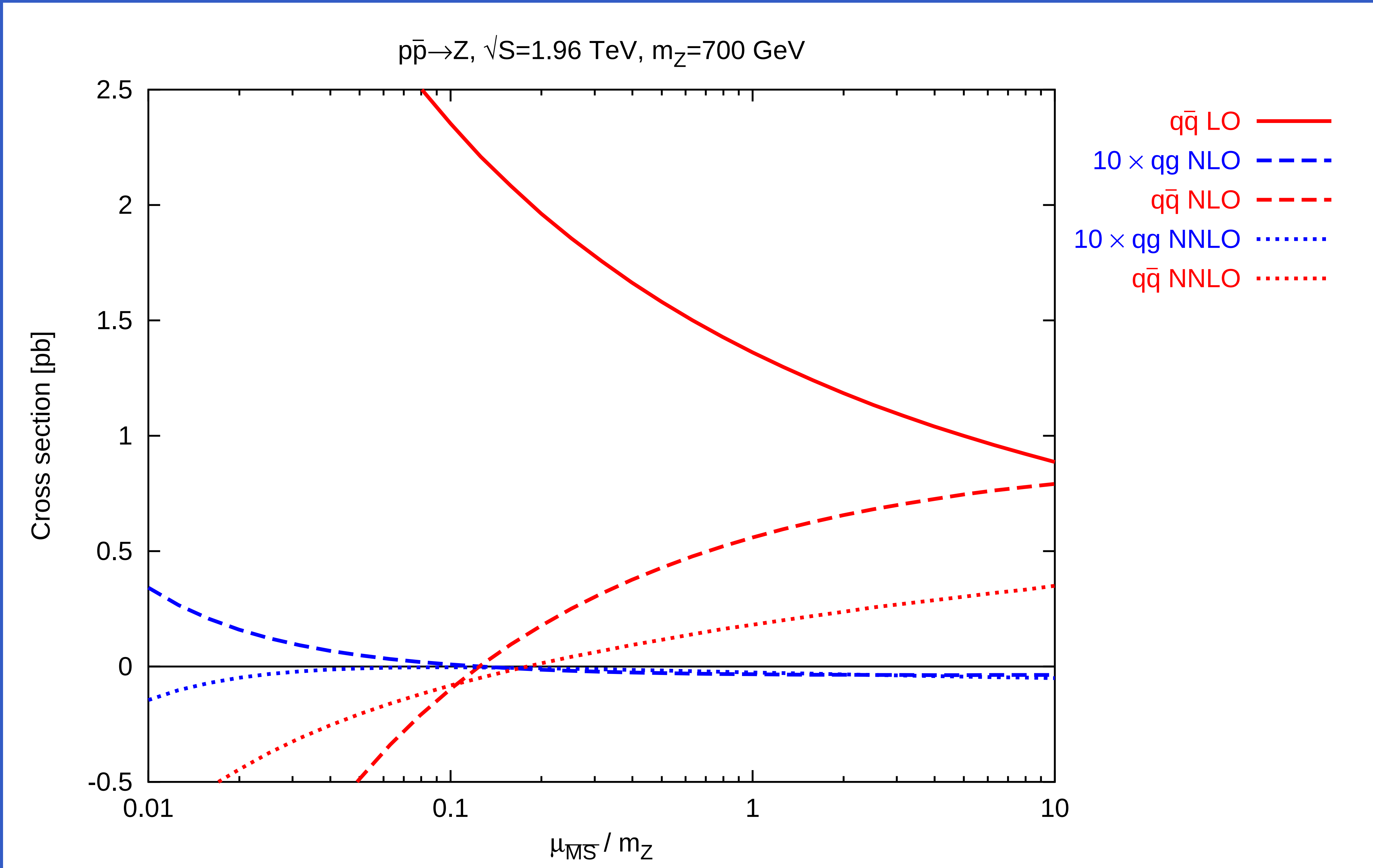

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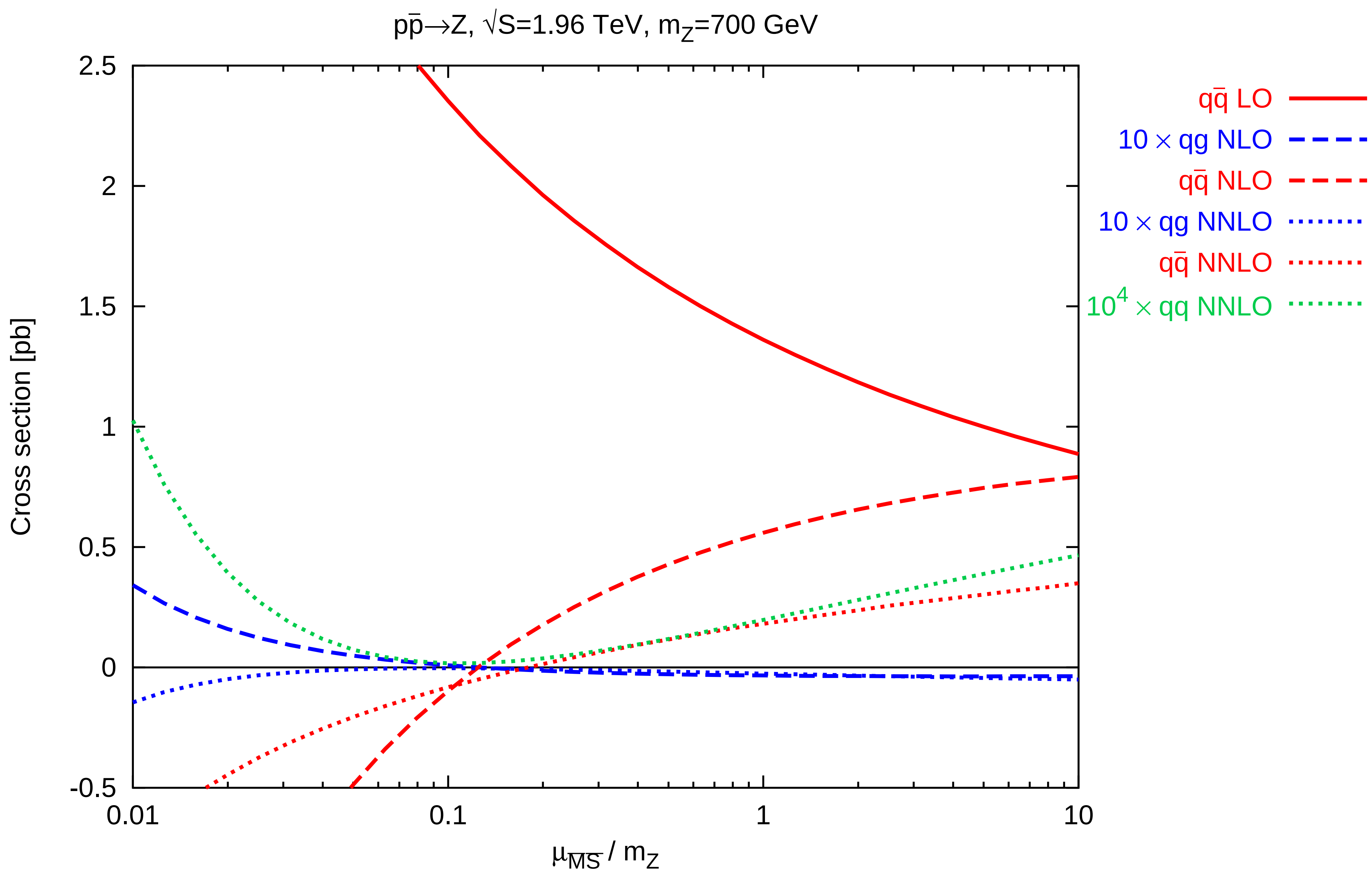

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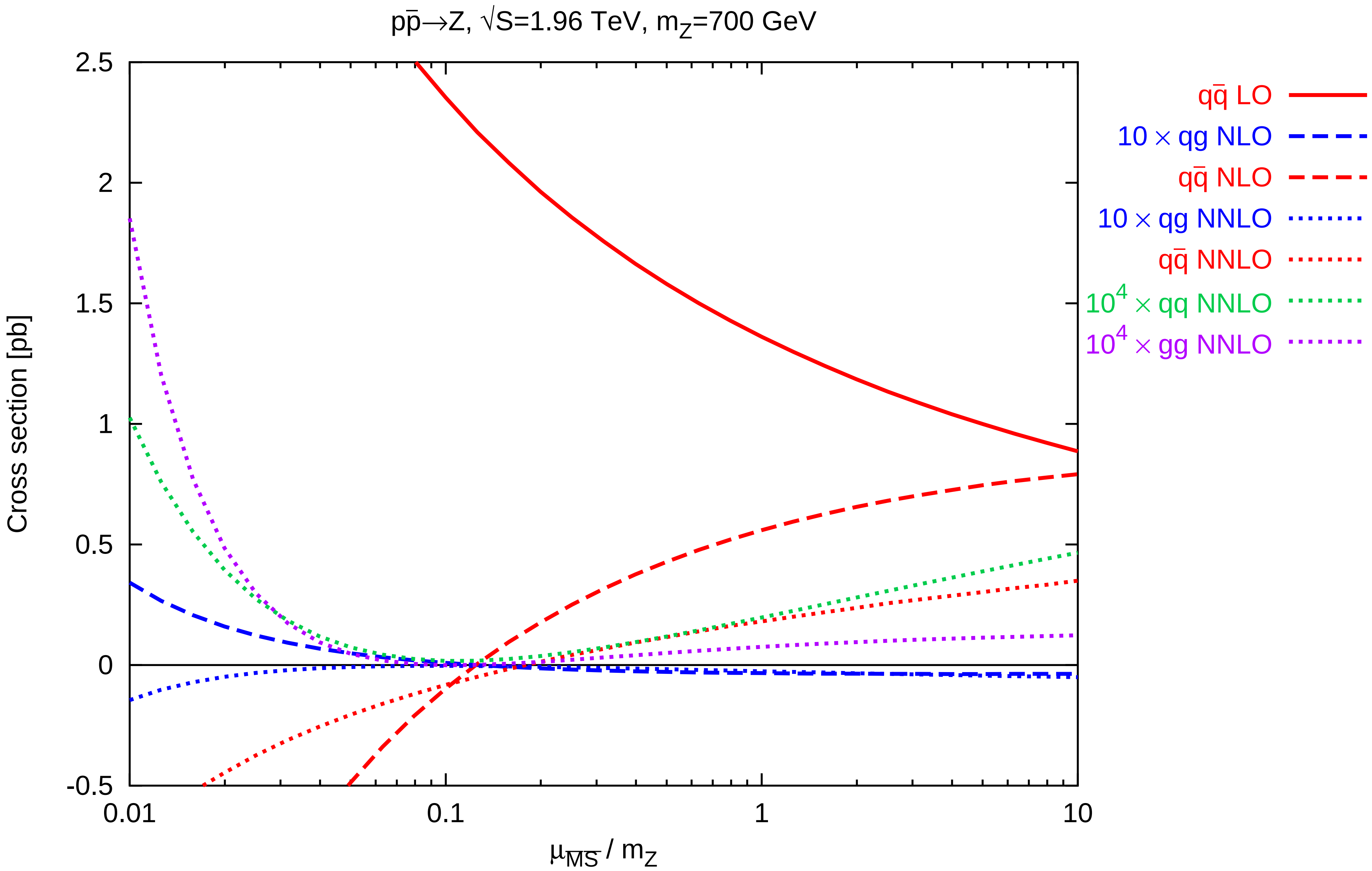

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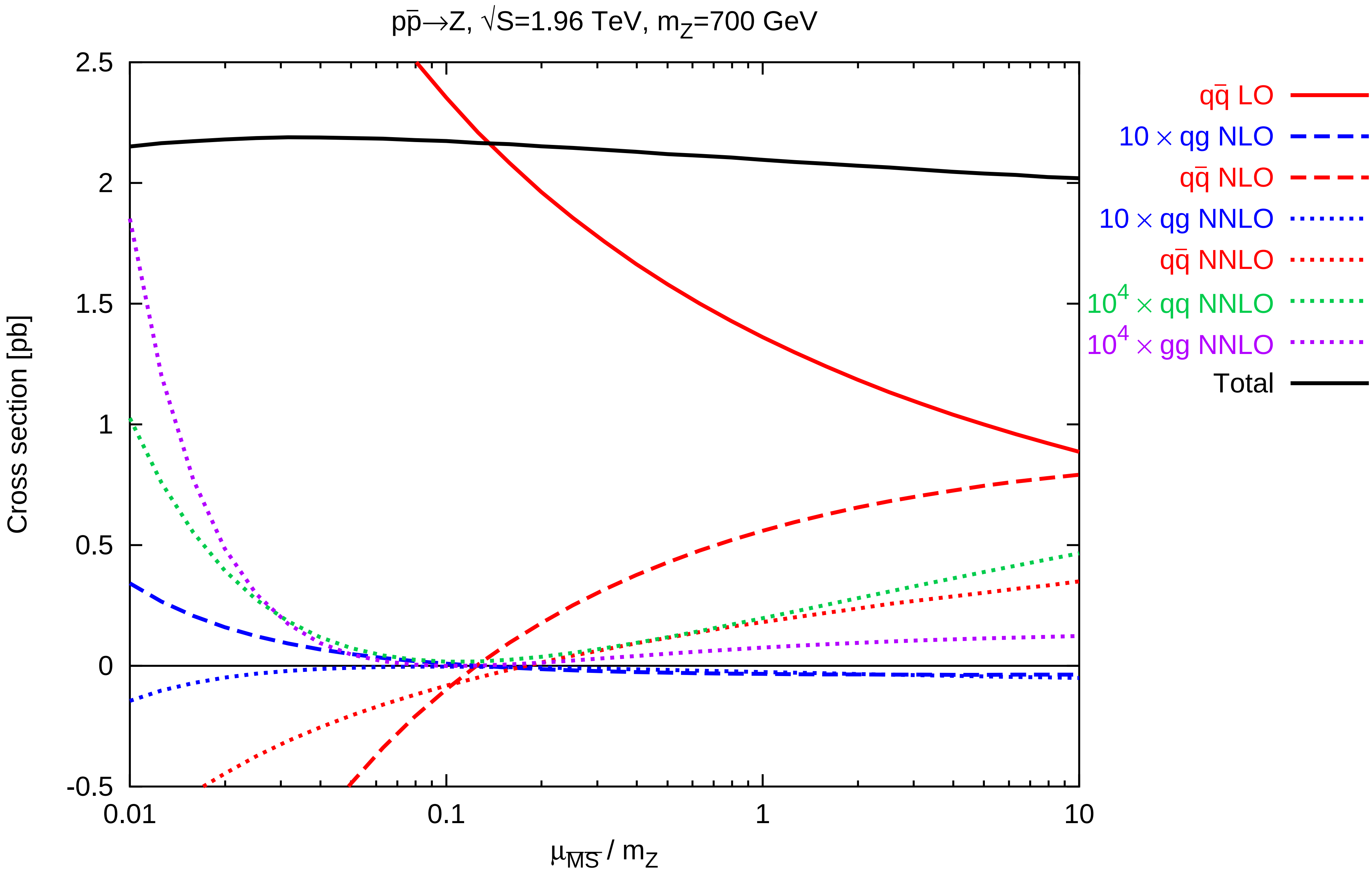

Some closing thoughts

- The collinear plateau provides a physically motivated choice of factorization scale.
- When everything points to the same $\overline{\text{MS}}$ scale, radiative corrections are small.
- NLO t-scheme calculation in progress . . .
- If only we had t-scheme PDFs . . .