### New Approaches to Electroweak Symmetry Breaking



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### Why Worry About EWSB? Loss of Unitarity in







### Problems with a fundamental Higgs Boson

- No fundamental scalars observed in nature!
- No explanation of Electroweak Symmetry Breaking
- Hierarchy and Naturalness Problem



• Triviality Problem  $\swarrow \beta = \frac{3\lambda^2}{2\pi^2} > 0$ 

# A Fork in the Road...

- (Make the Higgs Natural: Supersymmetry)
- Make the Higgs Composite
  - Little Higgs
  - Twin Higgs
- Eliminate the Higgs
  - Technicolor
  - "Higgsless" Models



"When you come to a fork in the road, take it!" — Yogi Berra



### The Little Higgs

**Collective Symmetry Breaking:** 



For weak springs, masses at end very weakly coupled!

$$\frac{\eta_2}{\eta_4} \simeq \frac{g^2}{16\pi^2}$$

$$m_h^2 \simeq \frac{g^2}{16\pi^2} f^2$$

Global Symmetries	Gauge Symmetries	$\operatorname{triplet}$	# Higgs
SU(5)/SO(5)	$[SU(2) \times U(1)]^2$	Yes	1
$SU(3)^{8}/SU(3)^{4}$	$SU(3) \times SU(2) \times U(1)$	Yes	2
SU(6)/Sp(6)	$[SU(2) \times U(1)]^2$	No	2
$SU(4)^{4}/SU(3)^{4}$	$SU(4) \times U(1)$	No	2
$SO(5)^{8}/SO(5)^{4}$	$SO(5) \times SU(2) \times U(1)$	Yes	2
SU(9)/SU(8)	$SU(3) \times U(1)$	No	2
$SO(9)/[SO(5) \times SO(4)]$	$SU(2)^3 \times U(1)$	Yes	1

Arkani-Hamed, Cohen, Georgi

Meade, hep-ph/0402036

Global Symmetry Extended to Third Generation

- Top Yukawa Large and breaks chiral symmetries
- Extra singlet quarks added
- Top mass results from seesaw like mixing between doublet and singlet fermions
- EWSB: radiatively induced

### Little Higgs : The Hierarchy









10 TeV +	UV completion ? sigma model cut-off
1 TeV –	colored fermion related to top quark new gauge bosons related to SU(2) new scalars related to Higgs
200 GeV	1 or 2 Higgs doublets, possibly more scalars

Cancellation of divergences by particles of same spin!

Schmaltz hep-ph/0210415



### Twin Higgs

• Global SU(4) Symmetry, H in fundamental

$$- V(H) = -m^2 H^{\dagger} H + \lambda (H^{\dagger} H)^2$$

- <H>, SU(4) breaks to SU(3); 7 GBs
- Weakly Gauge SU(2)<sub>W</sub> x SU(2)<sub>H</sub>,  $H=(H_W,H_H)$ 
  - 3 GBs eaten, 4 remaining are "higgs"

$$- \Delta V^{(2)} = \frac{9g_A^2 \Lambda^2}{64\pi^2} H_W^{\dagger} H_W + \frac{9g_B^2 \Lambda^2}{64\pi^2} H_H^{\dagger} H_H$$

- Z<sub>2</sub> symmetry: g<sub>A</sub>=g<sub>B</sub>
  - Accidental SU(4) symmetry of  $\Delta V^{(2)}$
  - No mass generated for higgs boson to  $O(g^2)$

Chacko, Go, and Harnick hep-ph/0506256

# Twin Higgs (cont'd)

- Self-coupling  $\Delta V^{(4)} \propto \frac{g^4}{16\pi^2} \log\left(\frac{\Lambda}{gf}\right) \left(|H_W|^4 + |H_H|^4\right)$
- Extend SU(4) global symmetry to top-quark sector
- EWSB: Radiatively induced
- Hierarchy : like Little Higgs



Goh, Argonne Workshop 2006



### Eliminate the Higgs...

### Technicolor: <u>Higgsless since 1976!</u>

Eliminate Scalars: Electroweak gauge symmetry broken by the nonzero expectation value of a fermion bilinear, driven by new strong interactions.

Understanding of strongly-interacting gauge theories is extremely limited  $\Rightarrow$  theories constructed by analogy!

### Technicolor Limits:

- Model Dependent
- Just Reaching interesting range!
- Run II & LHC will extend limits substantially

No Run II limits yet?

Narain, Womersley, RSC PDG review

Process	Excluded mass range	Decay channels	Ref
$p\overline{p} \to \rho_T \to W\pi_T$	$\begin{array}{l} 170 < m_{\rho_T} < 190 \; \mathrm{GeV} \\ \mathrm{for} \; m_{\pi_T} \approx m_{\rho_T}/2 \end{array}$	$\begin{array}{c} \rho_T \to W \pi_T \\ \pi_T^0 \to b \overline{b} \ \pi_T^{\pm} \to b \overline{b} \end{array}$	[16 c
$p\overline{p} \to \omega_T \to \gamma \pi_T$	$\begin{array}{l} 140 < m_{\omega_T} < 290 \ {\rm GeV} \\ {\rm for} \ m_{\pi_T} \approx m_{\omega_T}/3 \\ {\rm and} \ M_T = 100 \ {\rm GeV} \end{array}$	$\begin{array}{c} \omega_T \to \gamma \pi_T \\ \pi_T^0 \to b \overline{b} \\ \pi_T^{\pm} \to b \overline{c} \end{array}$	[18
$p\overline{p} \to \omega_T / \rho_T$	$\begin{array}{l} m_{\omega_T} = m_{\rho_T} < 203 \ {\rm GeV} \\ {\rm for} \ m_{\omega_T} < m_{\pi_T} + m_W \\ {\rm or} \ M_T > 200 \ {\rm GeV} \end{array}$	$\omega_T/\rho_T \to \ell^+ \ell^-$	[19
$e^+e^- \to \omega_T/\rho_T$	$\begin{array}{l} 90 < m_{\rho_T} < 206.7 \ {\rm GeV} \\ m_{\pi_T} < 79.8 \ {\rm GeV} \end{array}$	$ \begin{array}{l} \rho_T \to WW, \\ W\pi_T, \ \pi_T\pi_T, \\ \gamma\pi_T, \ \text{hadrons} \end{array} $	[20
$p\overline{p} \rightarrow \rho_{T8}$	$260 < m_{\rho_{T8}} < 480~{\rm GeV}$	$\rho_{T8} \rightarrow q\overline{q}, \ gg$	[22
$     \overline{pp} \to \rho_{T8} \\                                    $	$m_{ ho_{T8}} < 510 \text{ GeV} m_{ ho_{T8}} < 600 \text{ GeV} m_{ ho_{T8}} < 465 \text{ GeV}$	$\begin{aligned} \pi_{LQ} &\to c\nu \\ \pi_{LQ} &\to b\nu \\ \pi_{LQ} &\to \tau q \end{aligned}$	[25 [25 [24
$p\overline{p} \rightarrow g_t$	$0.3 < m_{g_t} < 0.6 \text{ TeV}$ for $0.3 m_{g_t} < \Gamma < 0.7 m_{g_t}$	$g_t \rightarrow b\overline{b}$	[30
$p\overline{p} \to Z'$	$m_{Z'} < 480 \text{ GeV}$ for $\Gamma = 0.012 m_{Z'}$ $m_{Z'} < 780 \text{ GeV}$ for $\Gamma = 0.04 m_{Z'}$	$Z' \to t\bar{t}$	[31

What about the S-parameter? Why are we still talking about technicolor?

- Technicolor may be there
  - No "computations" of S in non-QCD like theories
- Technicolor has interesting experimental signatures
  - Complementary to other BSM theories
- AdS/CFT Correspondence:
  - Some 4D strongly-coupled theories "dual" to weakly-coupled 5D theories
  - New model building ideas
  - Address S parameter issues



$$\widehat{A}_5^a = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} A_5^{an}(x_\nu) \sin\left(\frac{nx_5}{R}\right)$$

4-D gauge kinetic term contains  $\frac{1}{2} \sum_{n=1}^{\infty} \left[ M_n^2 (A_\mu^{an})^2 - 2M_n A_\mu^{an} \partial^\mu A_5^{an} + (\partial_\mu A_5^{an})^2 \right] \quad \text{i.e., } A_L^{an} \leftrightarrow A_5^{an}$ 



energy behavior through exchange of massive vector particles Can we apply this to W and Z?

RSC, H.J. He, D. Dicus

$$\begin{array}{|c|c|c|c|c|c|c|c|} & g^2C^{eab}C^{ecd} & g^2C^{eac}C^{edb} & g^2C^{ead}C^{ebc} \\ \hline (a) & 6c(x^4-x^2) & \frac{3}{2}(3-2c-c^2)x^4 & \frac{-3}{2}(3+2c-c^2)x^4 \\ & & -3(1-c)x^2 & +3(1+c)x^2 \\ \hline (b1) & -2c(x^4 \downarrow x^2) & & \\ (c1) & -4cx^4 & & \\ \hline (b2,3) & & \frac{-1}{2}(3-2c+c^2)x^4 & \frac{1}{2}(3+2c-c^2)x^4 \\ & & +3(1-c)x^2 & -3(1+c)x^2 \\ \hline (c2,3) & & (-3+2c+c^2)x^4 & (3+2c-c^2)x^4 \\ & & -8cx^2 & -8cx^2 \\ \hline & & 8cx^2 & -8cx^2 & -8cx^2 \Rightarrow 0 \\ \end{array}$$

# No Free Lunch

Non-renormalizability of 5-DYM implies lingering unitarity issues ... how is this manifest in KK scattering?

Consider a state composed of KK pairs with  $n \leq N_0$ 

$$|\psi^{ab}\rangle = \frac{1}{\sqrt{N_0}} \sum_{\ell=1}^{N_0} |A_L^{a\ell} A_L^{b\ell}\rangle$$

Find 4-D s-wave, gauge-singlet amplitude of  $|\psi^{aa}\rangle \rightarrow |\psi^{cc}\rangle$ 

$$a_{\psi}^{00} = \frac{N_0}{R} \frac{kg_5^2}{8\pi^2} \mathcal{O}(1)$$

Grows with N<sub>0</sub>!

### Moral: Unitarity can be delayed, but not avoided!

• unitary bound on  $a_{\psi}^{00}$  implies highest KK mode number is bounded from above:

(consistent with 5-d intuition)

 $\frac{N_0}{R} < \frac{\sqrt{32\pi^2}}{k} \frac{\mathcal{O}(1)}{n^2}$ 

- g<sub>SU(2)</sub>~I ; thus, one can potentially add a few vector mesons and delay unitarity onset
- Generalizes to a large class of 5-d manifolds and boundary conditions - Higgsless Models (Csaki, Grojean, Murayama, Pilo, Terning)

# Recipe for a Higgsless Model:

- Choose "bulk" gauge group, location of fermions, and boundary conditions
- Choose  $g(x_5)$
- Choose metric/manifold:  $g_{MN}(x_5)$
- Calculate spectrum & eigenfunctions
- Calculate fermion couplings
- Compare to Standard Model: S, T, U, ...

### **Deconstructed Higgsless Models**



- $SU(2)^N \times U(1)$ ; general  $f_j$  and  $g_k$
- Fermions sit on "branes" [sites 0 and N+1]
- Many 4-D/5-D theories are limiting cases... study them all at once!
- e.g., N=I equivalent to technicolor/one-Higgs

Foadi, et. al. & Chivukula et. al.

# Conflict of S & Unitarity

Heavy resonances must unitarize WW scattering (since there is no Higgs!)

This bounds lightest KK mode mass:  $m_{Z_1} < \sqrt{8\pi v}$ ... and yields  $\alpha S \ge \frac{4s_Z^2 c_Z^2 M_Z^2}{8\pi v^2} = \frac{\alpha}{2}$ 

#### Too large by a factor of a few!

Independent of warping or gauge couplings chosen...





Since Higgsless models with localized fermions are not viable, look at:

**Delocalized Fermions**, .i.e., mixing of "brane" and "bulk" modes

$$\mathcal{L}_f = \vec{J}_L^{\mu} \cdot \left(\sum_{i=0}^N x_i \vec{A}_{\mu}^i\right) + J_Y^{\mu} A_{\mu}^{N+1}$$

How will this affect precision EW observables?

Cacciapaglia et. al.

Foadi & Schmidt: see Schmidt talk

# Ideal Delocalization

- Choose delocalization related to W wavefunction:  $g_i x_i \propto v_i^W$
- NB:  $x_i = |\psi_f(i)|^2 > 0$
- W-wavefunction orthogonal to KK wavefunctions.
- No (tree-level) couplings to heavy modes!



$$\hat{S} = \hat{T} = W = 0$$
$$Y = M_W^2 (\Sigma_W - \Sigma_Z)$$

RSC, HJH, MK, MT, EHS hep-ph/0504114

# LEP II Constraints

# LEP II measurements of WWZ vertex yield $\Delta g_1^Z \le 0.028 @ 95\% {\rm CL}$

In a flat space SU(2) x SU(2) x U(1) model

$$\Delta g_1^Z = \frac{\pi^2}{12c^2} \left(\frac{M_W}{M_{W_1}}\right)^2 \left[\frac{1}{4} \cdot \frac{7+\kappa}{1+\kappa}\right]$$

$$M_{W_1} \ge 500 \text{ GeV} \cdot \sqrt{\frac{0.028}{\Delta g_{max}}} \left[\frac{1}{4} \cdot \frac{7+\kappa}{1+\kappa}\right]$$

# LHC Phenomenology



Birkedal, et.al., hep-ph/0412278

# Observations

- Our standards have changed
  - We are content with a low-energy effective theory valid to ~ few TeV
  - This is a good thing in preparation for the LHC ...
- Fine-tuning is in the eye of the beholder
  - S=O(I) in QCD-like technicolor; experimental bound O(0.1) - hence need 10% fine-tuning?
  - Dynamics matters: Inflation makes fine-tuning of flatness problem irrelevant.

# Conclusions

- Two new mechanisms to address hierarchy problem
  - Composite/Little/Twin Higgs
  - Higgsless Models
- Both predict new TeV Scale particles
  - Extended Electroweak Gauge Symmetries
  - Extended Fermion Sector
- Much Phenomenology Left to be done!

# LHC-TI Town Meeting





TODAY, 2:00pm -- 4:30pm CDT Pyle Center, Room 227 702 Langdon Street University of Wisconsin Madison, Wisconsin

# Extra Slides

### **Electroweak Parameters I**

EW corrections  $(S, T, \Delta \rho, \delta)$  defined from amplitudes for "on-shell" 4-fermion processes

$$\begin{split} -\mathcal{A}_{NC} &= e^2 \frac{\mathcal{Q}\mathcal{Q}'}{Q^2} + \frac{(I_3 - s^2 \mathcal{Q})(I_3' - s^2 \mathcal{Q}')}{\left(\frac{s^2 c^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2} - \alpha T\right)} \\ &+ \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2} I_3I_3' + 4\sqrt{2}G_F \left(\Delta\rho - \alpha T\right)\left(\mathcal{Q} - I_3\right)\left(\mathcal{Q}' - I_3'\right)}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2} \frac{(I_+I_-' + I_-I_+')}{2} \\ \\ -\mathcal{A}_{CC} &= \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2} \frac{(I_+I_-' + I_-I_+')}{2} \\ \\ -\mathcal{A}_{CC} &= \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2} \frac{(I_+I_-' + I_-I_+')}{2} \\ \\ -\mathcal{A}_{CC} &= \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2} \frac{(I_+I_-' + I_-I_+')}{2} \\ \\ -\mathcal{A}_{CC} &= \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2} \frac{(I_+I_-' + I_-I_+')}{2} \\ \\ -\mathcal{A}_{CC} &= \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2} \frac{(I_+I_-' + I_-I_+')}{2} \\ \\ -\mathcal{A}_{CC} &= \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2}\frac{(I_+I_-' + I_-I_+')}{2} \\ \\ -\mathcal{A}_{CC} &= \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2}\frac{(I_+I_-' + I_-I_+')}{2} \\ \\ -\mathcal{A}_{CC} &= \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} \\ \\ + \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} \\ \\ - \frac{(I_+I_+' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{1}{4\sqrt{2}G_F}\right)} \\ \\ - \frac{(I_+I_+' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{1}{4\sqrt{2}G_F}\right)} \\ \\ - \frac{(I_+I_+' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{1}{4\sqrt{2}G_F}\right)} \\ \\ -$$

### **Electroweak Parameters II**

Alternative formulation defined at zero momentum

$$\hat{S} = \frac{1}{4s^2} \left( \alpha S + 4c^2 (\Delta \rho - \alpha T) + \frac{\alpha \delta}{c^2} \right)$$
$$\hat{T} = \Delta \rho$$
$$Y = \frac{c^2}{s^2} (\Delta \rho - \alpha T)$$
$$W = \frac{\alpha \delta}{4s^2 c^2}$$

Barbieri, Pomarol, Rattazzi, Strumia

### Aside: Moose notation



Reveals symmetry (breaking) structure at a glance A familiar example:

$$SU(2)_W \times U(I)_B \longrightarrow U(I) \iff 2_g \xrightarrow{w} (1)_{g'}^B$$

**XX**2

Each circle represents a global SU(2) of which all (solid, left) or a U(1) subgroup (dashed, right) is gauged Low-energy  $\mathcal{L}_{eff}$  description of symmetry-breaking sector employs non-linear sigma-model fields  $\Sigma$ A solid line linking two circles is an [SU(2) x SU(2) / SU(2)] non-linear sigma model field; at the scale v this breaks the gauged or global symmetries of the attached circles Note:  $\Sigma$  is a 2x2 matrix field transforming as  $\Sigma \to L\Sigma R^{\dagger}$ under the SU(2) groups which it connects. An SU(2)xSU(2)xU(1)xU(1) model with the following symmetry-breaking pattern:

 $SU(2)_L \times SU(2)_W \times U(1)_B \times U(1)_R$ 

SU(2)<sub>weak</sub> x U(1)<sub>Y</sub>

**∀** <sub>V</sub> U(1)<sub>EM</sub>

Can be represented compactly in Moose notation



#### Mass Matrix & Spectrum g<sub>2</sub> 9<sub>1</sub> g<sub>N+1</sub> g<sub>0</sub> g<sub>N</sub> $\begin{array}{c|ccc} g_0^2 f_1^2 & -g_0 g_1 f_1^2 \\ \hline -g_0 g_1 f_1^2 & g_1^2 (f_1^2 + f_2^2) \\ \hline & -g_1 g_2 f_2^2 \end{array}$ $\frac{-g_1g_2f_2^2}{g_2^2(f_2^2+f_3^2)}$ $-g_2g_3f_3^2$ $M_Z^2 = \frac{1}{4}$

- $M_Z^2$  as above; Spectrum: Photon, Z, heavy Z's
- $M_W^2$  has  $g_{N+1} = 0$ ; Spectrum: W, heavy W's
- EM coupling as expected:  $\frac{1}{e^2} = \frac{1}{g_0^2} + \frac{1}{g_1^2} + \ldots + \frac{1}{g_{N+1}^2}$

# **Correlation Functions I**



e.g., weak-hypercharge correlation function for Z exchange is within

$$[G(Q^2)]_{0,N+1} \equiv g_0 g_{N+1} \langle 0 | \frac{1}{Q^2 + M_Z^2} | N+1 \rangle$$



By considering the (0, N+1) co-factor, we deduce the form of the correlation function

$$\begin{bmatrix} G(Q^2) \end{bmatrix}_{0,N+1} = \frac{C}{Q^2(Q^2 + M_Z^2) \prod_{n=1}^N (Q^2 + m_{Z_n}^2)}$$

We know the residue of  $Q^2=0$  pole must be  $e^2$ ...

$$[G(Q^2)]_{0,N+1} = \frac{e^2 M_Z^2 \prod_{n=1}^N \mathsf{m}_{Z_n}^2}{Q^2 (Q^2 + M_Z^2) \prod_{n=1}^N (Q^2 + \mathsf{m}_{Z_n}^2)}$$

Other residues are also informative.

**S** parameter: Brane Fermions Correlation function residue at  $Q^2 = -M_Z^2$ gives "J<sub>3</sub> J<sub>y</sub>" coupling of light Z-boson

$$[\xi_Z]_{WY} = -e^2 \prod_{n=1}^N \frac{1}{1 - \frac{M_Z^2}{\mathsf{m}_{Z_n}^2}} = -e^2 \left( 1 + \frac{\alpha S}{4s_Z^2 c_Z^2} \right)$$

 $\begin{array}{ll} \mbox{Requiring } \mathbf{M_{Z}} << \mathbf{m_{Zn}} \mbox{ yields} \\ \\ \alpha S \approx 4 s_{Z}^{2} c_{Z}^{2} \sum_{n=1}^{N} \frac{M_{Z}^{2}}{\mathbf{m}_{Z_{n}}^{2}} \end{array}$ 

for the entire class of models!