Fermion masses without Yukawa hierarchies

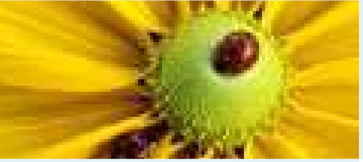
William A. Ponce
Instituto de Física.
Universidad de Antioquia.
Medellín Colombia

PHENO 06/ May 16 2006

3-3-1

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X$$

- Free of gauge anomalies **iff** N_F multiple of 3.
- Peccei-Quinn symmetry can be easily implemented.
- Third quark family different from other two.
- Scalar sector with good candidates for dark matter.
- Suitable for explaining some neutrino properties.
- Strong hierarchy in the Yukawas can be avoided.



Electric Charge Operator

$$Q = \frac{\lambda_{3L}}{2} + \frac{b\lambda_{8L}}{\sqrt{3}} + XI_3$$

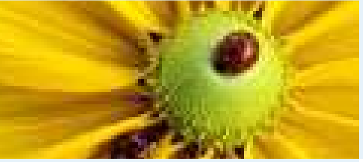
$b = 1/2, 3/2, 5/2, \ldots n/2$

- $b = 1/2$ models without exotic electric charges.
- $b = 3/2$ the Pleitez-Frampton model.

For $b = 1/2$ there are:

- 2 one family models reported in the literaure.
- 4 different models reported in the literature.
- 4 more three family models.

If exotic electric charges are allowed, there are an ∞ number of models.



Model with Right-Handed neutrinos

Quarks: $Q^i_L = (u^i, d^i, D^i)_L \sim (3,3,0)$, $i = 1,2$. Two families.

$Q^3_L = (d^3, u^3, U)_L \sim (3, 3^*, 1/3)$. Third family.

D^i_L, $i = 1,2$ two exotic Down quarks.
U_L an exotic Up quark.

Right-Handed quarks $u^{ac}_L \sim (3^*, 1, -2/3)$, $d^{ac}_L \sim (3^*, 1, 1/3)$
$a = 1,2,3,$, $D^{ic}_L \sim (3^*, 1, 1/3)$, $i = 1,2$, $U^c_L \sim (3^*, 1, -2/3)$.

Leptons $L_{lL} = (l^-, \nu^0_l, \nu^{0c}_l)_L \sim (1, 3^*, -1/3)$, $l = e, \mu, \tau$.

Singlets $l^+_L \sim (1,1,1)$.
ν^0_{lL} neutrino related to lepton l^-_L.
ν^{0c}_{lL} the right-handed neutrinos.

R.Foot, H.N.Long and T.A.Tran, Phys. Rev. D50, R34 (1994)

The scalar sector

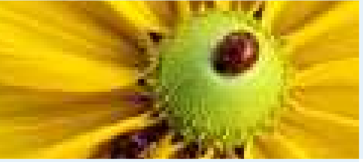
$$\begin{aligned}
\langle \phi_1^T \rangle &= \langle (\phi_1^+, \phi_1^0, \phi_1^{\prime 0}) \rangle = \langle (0, 0, v_1) \rangle \sim (1, 3, 1/3) \\
\langle \phi_2^T \rangle &= \langle (\phi_2^+, \phi_2^0, \phi_2^{\prime 0}) \rangle = \langle (0, v_2, 0) \rangle \sim (1, 3, 1/3) \\
\langle \phi_3^T \rangle &= \langle (\phi_3^0, \phi_3^-, \phi_3^{\prime -}) \rangle = \langle (v_3, 0, 0) \rangle \sim (1, 3, -2/3) \\
\langle \phi_4^T \rangle &= \langle (\phi_4^+, \phi_4^0, \phi_4^{\prime 0}) \rangle = \langle (0, 0, V) \rangle \sim (1, 3, 1/3),
\end{aligned}$$

with the hierarchy $v_1 \sim v_2 \sim v_3 \sim 10^2$ GeV $<< V \sim$ a few TeV.

The anomaly free discrete Z_2 symmetry:

$$\begin{aligned}
Z_2(Q_L^a, \phi_2, \phi_3, \phi_4, u_L^{ic}, d_L^{ac}) &= 1 \\
Z_2(\phi_1, u_L^{3c}, U_L^c, D_L^{ic}, L_{lL}, l_L^+) &= 0.
\end{aligned}$$

W.A.P. C-G.C. H.F. Europhys. Lett. 72, 733 (2005)

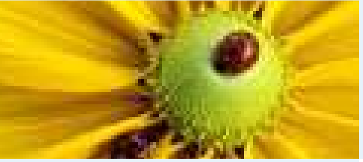


Up quark sector 1

In the basis (u^1, u^2, u^3, U)

$$M_u = \begin{pmatrix} 0 & 0 & h^u_{13} v_3 & h^u_{14} v_3 \\ 0 & 0 & h^u_{23} v_3 & h^u_{24} v_3 \\ 0 & 0 & h^u_{33} v_2 & h^u_{34} v_2 \\ h^u_{41} v_1 & h^u_{42} v_1 & h^u_{43} V & h^u_{44} V \end{pmatrix}$$

$h_{ij} \approx 1$
$v_1 = v_2 = v_3 \equiv v << V$. A see-saw type mass matrix.
$v = 123$ GeV, fixed by $M^2_{W^\pm} = g_3^2 v^2, (g_3 = g_2)$
$h_{ij} \equiv h_t, \quad \delta = v/V$



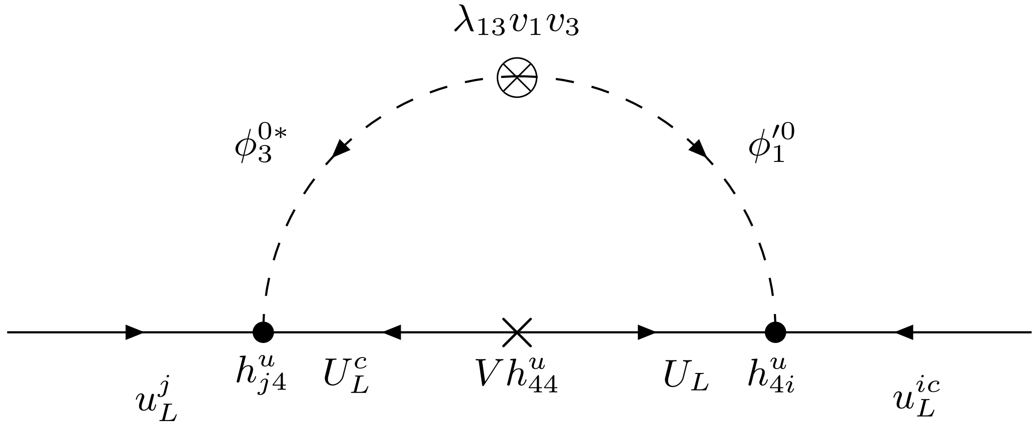
Up quark sector 2

$$M_u M_u^\dagger$$

- A tree-level top quark mass $m_t^2 \approx (h_t - 1)^2 v^2/2 \longrightarrow |1 - h_t| \approx 2$
- A see-saw mass $m_c^2 \approx 4h_t^2 v^2 \delta^2 \longrightarrow V \geq 25$ TeV.
- A zero eigenvalue $(1, -1, 0, 0)/\sqrt{2}$ (The up quark).
- A heavy U quark (mass $\approx V$).

u quark mass

Break the symmetry $u^1 \leftrightarrow u^2$



$\otimes\ \lambda_{13}(\phi_1^*\phi_1)(\phi_3^*\phi_3)$; $\lambda_{13} \sim 1$.
$h^u_{41} = h_t(1-k)$ and $h^u_{13} = h_t(1+k)$.

$$m_u \approx -\lambda_{13} v k^2 \delta \ln \delta / 8\pi^2 \longrightarrow k \approx 0.1$$

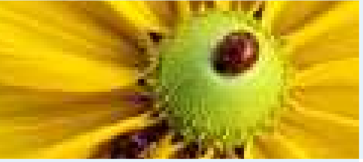
Down quark sector 1

In the basis $(d^1, d^2, d^3, D^1, D^2)$.

$$M_d = \begin{pmatrix} 0 & 0 & 0 & h^d_{14}v_2 & h^d_{15}v_2 \\ 0 & 0 & 0 & h^d_{24}v_2 & h^d_{25}v_2 \\ 0 & 0 & 0 & h^d_{34}v_3 & h^d_{35}v_3 \\ h^d_{41}v_1 & h^d_{42}v_1 & h^d_{43}v_1 & h^d_{44}V & h^d_{45}V \\ h^d_{51}v_1 & h^d_{52}v_1 & h^d_{53}v_1 & h^d_{54}V & h^d_{55}V \end{pmatrix}$$

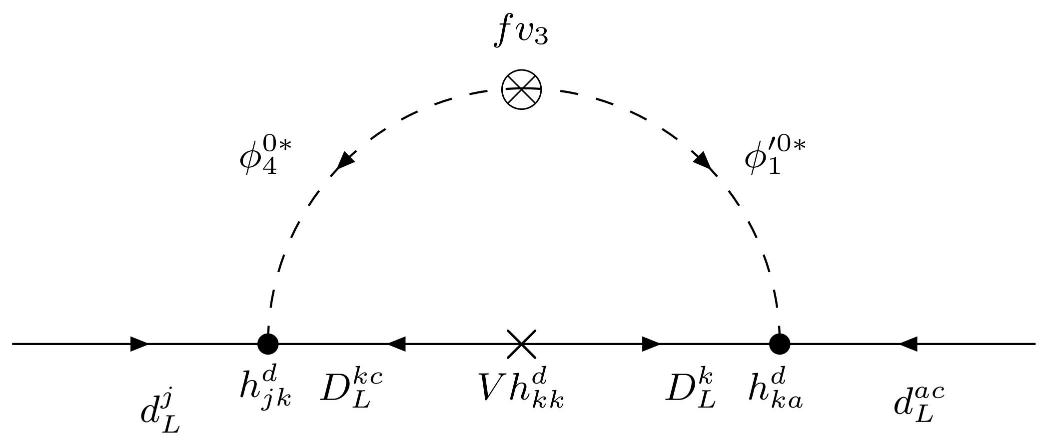
$h^d_{45} = h^d_{54} = 0$ to avoid flavor democracy in the heavy sector.
$h^d_{ks} \equiv h_b \sim 1$

- Two heavy eigenvalues $(\sim V)$.
- A see-saw eigenvalue $m_b \approx 6v\delta h_b \sim 3m_c h_b/h_t$
- A Null space $(1,-1,0,0,0)/\sqrt{2},\ (1,1,-2,0,0)/\sqrt{6}$.



Down quark sector 2

Break the $d^1 \leftrightarrow d^2 \leftrightarrow d^3$ symmetry.
$h^d_{35} = h^d_{53} \equiv h_s = h_b(1 + k_s)$.
There is a new see-saw eigenvalue
$m_s \approx 2h_b v k_s^2/3, \longrightarrow k_s \approx 3\sqrt{m_s/m_b} \approx 0.47$



Break the $d^1 \leftrightarrow d^2$ symmetry. $f\phi_1\phi_2\phi_3, \quad f \sim v.$
$h^d_{14} \approx h_b(1 + k_d), \; h^d_{11} \approx h_b(1 - k_d)$

$$m_d \approx \frac{h_h^2 k_d^2}{h_t^2 k_u^2} m_u, \quad k_d \approx 1.4 k_u \sim 0.14$$

The charged lepton sector 1

New ingredients: E^a, $a = 1, 2, 3$ with quantum numbers
$E_L^{a-} \oplus E_L^{a+} \sim (1, 1, -1) \oplus (1, 1, 1)$.
$\phi_5^0 \sim (1, 1, 0)$ with VEV $\langle \phi_5^0 \rangle = v_5 \approx v$

In the basis $(e,\ \mu,\ \tau,\ E^1,\ E^2,\ E^3)$

$$M_l = \begin{pmatrix} 0 & 0 & 0 & h_{e1}v_3 & h_{e2}v_3 & h_{e3}v_3 \\ 0 & 0 & 0 & h_{\mu 1}v_3 & h_{\mu 2}v_3 & h_{\mu 3}v_3 \\ 0 & 0 & 0 & h_{\tau 1}v_3 & h_{\tau 2}v_3 & h_{\tau 3}v_3 \\ h_{1e}v_5 & h_{2e}v_5 & h_{3e}v_5 & h_{11}M & h_{12}M & h_{13}M \\ h_{1\mu}v_5 & h_{2\mu}v_5 & h_{3\mu}v_5 & h_{21}M & h_{22}M & h_{23}M \\ h_{1\tau}v_5 & h_{2\tau}v_5 & h_{3\tau}v_5 & h_{31}M & h_{32}M & h_{33}M \end{pmatrix}$$

The charged lepton sector 2

With the symmetry $e \leftrightarrow \mu \leftrightarrow \tau$

Making $h_{ab} = h_\tau \delta_{ab},\ h_{al} = h_{la} = h_\tau,\ v_3 = v_5 = v << M \equiv V$

- 3 heavy states related with the exotics E^a.
- One see-saw $m_\tau \approx 9 h_\tau v \delta, \longrightarrow h_\tau \approx h_t/3$.
- see-saw eigenvector $(1,1,1,-3\delta,-3\delta,-3\delta)/\sqrt{3+27\delta^2}$
- Two zero eigenvalues.
- Eigenvectors $(1,-1,0,0,0,0)/\sqrt{2},\ (1,1,-2,0,0,0)/\sqrt{6}$.

Break the $\mu \leftrightarrow \tau$ symmetry (keep the $e \leftrightarrow \mu$)

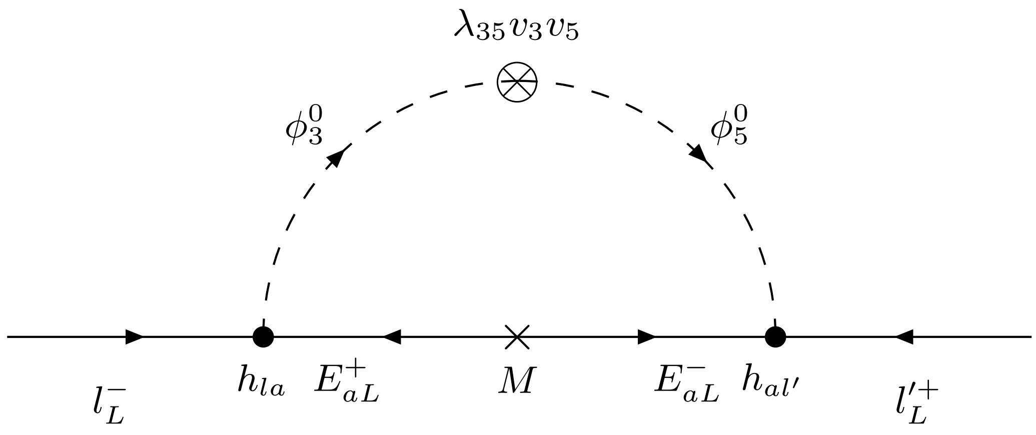
$h_{\tau 3} = h_{3\tau} = h_\mu \neq h_\tau$

A new see-saw eigenvalue $\longrightarrow h_\mu \approx 2.87 h_\tau$.

A zero eigenvalue (the e) with eigenvector $(1,-1,0,0,0,0)/\sqrt{2}$.

The charged lepton sector 3

Generate e mass via radiative corrections.



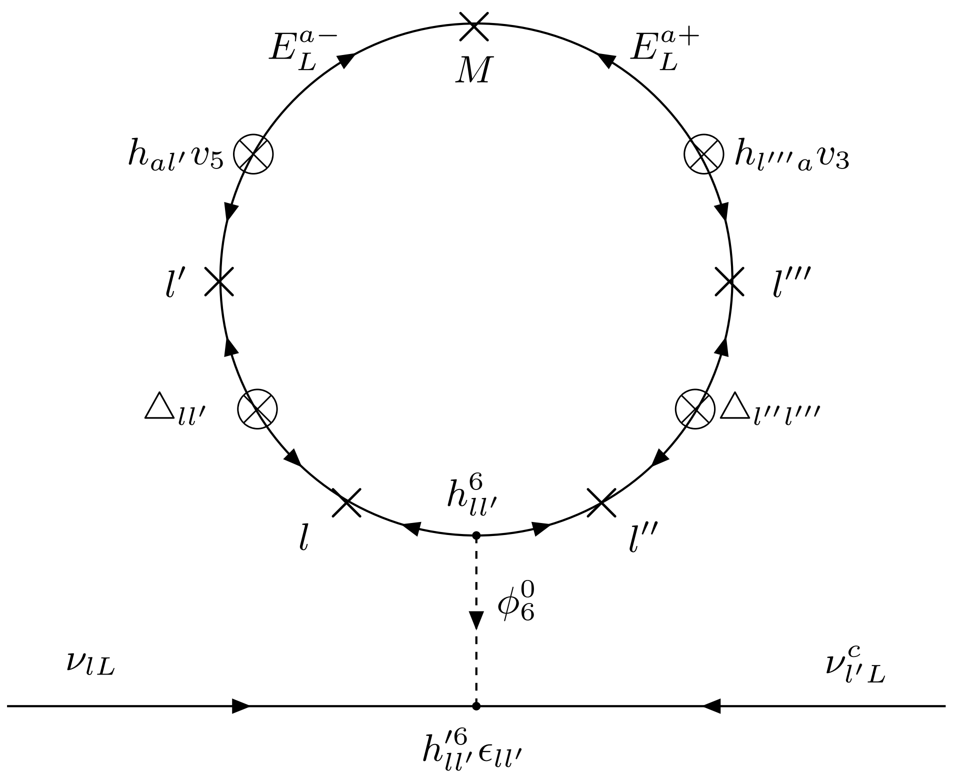
Break $e \leftrightarrow \mu$ symmetry. $\lambda_{35}|\phi_3|^2|\phi_5|^2, \ \lambda_{35} \sim 1.$

$h_{e1} \approx h_\tau(1-k_e), \ h_{1e} \approx h_\tau(1+k_e)$

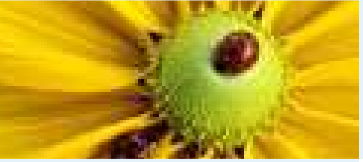
$$k_e \approx 0.1$$

Neutrinos 1

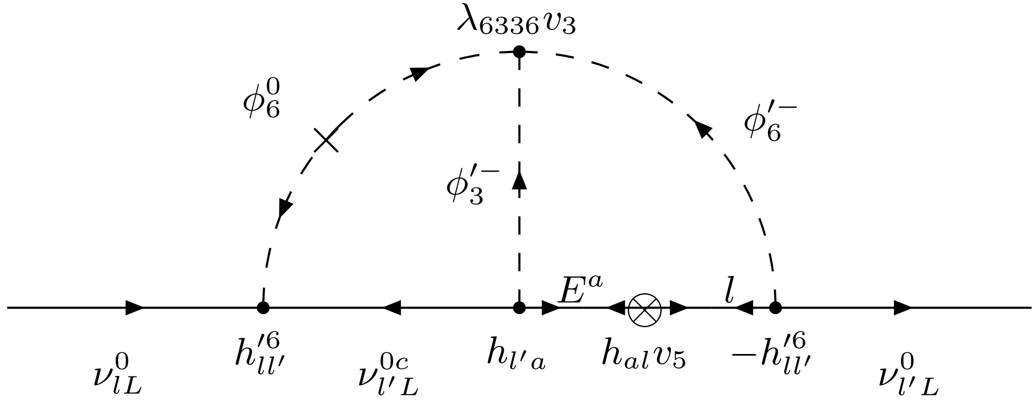
No tree-level mass terms for ν_l. No radiative corrections.
New ingredient:
$\phi_6^T = (\phi_6^0, \phi_6^-, \phi_6'^-) \sim (1, 3, -2/3), \; <\phi_6>= 0, \; Z_2(\phi_6) = 0$



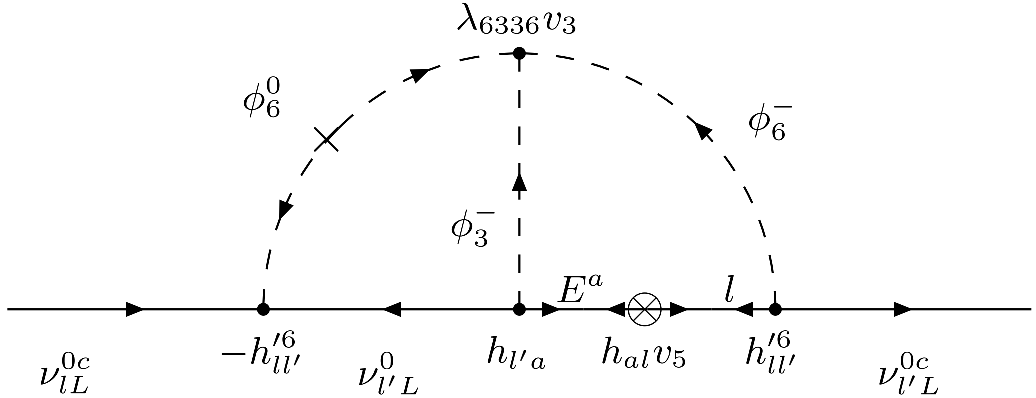
Radiatively induced VEV. Dirac masses $\sim 0.1 - 1$ eV



Neutrinos 2

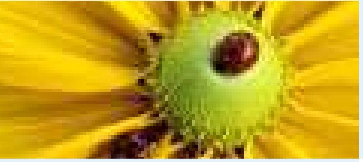


(a)



(b)

Majorana masses.



Conclusions

- Fermion mass spectrum withoutYukawa hierarchies.
- No predictive power.
- Unitarity violation of the CKM mixing matrix: $\sim \delta^4$.
- FCNC: $\sim \delta^4$
- High price payed: 6 Higgs scalars + 3 exotic leptons + Alignment of the vacuum state.