Constraints on Effective Field Theory Couplings Using 311.2 days of LUX Data

2	D.S. Akerib, ^{1,2} S. Alsum, ^{3,*} H.M. Araújo, ⁴ X. Bai, ⁵ J. Balajthy, ⁶ J. Bang, ⁷ A. Baxter, ⁸ E.P. Bernard, ⁹				
3	A. Bernstein, ¹⁰ T.P. Biesiadzinski, ^{1,2} E.M. Boulton, ^{9,11,12} B. Boxer, ⁸ P. Brás, ¹³ S. Burdin, ⁸ D. Byram, ^{14,15}				
4	M.C. Carmona-Benitez, ¹⁶ C. Chan, ⁷ J.E. Cutter, ⁶ L. de Viveiros, ¹⁶ E. Druszkiewicz, ¹⁷ A. Fan, ^{1, 2} S. Fiorucci, ^{11, 7}				
	R.J. Gaitskell, ⁷ C. Ghag, ¹⁸ M.G.D. Gilchriese, ¹¹ C. Gwilliam, ⁸ C.R. Hall, ¹⁹ S.J. Haselschwardt, ²⁰ S.A. Hertel, ^{21,11}				
5					
6	D.P. Hogan, ⁹ M. Horn, ^{15,9} D.Q. Huang, ⁷ C.M. Ignarra, ^{1,2} R.G. Jacobsen, ⁹ O. Jahangir, ¹⁸ W. Ji, ^{1,2}				
7	K. Kamdin, ^{9,11} K. Kazkaz, ¹⁰ D. Khaitan, ¹⁷ E.V. Korolkova, ²² S. Kravitz, ¹¹ V.A. Kudryavtsev, ²² E. Leason, ²³				
8	B.G. Lenardo, ^{6,10} K.T. Lesko, ¹¹ J. Liao, ⁷ J. Lin, ⁹ A. Lindote, ¹³ M.I. Lopes, ¹³ A. Manalaysay, ^{11,6}				
9	R.L. Mannino, ^{24,3} N. Marangou, ⁴ D.N. McKinsey, ^{9,11} DM. Mei, ¹⁴ J.A. Morad, ⁶ A.St.J. Murphy, ²³				
10	A. Naylor, ²² C. Nehrkorn, ²⁰ H.N. Nelson, ²⁰ F. Neves, ¹³ A. Nilima, ²³ K.C. Oliver-Mallory, ^{4,9,11}				
11	K.J. Palladino, ³ C. Rhyne, ⁷ Q. Riffard, ^{9,11} G.R.C. Rischbieter, ^{25,†} P. Rossiter, ²² S. Shaw, ^{20,18} T.A. Shutt, ^{1,2}				
12	C. Silva, ¹³ M. Solmaz, ²⁰ V.N. Solovov, ¹³ P. Sorensen, ¹¹ T.J. Sumner, ⁴ N. Swanson, ⁷ M. Szydagis, ²⁵				
	DI Territor 15 D. Territor 4 W.C. Territor 7 D.D. Territor 12 D.A. Territor 24 D.D. Territor 19 W.H. T. 26				
13	L. Tvrznikova, ^{9,11,12} U. Utku, ¹⁸ A. Vacheret, ⁴ A. Vaitkus, ⁷ V. Velan, ⁹ R.C. Webb, ²⁴ J.T. White, ²⁴				
14					
15	T.J. Whitis, ^{1, 2} M.S. Witherell, ¹¹ F.L.H. Wolfs, ¹⁷ D. Woodward, ¹⁶ X. Xiang, ⁷ J. Xu, ¹⁰ and C. Zhang ¹⁴				
16	(LUX Collaboration)				
17	¹ SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94205, USA				
18	2 Kavli Institute for Particle Astrophysics and Cosmology,				
19	Stanford University, 452 Lomita Mall, Stanford, CA 94309, USA				
20	³ University of Wisconsin-Madison, Department of Physics,				
21	1150 University Ave., Madison, WI 53706, USA				
22	⁴ Imperial College London, High Energy Physics, Blackett Laboratory, London SW7 2BZ, United Kingdom				
23	⁵ South Dakota School of Mines and Technology, 501 East St Joseph St., Rapid City, SD 57701, USA				
24	⁶ University of California Davis, Department of Physics, One Shields Ave., Davis, CA 95616, USA				
25	⁷ Brown University, Department of Physics, 182 Hope St., Providence, RI 02912, USA				
26	⁸ University of Liverpool, Department of Physics, Liverpool L69 7ZE, UK				
27	⁹ University of California Berkeley, Department of Physics, Berkeley, CA 94720, USA				
28	¹⁰ Lawrence Livermore National Laboratory, 7000 East Ave., Livermore, CA 94551, USA				
29	¹¹ Lawrence Berkeley National Laboratory, 1 Cyclotron Rd., Berkeley, CA 94720, USA				
30	¹² Yale University, Department of Physics, 217 Prospect St., New Haven, CT 06511, USA				
31	¹³ LIP-Coimbra, Department of Physics, University of Coimbra, Rua Larga, 3004-516 Coimbra, Portugal ¹⁴ University of South Dakota, Department of Physics, 414E Clark St., Vermillion, SD 57069, USA				
32	¹⁵ South Dakota Science and Technology Authority,				
33	Sanford Underground Research Facility, Lead, SD 57754, USA				
34 35	¹⁶ Pennsylvania State University, Department of Physics,				
36	104 Davey Lab, University Park, PA 16802-6300, USA				
37	¹⁷ University of Rochester, Department of Physics and Astronomy, Rochester, NY 14627, USA				
38	¹⁸ Department of Physics and Astronomy, University College London,				
39	Gower Street, London WC1E 6BT, United Kingdom				
40	¹⁹ University of Maryland, Department of Physics, College Park, MD 20742, USA				
41	²⁰ University of California Santa Barbara, Department of Physics, Santa Barbara, CA 93106, USA				
42	²¹ University of Massachusetts, Amherst Center for Fundamental				
43	Interactions and Department of Physics, Amherst, MA 01003-9337 USA				
44	²² University of Sheffield, Department of Physics and Astronomy, Sheffield, S3 7RH, United Kingdom				
45	²³ SUPA, School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3FD, United Kingdom				
46	²⁴ Texas A & M University, Department of Physics, College Station, TX 77843, USA				
47	²⁵ University at Albany, State University of New York, Department of Physics, 1400 Washington Ave., Albany, NY 12222, USA				
48	²⁶ California State University Stanislaus, Department of Physics, 1 University Circle, Turlock, CA 95382, USA				
49	(Dated: February 4, 2021)				
50	(Darca. 1 columly 4, 2021)				
51	We report here the results of an Effective Field Theory (EFT) WIMP search analysis using LUX				
52	data. We build upon previous LUX analyses by extending the search window to include nuclear recoil				
53	energies up to $\sim 180 \text{ keV}_{nr}$, requiring a reassessment of data quality cuts and background models.				
54	In order to use a binned Profile Likelihood statistical framework, the development of new analysis				
55	techniques to account for higher-energy backgrounds was required. With a 3.14×10^4 kg day exposure				
56	using data collected between 2014 and 2016, we set 90% C.L. exclusion limits on non-relativistic EFT				
57	WIMP couplings to neutrons and protons, providing the most stringent constraints on a significant				
58	fraction of the possible EFT WIMP interactions. Additionally, we report world-leading exclusion				

60

I. INTRODUCTION

Over the last century, an abundance of evidence sug-61 gests that non-baryonic, non-luminous "dark matter" 62 comprises approximately 25% of the universe's energy 63 density [1–5]. A popular dark matter candidate has 64 been the Weakly Interacting Massive Particle (WIMP) 65 with masses between 10 GeV and several TeV [6]. How-66 ever, non-gravitational interactions with dark matter 67 have never been definitively observed, despite many ded-68 icated experiments over the last several decades. 69

In an attempt to detect dark matter, the Large Un-70 derground Xenon Experiment (LUX) collected data be-71 tween 2013 and 2016, while being hosted 4850 feet under-72 ground in the Davis Cavern at the Sanford Underground 73 Research Facility (SURF) in Lead, South Dakota. The 74 LUX detector was a dual-phase Time Projection Cham-75 ber (TPC) equipped with an active xenon mass of 250 kg 76 to detect the possible interactions between WIMP dark 77 matter and Standard Model nucleons. Liquid xenon is 78 a promising target medium for dark matter searches, as 79 they constitute dense, stable targets with well-developed 80 purification techniques. LUX set world-leading limits in 81 the mass range of $\mathcal{O}(\text{GeV})$ - $\mathcal{O}(\text{TeV})$ for Spin-Independent 82 (SI) WIMP interactions and Spin-Dependent (SD) inter-83 actions with neutrons [7-10]. These results were con-84 firmed and improved upon by other Xe TPC-based ex-85 periments: XENON1T and PandaX [11, 12]. 86 109

In this paper, following theoretical work by Fitzpatrick₁₁₀ 87 et al. [13] and conventions set by Anand et al. [14], we₁₁₁ 88 extend prior analyses by utilizing a generalized Effec-112 89 tive Field Theory (EFT) approach going beyond simple₁₁₃ 90 SI and SD couplings, with the inclusion of momentum-114 91 dependent and velocity-dependent operators. All opera-115 92 tors in the WIMP-nucleon interaction, under momentum₁₁₆ 93 conservation and Galilean invariance, can be reduced to_{117} 94 a basis of four Hermitian quantities: 95 118

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^\perp \equiv \vec{v} + \frac{\vec{q}}{2\mu}, \quad \vec{S}_\chi, \quad \vec{S}_N$$
 (1)¹²⁰

119

131

132

133

134

135

136

where \vec{q} is the momentum transferred from the WIMP to the nucleus, m_N is the nucleon mass, \vec{v}^{\perp} is the component of the relative incoming velocity between the WIMP and the nucleon perpendicular to that momentum transfer, \vec{S}_{χ} is the spin of the WIMP, and \vec{S}_N the spin of the relevant nucleon.

¹⁰² These quantities are combined into fifteen independent₁₃₀

¹⁰³ and dimensionless EFT operators:

$$\begin{aligned}
\mathcal{O}_{1} &= \mathbf{1}_{\chi} \mathbf{1}_{N} \\
\mathcal{O}_{2} &= \left(v^{\perp}\right)^{2} \\
\mathcal{O}_{3} &= i\vec{S}_{N} \cdot \left(\frac{\vec{q}}{m_{N}} \times \vec{v}^{\perp}\right) \\
\mathcal{O}_{4} &= \vec{S}_{\chi} \cdot \vec{S}_{N} \\
\mathcal{O}_{5} &= i\vec{S}_{\chi} \cdot \left(\frac{\vec{q}}{m_{N}} \times \vec{v}^{\perp}\right) \\
\mathcal{O}_{6} &= \left(\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}}\right) \left(\vec{S}_{N} \cdot \frac{\vec{q}}{m_{N}}\right) \\
\mathcal{O}_{7} &= \vec{S}_{N} \cdot \vec{v}^{\perp} \\
\mathcal{O}_{8} &= \vec{S}_{\chi} \cdot \vec{v}^{\perp} \\
\mathcal{O}_{9} &= i\vec{S}_{\chi} \cdot \left(\vec{S}_{N} \times \frac{\vec{q}}{m_{N}}\right) \\
\mathcal{O}_{10} &= i\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \\
\mathcal{O}_{12} &= \vec{S}_{\chi} \cdot \left(\vec{S}_{N} \times \vec{v}^{\perp}\right) \\
\mathcal{O}_{13} &= i \left(\vec{S}_{\chi} \cdot \vec{q}^{\perp}\right) \left(\vec{S}_{N} \cdot \vec{v}^{\perp}\right) \\
\mathcal{O}_{15} &= - \left(\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}}\right) \left(\left(\vec{S}_{N} \times \vec{v}^{\perp}\right) \cdot \frac{\vec{q}}{m_{N}}\right),
\end{aligned}$$
(2)

dividing each \vec{q} by m_N leaves each operator conveniently dimensionless without compromising the operator's hermiticity. We neglect operator \mathcal{O}_2 in this analysis, as it cannot arise in non-relativistic scenarios. Each of these operators can in principle be coupled differently to protons versus neutrons (or equivalently, to isoscalars versus isovectors); therefore, we consider 28 different couplings in this analysis. In an actual experiment the dark matter would not couple to an individual nucleon, but to a composite nucleus. This leads to a series of nuclear responses that can vary by target isotope causing certain targets to be better at probing certain operator couplings. Additionally, while the recoil energy spectrum for momentum-independent interactions peaks at zero energy due to kinematics, momentum-dependent operators can have significant contributions at energies well above nuclear recoil energies of 100 keV, motivating analysis of a larger energy window than that used in other LUX analyses [7–10]. Figure 1 shows the differential rate spectra for each of the non-relativistic operators.

A previous EFT analysis was conducted on LUX's first WIMP search (WS) *i.e.* WS2013 [15], consisting of 95 live-days of data collected in 2013. In our current analysis, however, we utilize the longer-duration WS2014–16: 332 live-days collected between 2014 and 2016. We focus solely on WS2014–16 data, as the detector experienced significantly different data-collection conditions between the two science runs, as described in the following section. This creates different systematics and independent analysis frameworks between the two runs, making it difficult to combine both science runs in a single analysis. While a typical WIMP search region is restricted to lower energies, such as ~40 keV_{nr} in LUX's SI and SD anal-

^{*} salsum@wisc.edu

[†] grischbieter@albany.edu

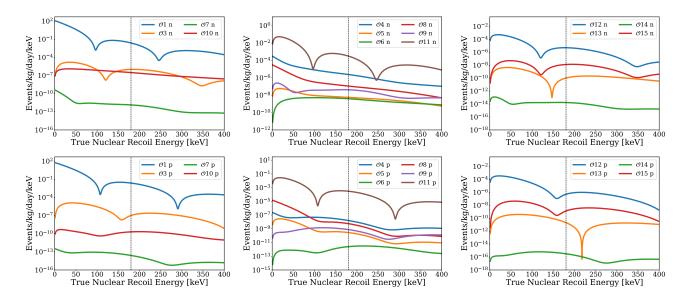


FIG. 1. Differential event rates versus true nuclear recoil energy for the fourteen non-relativistic EFT operators. This example is a 400 GeV WIMP. From left to right: \vec{S}_{χ} -independent operators, \vec{S}_{χ} -dependent operators, and \vec{S}_{χ} -dependent operators that arise only in interactions which do not involve exchange of a spin-0 or spin-1 mediator. Plots on the top row are WIMP-n rates, while the bottom consists of WIMP-p spectra. Vertical dashed black lines correspond to the energy above which the detection efficiency for the analyses presented here falls below 50% (see Fig. 3). For each spectrum, it is assumed that WIMPs only interact with the relevant nucleon through a single operator with the coupling strength set to unity, ignoring the possibility of interference between different operators.

173

174

yses [9, 10], this analysis extends the Region of Inter-164 137 est (ROI) to approximately 180 $\mathrm{keV}_{nr},$ corresponding to $_{165}$ 138 detected scintillation signals (S1) of up to 300 detected₁₆₆ 139 photons (phd). As reported in [15], the extension of the₁₆₇ 140 WIMP ROI leads to the inclusion of backgrounds con-168 141 sidered negligible in the traditional WIMP paradigm. In₁₆₉ 142 this work, we describe in detail the necessary steps to₁₇₀ 143 take these backgrounds into account. 144 171

145

II. THE LUX EXPERIMENT

As a two-phase TPC utilizing both liquid and gaseous 146 Xe, LUX measures signals by extracting electrons and¹⁷⁵ 147 collecting light released by the Xe target after a recoil¹⁷⁶ 148 event. The initial interaction excites and ionizes elec-177 149 trons from multiple Xe atoms; some ionized electrons re-178 150 combine with Xe ions, producing additional scintillation179 151 light, while others are extracted to the gas layer by an180 152 applied electric field where they produce an electrolumi-181 153 nesence signal. Initial scintillation light production takes182 154 place on timescales of $\mathcal{O}(10 \text{ ns})$, while the electron drift¹⁸³ 155 takes 0–325 μ s, creating two distinct signals: S1 and S2,¹⁸⁴ 156 respectively. LUX detected the emitted photons via 122₁₈₅ 157 photomultiplier tubes (PMTs) separated into two arrays186 158 at the top and bottom of the detector, with a photon de-187 159 tection efficiency of $\sim 10\%$. The hit-pattern of S2 light in₁₈₈ 160 the top PMT array provides $\{x,y\}$ coordinate reconstruc-189 161 tion of the original event, while the drift time between the₁₉₀ 162 S1 and S2 signals provides information regarding event₁₉₁ 163

depth.

It is important to note that the amount of primary and secondary scintillation light collected for a given event depends on the location in the detector in which the energy deposition occurred. Because of this, ^{83m}Kr dissolved in the LXe (providing a spatially uniform, effectively monoenergetic 41.5 keV electron recoil calibration) was used to construct S1 and S2 detection maps in order to correct for the position-dependence in the observed S1 and S2 signals [16]. This allows us to take advantage of the following linear conversions:

$$S1_c = g_1 \cdot n_\gamma; \qquad S2_c = g_2 \cdot n_e, \tag{3}$$

where $S1_c$ and $S2_c$ are the position-corrected S1 and S2 signals, n_{γ} and n_e are the initial numbers of photons and electrons leaving the interaction site, and g_1 and g_2 are the scintillation and electroluminescence gains, respectively. We note that while g_1 is simply a geometric light collection efficiency multiplied by PMT quantum efficiency for the prompt scintillation light S1, g_2 is a product of the efficiency to extract electrons from the liquid to gaseous xenon, photons produced per extracted electron in the gas layer, and the S2 photon detection efficiency in the gas [17].

Discrimination between electronic recoil (ER) and nuclear recoil (NR) interactions is possible in a dual-phase xenon TPC, as the total produced quanta, the ratio between excited and ionized electrons for an energy deposition, as well as the recombination probability for ionized electrons, all differ between the two interaction

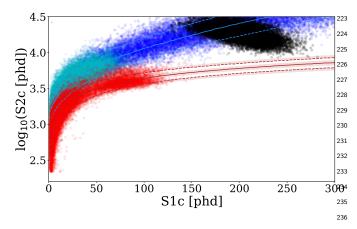


FIG. 2. A sample of single-scatter calibration events taken²³⁷ near the end of WS2014–16 with drift times between 40-105 μ s. Cyan points correspond to the ³H β ER calibration; blue points correspond to the ¹⁴C β ER calibration; red mark-₂₃₈ ers are events associated with the D-D NR calibration; and black markers are ^{83m}Kr events. Each population consists of a random selection of 20,000 events. The light blue solid²³⁹ and dashed lines show the expected mean and 90% C.L. ER²⁴⁰ response region, while red solid and dashed lines show the ex-²⁴¹ pected mean and 90% C.L. NR response from NEST v2.1.0.²⁴² The shaded red region shows the uncertainty in the NR ex-²⁴³ pectation based on *ex situ* NR calibrations reported in the₂₄₄ literature (see Sec. IV).

types. However, discrimination is not 100% efficient, as₂₄₇
ER events with a stochastically lower charge-to-light ra-₂₄₈
tio can "leak" into the expected NR signal region in {S1,₂₄₉
S2} space. As we expect WIMPs to primarily produce₂₅₀
NR, it is paramount that we minimize ER leakage, while₂₅₁
fully characterizing all backgrounds, in order to distin-₂₅₂
guish a possible WIMP signal from them.

To characterize the $\{S1, S2\}$ response of LXe in LUX₂₅₄ 199 for both ER and NR interactions, LUX underwent peri-255 200 odic calibrations. For ER, tritiated methane (0-18.6 keV₂₅₆ 201 β decay) was injected into the detector several times₂₅₇ 202 over LUX's lifetime, providing the LXe response for₂₅₈ 203 energies relevant to most typical lower-energy WIMP₂₅₉ 204 searches [18]. Additionally, at the end of LUX's tenure₂₆₀ 205 in the Davis Cavern, a ${}^{14}C$ calibration took place (0-261 206 156 keV β decay), allowing for characterization of the₂₆₂ 207 ER response out to much higher energies [19]. For NR,263 208 an external deuterium-deuterium (D-D) fusion neutron₂₆₄ 209 generator was used to provide in situ characterization₂₆₅ 210 of nuclear recoils between $0.7-74 \text{ keV}_{nr}$ [20]. We note₂₆₆ 211 here that a nuclear recoil with a given energy produces₂₆₇ 212 smaller S1 and S2 signals than an ER event of the same₂₆₈ 213 energy: this is due to the fraction of energy from a be-269 214 ing transferred to the electrons to produce ionized and₂₇₀ 215 excited atoms being smaller for NRs than ERs. Figure 2_{271} 216 shows a sample of the {S1, S2} response for LUX's cal-272 217 ibrations compared to expected ER and NR responses₂₇₃ 218 from simulation. 219 274

Before WS2014–16, LUX underwent a grid condition-275
 ing campaign to significantly increase the allowed applied276
 drift field and extraction efficiency. However, this had the277

unintended consequence of creating a significant amount of trapped charge on the inner walls of the TPC, creating a spatially-distorted and temporally-varying drift field, varying between 50-550 V/cm as function of time and position. 3-D electrostatic models of the built-up charge density were created using the COMSOL Multiphysics software [21], providing a spatial map of the electric field configuration. Field and charge maps were updated monthly, which allows for a robust understanding of the temporal features of the applied drift field. More details are reported in Ref [22]. Additionally, WS2014– 16 data were collected with temporally changing gain factors, where g_1 gradually decreased from 0.100 ± 0.002 to 0.097 ± 0.001 phd/photon and g_2 varied between 18.9 ± 0.8 and 19.7 ± 0.2 phd/e⁻ [9].

III. DATA SELECTION

For this analysis, data from WS2014–16 are used. Despite the challenges from the temporally varying gain factors $(g_1 \text{ and } g_2)$ and electric field distortions, WS2014– 16 has been well-characterized by multiple analyses since LUX's decommissioning [17, 22, 23]. As the EFT ROI is significantly larger in $\{S1, S2\}$ space than in the SI and SD WIMP analyses, data quality cuts are crucial for removing backgrounds, including: events with poor position reconstruction; multiple scatters with merged S2 signals; events with gaseous xenon interactions classified as the event's S2; and events with an overabundance of non-S1 and non-S2 pulses such as single photons and electrons not associated with the observed S1 or S2. To minimize potential bias when creating these data quality cuts (described in more detail below), the WS2014–16 data were "salted" with artificial WIMP-like events at early stages of the data-processing pipeline. Details of the salting procedure are described in Ref. [9]. These events are only removed from the data set after all data quality cuts and models (described in Sec. IV) had been finalized. Additionally, energy depositions from LUX's ^{83m}Kr calibrations fall into this extended-energy ROI. To combat the additional leakage from the regular highstatistics calibration injections, data acquisition corresponding to significant ^{83m}Kr contamination were omitted from this analysis. A similar exclusion was reported in Ref. [24], however, this resulted in a significant loss of live-time. To increase the exposure of this analysis while also maintaining low ^{83m}Kr activity, each exclusion period was reduced by 17^{83m} Kr half-lives (31.1 hours). The final amount of exposure excluded was 20.8 live-days, resulting in a 311.2-day science run.

To account for the temporal and spatial variation of the detector response, the WS2014–16 data are divided into four temporal bins, each further subdivided into four spatial bins corresponding to 65 μ s windows of drift time. Selecting periods when the field configuration was approximately static, we approximate each of the resulting 16 bins as temporally static with near-uniform electric

field distribution. This results in negligible loss of ac-278 curacy for reproduction of light and charge yields. This 279 same division of the data set into 16 date and drift bins 280 was is further described in Refs. [9, 17, 22]. Bins near 281 the bottom of the detector experienced weaker electric 282 fields (50-100 V/cm), while the strongest fields were in 283 the topmost portion of LXe (400–550 V/cm). The four 284 temporal bins result in unequal live times: 43.9, 43.8, 285 85.8, and 137.7 days. An illustration of the data divided 286 into these 16 time and drift time bins is provided in Ap-287 pendix A. 288

The fiducial volume is defined as the region for which 289 the electron drift time (vertical coordinate) lies between 290 40 and 300 μ s and (in the radial dimension) the region 291 that is greater than 3 cm inward from the TPC wall. 292 The distorted electric field also caused the electron drift 293 paths to bend significantly inward as the electrons drift 294 from the interaction vertex to the liquid surface. This 295 effect is strongest for events originating near the bottom 296 of the TPC. As a result, near-wall events at the bottom 297 of the TPC have more centralized S2 hit-patterns in the 298 top PMT array than near-wall events at the top of the₃₃₆ 299 TPC. Effectively, this moves the observed wall position₃₃₇ 300 inward at the bottom of the TPC, requiring that the fidu-338 301 cial LXe target volume is reduced as a function of drift₃₃₀ 302 time. In temporal order, the resultant fiducial $masses_{340}$ 303 for each WS2014–16 date bin are: 105.4 ± 5.3 , 107.2 ± 5.4 ,₃₄₁ 304 99.2 ± 5.0 , and 98.4 ± 4.9 kg. These volumes are deter-305 mined by counting remaining ^{83m}Kr events after apply-₃₄₃ 306 ing fiducial cuts, and using the knowledge that the full 307 TPC volume contains 250 kg of LXe. The total exposure $_{_{345}}$ 308 used in this analysis therefore is 3.14×10^4 kg·days. 309 346

To remove adverse events that are would be incorrectly₃₄₇ 310 classified as single scatters from the data set, a series of_{348} 311 data quality cuts are applied. Events with an overabun-₃₄₉ 312 dance of pulses preceding or following either the S1 or_{350} 313 S2 — such as single photons or single electrons emitted₃₅₁ 314 from the detector's grids or delayed releases from impuri-315 ties [25] — were removed, as these events are more likely₃₅₃ 316 to have misidentified S1 or S2 signals. Cuts are applied₃₅₄ 317 based on the S1 PMT hit-patterns as well as the shape $\mathrm{of}_{\scriptscriptstyle 355}$ 318 the S1 pulse; these remove events where S1s may origi- $_{356}$ 319 nate from light leaking in from outside the TPC walls and $_{357}$ 320 misidentified S1s, respectively. For S2s, cuts are applied₃₅₈ 321 based on the pulse width and shape as a function of $\operatorname{area}_{359}$ 322 and drift time. As bulk S2s are expected to be approx- $_{360}$ 323 imately Gaussian in shape [26], a cut on the goodness $_{361}$ 324 of a Gaussian fit to the pulse shape was implemented.₃₆₂ 325 The mean single-scatter selection efficiency of these $cuts_{363}$ 326 based on ER and NR calibration data and simulations $_{364}$ 327 is 96% between 0-300 phd, while the full NR detection₃₆₅ 328 efficiency is shown in Figure 3. 329 366

Unmodeled backgrounds in and below the signal re- $_{367}$ gion were reported in Ref [15]. There is a 5.6 cm gap₃₆₈ between the cathode and the bottom PMT array; scintil- $_{369}$ lation produced in this region is visible to the PMTs, but₃₇₀ emitted electrons are carried downward (instead of up- $_{371}$ ward to produce an S2 signal). If a γ -ray scatters in this₃₇₂

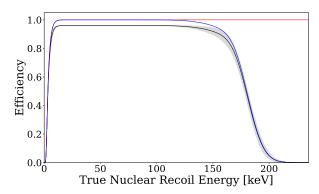


FIG. 3. Nuclear Recoil detection and selection efficiencies based on calibration data and simulations. Red corresponds to S1+S2 detection efficiency in the fiducial target without any data selection cuts applied, while blue corresponds to the mean detection efficiency in the EFT {S1, S2} ROI. The black curve corresponds to the mean overall efficiency after data quality cuts are applied, and the grey band signifies the standard deviation of the efficiency due to differing temporal and spatial detector conditions.

sub-cathode region in addition to the fiducial volume, both scatters contribute to the S1, while only the fiducial scatter produces S2 light. The result is a " $\gamma - X$ " event with an S1–S2 ratio anomalously low for an ER event. Combined with the reduced recombination due to having the weakest electric fields at the bottom of the fiducial region, these events could significantly increase the leakage of ER events into the NR signal region.

 $\gamma - X$ events pose a unique challenge because they can appear as typical single scatters. Any hints of their anomalous behavior could in principle be captured in the S1 signal. However, due to the timescales at which light collection takes place ($\mathcal{O}(10 \text{ ns})$) being longer than transit time between scatters (typically less than 1 ns), these S1s are not separable from single scatters using simple shape cuts, such as those described above. Instead, a six-dimensional parameter space is utilized, with the intent of using a Boosted Decision Tree (BDT) machine learning event classifier to identify and remove $\gamma - X$ -like events. BDTs are becoming more commonly used in particle physics analyses, and they provide an efficient way to draw distinctions between two populations in higherdimensionality phase spaces [26, 27]. The six features used are: S1c; S2c; electron drift time; the mean size of the hit pattern in the bottom PMT array, or "cluster" size; the fraction of the total S1 light detected by the PMT registering the largest contribution to the S1; and the ratio of collected scintillation light in the top and bottom PMT arrays. By utilizing information from S1 hit-patterns in the bottom PMT array with position reconstruction information from the S2, we hope to separate $\gamma - X$ events from true single scatters. To train the $\gamma - X$ classifier, a model was made by characterizing double-scattering events near the cathode but using simulation to extrapolate these double scatters into $\gamma - X$ events with a sub-cathode energy deposit. More details of the $\gamma - X$ model are discussed in Sec IV.

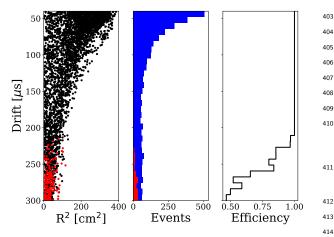


FIG. 4. Left: Spatial distribution of the background data. Red markers correspond to events removed by the BDT $\gamma - X$ cut. Center: Histograms of events as a function of detector depth. Blue corresponds to all events, while red corresponds to the events removed with application of the BDT₄₁₅ cut. Right: Efficiency as a function of event depth. While₄₁₆ the efficiency is the poorest near the bottom of the detector,₄₁₇ the background data are mostly concentrated near the top,⁴¹⁸ resulting in a 99.2% overall efficiency from the BDT cut for ⁴¹⁹ the background data.

The cut using the trained BDT rejects 89.2% of simu-⁴²¹ 373 lated $\gamma-X$ events, while only removing 1.7% of simulated $^{\scriptscriptstyle 422}$ 374 single scatters, reducing our signal detection efficiency to⁴²³ 375 94.4%. The efficiency is highly position-dependent, with $_{424}$ 376 100% efficiency throughout most of the volume, but the $_{425}$ 377 largest loss of efficiency in the bottom-most 20% of the $_{_{426}}$ 378 fiducial target (near 50% at the poorest). Figure 4 details 379 the $\gamma - X$ cut efficiency in the background data. 380

381

IV. MODELING

Using the Profile Likelihood Ratio (PLR) construction 382 (described in Sec. V), we use statistical inference to quan-427 383 tify the level of sensitivity of our detector to identify or₄₂₈ 384 constrain the possibility of WIMPs interacting under a⁴²⁹ 385 given EFT operator. A likelihood ratio test provides a⁴³⁰ 386 strong statistical framework when dealing with higher-431 387 dimensionality parameter space, and it requires a good₄₃₂ 388 model of both the null and alternative hypotheses to be433 389 valid. In this section, we describe the construction of each⁴³⁴ 390 of the models used in the PLR framework. We identified 435 391 and constructed five-dimensional models $(S1_c, S2_c, ra-$ 392 dius, drift time (d), and azimuthal angle (ϕ) about the 393 TPC's central axis) for the sources that could lead to 394 events in our ROI: EFT WIMPs; ER single scatters; re-395 maining ^{83m}Kr after the calibration injections; degraded 396 events and ion recoils from the TPC walls; $\gamma - X$; and 397 accidental coincidences of unrelated S1-only or S2-only 398 events. After separation of the data into the 16 date 399 and drift time bins, we make the assumption that the 400 field variation in each drift time bin has minimal im-401 pact on the S1 and S2 distributions. Accordingly, we 402

make the simplification of separating the spatial and energetic components of most models, resulting in Probability Density Functions (PDFs) that are the direct product of two $(S1_c \text{ and } S2_c)$ and three $(r, \phi, \text{ and } d)$ dimensions. However, the model for degraded wall events and ions has no such separation as the energy and spatial observables are highly correlated even after separation into 16 drift time bins (see Sec. IV D).

A. Signal Modeling

Signal spectra are obtained using the Mathematica package developed by Anand et al. [14]. This gives the differential rate of nuclear recoils per recoil energy, E_R :

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = N_T \frac{\rho_0 m_N^2}{2\pi m_\chi m_A} \int_{v > v_{\min}} \frac{f(\vec{v})}{v} |\mathcal{M}|^2 \,\mathrm{d}^3 v, \quad (4)$$

where N_T is the number of target nuclei, ρ_0 is the local dark matter density, m_{χ} is the mass of a WIMP, m_A is the target nucleus mass, and $f(\vec{v})$ is the galactic WIMP velocity distribution for which we assume a Maxwell-Boltzmann distribution following the Standard Halo Model: characteristic velocity $v_0 = 220$ km/s and escape velocity $v_{esc} = 544$ km/s. The spin-averaged matrix element $|\mathcal{M}|^2$ is calculated via a combination of WIMP velocity and momentum-transfer dependent form factors $F_{ij}^{(N,N')}(v^2,q^2)$ presented in Appendix A.2 of [13], scaled based on the value of the EFT coupling constants $c_i^{(N)}$:

$$\frac{1}{2j_{\chi}+1} \frac{1}{2j+1} \sum_{\text{spins}} |\mathcal{M}|^2 \equiv \frac{m_A^2}{m_N^2} \sum_{i,j=1}^{15} \sum_{N,N'=p,n} c_i^{(N)} c_j^{(N')} F_{ij}^{(N,N')} \left(v^2, q^2\right),$$
(5)

where j and j_{χ} are the spins of the nucleus and WIMP, respectively. Note that this representation of the amplitude differs from Ref [13] by a factor of $(4m_{\chi}m_N)^2$, accounting for the different normalization conventions and dimensionality of the c_i used in the Mathematica package [14]. The form factors are also affected by differing conventions and are scaled to account for this¹. Putting equations 4 and 5 together, the differential rate spectrum becomes

$$\frac{dR}{dE_R} = N_T \frac{\rho_0 m_A}{2\pi m_\chi} \int_{v > v_{min}} \left[\frac{f(v)}{v} + \sum_{i,j} \sum_{N,N'=n,p} c_i^N c_j^{N'} F_{i,j}^{(N,N')} \left(v^2, q^2 \right) \mathrm{d}v \right].$$
(6)

¹ Specifically, factors of factors of \vec{q} have been normalized by factors of m_N , similar to the normalization used in Equations 1 and 2.

Note that one can just as easily use isoscalars and₄₉₂ 436 isovectors in place of the p and n for the proton and neu-493 437 tron. This is also a valid approach, and has been $done_{494}$ 438 in analyses by several other experiments as it allows for $_{_{495}}$ 439 direct comparisons between experiments with different 440 target compositions [28–30]. However, the $\{n,p\}$ basis is 441 used in this analysis, as it provides a more natural rep-442 resentation of the physical interactions that this analysis 443 attempts to identify or constrain. Additionally, due to 444 the presence of two couplings in each term, the possibil-445 ity for destructive interference exists. For this analysis, 446 we ignore the possibility of interference and make the as-497 447 sumption that one coupling is dominant over all others.⁴⁹⁸ 448 As such, the signal spectra that we obtain are the result⁴⁹⁹ 449 of setting all but one of the couplings $c_i^{(N)}$ to 0. The⁵⁰⁰ resulting differential event rate scales linearly with the⁵⁰¹ 450 451 remaining non-zero coupling $c_i^{(N)^2}$:

452

$$\frac{dR}{dE_R} = N_T \frac{\rho_0 m_A}{2\pi m_\chi} \int_{v > v_{min}} \frac{f(v)}{v} c_i^{(N)^2} F_{i,i}^{(N,N)} \left(v^2, q^2\right) \mathrm{d}v, \quad (7)$$

Due to the linear relation between differential rate and $_{503}$ 453 $c_i^{(N)^2}$, the spectrum for any value of the coupling constant $_{504}$ 454 can be easily determined by calculating the $c_i^{(N)^2} = m_n^{-2505}$ 455 case and then scaling appropriately. We use the bench-506 456 mark value m_v^{-2} , where $m_v = 246.2$ GeV and is the Higg's⁵⁰⁷ 457 vacuum expectation value, as this is the chosen scaling 458 factor used internally by [14]. 459

To generate the detector response to the resultant nu-⁵⁰⁸ 460 clear recoil energy spectra, a recent release of the Noble 461 Element Simulation Technique (NEST v2.1.0) was uti-509 462 lized [31], chosen prior to unsalting. An empirical fit to₅₁₀ 463 all existing nuclear recoil data in LXe, NEST provides⁵¹¹ 464 precise light and charge yields resulting from an energy₅₁₂ 465 deposition. While the D-D NR calibrations characterize513 466 the detector response out to 74 keV_{nr} (\sim 150 phd), NEST₅₁₄ 467 allows for extrapolation to higher energies using reported515 468 yields in the literature extending to 330 keV_{nr} from other₅₁₆ 469 sources such as AmBe [32]. This provides an understand-517 470 ing of the signal region beyond where the detector NR_{518} 471 response was directly calibrated. Uncertainty in the sig-519 472 nal region for energies beyond the in situ D-D calibration₅₂₀ 473 was calculated by allowing the NEST v2.1.0 NR model₅₂₁ 474 (largely unchanged between versions 2.0.1 through 2.2) to₅₂₂ 475 fluctuate within the uncertainties for the total reported₅₂₃ 476 quanta of the highest energy data used to fit the model;524 477 for 300 phd S1s, the resultant uncertainty of the location₅₂₅ 478 of the NR band mean in S2-space is approximately 7.5%, 526 479 corresponding to a change in S2 size of roughly 540 phd.527 480 Ultimately, the NR band is sufficiently far from the ER_{528} 481 band in any scenario to make this difference negligible. 529 482

Recoil spectra for different operator-mass combina-530 483 tions are simulated using the LUX Legacy Analysis531 484 Monte Carlo Application (LLAMA) [17]. LLAMA532 485 uses spatial and temporal interpolation between the 16533 486 approximately-static WS2014-16 drift time bins, utiliz-534 487 ing the NR response from NESTv2.1.0 and the three-535 488 dimensional field maps described in Ref. [22]. Signal⁵³⁶ 489 spectra are generated homogeneously throughout the de-537 490 tector. 538 491

Generation of the inelastic EFT WIMP-nucleon signal models used recoil spectra from a modified version of the Anand et al. Mathematica package developed by Barello et al. [33]. This version introduces an additional energy conservation requirement,

$$\delta_m + \vec{v} \cdot \vec{q} + \frac{|\vec{q}|^2}{2\mu_N} = 0, \tag{8}$$

where δ_m is the mass splitting term between the incoming and outgoing WIMP ($\delta_m = m_{\chi_2} - m_{\chi_1}$). This requirement is included into our basis of Hermition quantities by an additional term in the perpendicular velocity proportional to δ_m ,

$$\vec{v}_{inel}^{\perp} \equiv \vec{v} + \frac{\vec{q}}{2\mu} + \frac{\delta_m}{|\vec{q}|^2} \vec{q} = v^{\perp} + \frac{\delta_m}{|\vec{q}|^2} \vec{q}.$$
 (9)

Signal models with a range of δ_m from 0–200 keV were generated for all operators in the isoscalar basis using a WIMP mass of 1 TeV/ c^2 . Other parameters, including astrophysical and nuclear, remain unaltered from those used for the elastic signal models, and the same procedure was applied.

в. Standard ER Backgrounds

We expect the overwhelming majority of backgrounds to originate from ER-producing contaminants within the LXe, namely ²²²Rn and ²²⁰Rn and their charged daughter isotopes plating-out on the detector surfaces, as well as decays from radioisotopes in the detector components. Decays from the detector components are mostly isotopes originating from 238 U, 232 Th, 60 Co, and 40 K, producing β , γ , and α radiation at a wide range of energies. A dedicated modeling campaign for reproducing the LXe ER response in LUX was reported in Ref. [17]. To summarize, utilization and tuning of NEST ER response models allowed for accurate characterization of the temporal and spatial features of the WS2014-16 detector, and precise reproduction of all available LUX ¹⁴C and CH₃T ER calibration data. While NEST is a global fit to xenon light and charge yields, this LUX-specific version allows for efficient creation of high-statistics LUX ER simulated data for all 16 WS2014–16 drift and date bins for all relevant energies.

Assays of LUX components provide initial expectations for the expected radioactivity from the detector leading to ER backgrounds. However, due to uncertainties in the assay measurements and the modeled response of each detector component and their geometries, the simulated energy depositions from each contributing detector component and radiogenic source was fit to high-energy data, including multiply-scattering events, allowing for effective activities from each source. Data below 80 keV were excluded when fitting the effective activities. LXe light and charge responses for each source were then simulated

using the LUX-specific version of NESTv2.1.0, providing₅₉₁ S_{40} S_{1c} and S_{2c} distributions for each expected ER source. ₅₉₂

593

594

595

596

628

629

630

631

C. The ^{83m}Kr Model

^{83m}Kr was injected into the TPC on a weekly basis to⁵⁹⁷ 542 ensure proper position corrections. This source decays⁵⁹⁸ 543 in two transitions: 32.1 keV followed by 9.4 keV. Most⁵⁹⁹ 544 often, these de-excitations occur via internal conversion⁶⁰⁰ 545 electrons or Auger electrons. The time between the two⁶⁰¹ 546 emissions ranges from $\mathcal{O}(10 \text{ ns})$ to $\mathcal{O}(1 \mu \text{s})$, and those⁶⁰² 547 on shorter timescales appear as 41.5 keV single scatters,⁶⁰³ 548 having only a single detectable S1 and S2. These quasi-604 549 monoenergetic depositions are of high enough energy to⁶⁰⁵ 550 be cut from a typical momentum-independent analysis,⁶⁰⁶ 551 leading to no loss of exposure time. However, in an analy-607 552 sis reaching to higher energies, ^{83m}Kr events can interfere⁶⁰⁸ 553 with the signal region. 554

As a high-statistics monoenergetic peak, ^{83m}Kr yields 555 are observed with wide recombination fluctuations in the 556 $\frac{S2}{S1}$ ratio, resulting in events near the NR signal region₆₁₀ 557 at energies where most other ER backgrounds are well-558 discriminated (see Fig. 2). Additionally, this proximity $_{611}$ 559 of ^{83m}Kr events to the signal region worsens for weaker 560 fields, as the ER and NR bands are less separated than $\mathrm{at}_{_{613}}$ 561 stronger electric fields. As stated in the Sec. III, 20.8 live- $_{614}$ 562 days were excluded from WS2014–16 that correspond $\rm to_{_{615}}$ 563 periods of significant ^{83m}Kr contamination in order to₆₁₆ 564 omit most of these events from this analysis. 565 617 Despite this, some 83m Kr events are expected in the₆₁₈ 566

data set; ^{83m}Kr has a 1.83 hour half-life, resulting in₆₁₉ 567 lingering decays after the injections end. Therefore, a_{620} 568 robust characterization of these events was required. For₆₂₁ 569 this, the remaining 83m Kr data excluded from the final₆₂₂ 570 search data were used to construct a model for these 623 571 events. The expected number of events was calculated $_{624}$ 572 by measuring the rate of 83m Kr events at the end of each₆₂₅ 573 data exclusion period and extrapolating using the known $_{626}$ 574 half-life. 575 627

576

541

D. The Wall Model

Similarly to previous LUX analyses [9, 34], we con-632 577 struct a model characterizing energy depositions in close₆₃₃ 578 proximity to the inner TPC walls. The electron extrac-634 579 tion efficiency near the walls is poorer than in the bulk $_{635}$ 580 LXe, resulting in degraded S2 signals. Additionally, nu-636 581 clear recoils from 206 Pb (a daughter of 210 Po α -decay) on₆₃₇ 582 the inner TPC walls leads to events with naturally low $\frac{S2}{S1}$ 638 583 ratios compared to ER backgrounds, resulting in a popu-639 584 lation of events well-below the signal region in $\{S1_c, S2_c\}_{640}$ 585 space. 586

As mentioned briefly at the beginning of this section, the reconstructed position of the detector wall depends on the drift time d, azimuthal angle ϕ , and acquisition time [22] due to the radial field. We observed that the the reconstructed position of the events fluctuates around the position of the wall according to a Gaussian distribution with width proportional to $1/\sqrt{S2_{raw}}$ [35]. Therefore, the wall events have a larger uncertainty for the same deposited energy due to the smaller S2 size, allowing for a fraction of these events to appear within the fiducial volume.

To characterize this background, we selected WS2014– 16 events with reconstructed positions beyond the measured position of the TPC wall, counting the number of events for a specific bin in drift time, azimuth, and acquisition time, as the fluctuations in reconstructed position about the wall should be equal both inside and outside the wall position. Integrating the tail of this empirical fit provides an understanding of the expected number of wall events that leak into the fiducial volume. Since this leakage depends heavily on the observed S2 size, the energy and spatial PDFs are significantly correlated, making the wall model a true five-dimensional PDF.

E. The $\gamma - X$ model

As described in Sec. III, we consider the possibility of multiply-scattering γ -rays with only a single detectable ionization signal due to one or more sub-cathode energy depositions: $\gamma - X$. With only a single S2, these events appear as single scatters, as the individual energy depositions occur at short-enough timescales where the primary scintillation from each interaction blends into a single S1 signal. However, the detected S2 corresponds to only a fraction of the recoil energy that corresponds to the S1. This creates the possibility of ER events having observed $\frac{S2}{S1}$ ratios similar to nuclear recoils. These events would be observed near the bottom of the fiducial volume, where the electric field values are the weakest and the ER/NR discrimination is the poorest, creating the possibility of excessive ER leakage.

The volume of LXe between the cathode and bottom PMT array is referred to as the "Reverse Field Region" (RFR), where the mean applied electric field is antiparallel to the above-cathode region, causing electrons to drift downwards away from the extraction region. Gammas coming from radiogenic impurities in the bottom PMT array and RFR TPC walls – namely ²³⁸U, ²³²Th, ⁶⁰Co, ⁴⁰K and their daughters – may deposit energy in the RFR before scattering in the fiducial target. Additionally, back-scattering events originating from the cathode grid wires can also contribute to the $\gamma - X$ rate. The RFR field magnitude is of $\mathcal{O}(1 \text{ kV/cm})$, which results in significantly lower light yields for a given energy deposition compared to the bottom of fiducial volume: a reduction to 75% for 50 keV γ -rays [31]. This results in higher-energy γ -rays (which are more likely to traverse a significant portion of the RFR xenon) producing S1s below our 300 phd threshold that would normally be excluded if that interaction occurred in the bulk LXe. These events become more likely as the search window

extends to higher energies, as multiply-scattering γ back-701 grounds become more prevalent. Therefore, characteriz-702 ing and modeling these events becomes necessary for an703 analysis with an extended-energy ROI. 704

Because $\gamma - X$ events appear superficially as normal⁷⁰⁵ 650 single scatters, we are unable to obtain a data set of⁷⁰⁶ 651 known $\gamma - X$ events. However, the presence of double-707 652 scatter events near the cathode provides information on 653 multiply-scattering γ -rays near the RFR. We selected a 654 set of double-scatter events that had: at least 3 cm of 655 vertical separation between the two reconstructed inter-656 action locations; $S1_c$ less than 300 phd; the lower-most 657 S2 within 4 cm of the cathode; and the top-most energy 658 deposit within the fiducial radius. The distance between⁷⁰⁸ 659 the cathode and the fiducial volume is approximately 3 660 cm, thus the first condition reproduces the minimum ver-661 tex separation for $\gamma - X$ events that may pass other data 662 quality cuts. The remaining criteria allow for selection 663 of events with uppermost S2s similar to single scatters⁷¹¹ 664 in the background data (as those would be the observed⁷¹² 665 S2s for $\gamma - X$ events). Seventeen of these "near-miss"⁷¹³ 666 double-scatters were found in WS2014–16, and a model⁷¹⁴ 667 was created that replicated the behavior of these near-668 miss events using the LUX-specific NEST framework. By 669 translating this model 4 cm downwards, guaranteeing the 670 first simulated scatter to be sub-cathode, we were able 671 to generate simulated $\gamma - X$ events based on LUX data. 672 Additionally, this model was used to train the BDT de-673 scribed in Sec. III in an attempt to remove most $\gamma - X$ 674 events in the data. While characterizing the rate of ex-675 pected $\gamma - X$ events proves challenging, we make the₇₁₇ 676 assumption that it should be similar to the rate of near-718 677 miss double-scatter events. Taking the efficiency of the $_{719}$ 678 BDT cut into account with respect to simulated $\gamma - X_{,_{720}}$ 679 we therefore expect $\mathcal{O}(1) \gamma - X$ events. 680 721

681

Accidental Coincidences

F.

Lastly, we take into consideration the coincidental pair 682 ing of unrelated S1-only and S2-only events, forming an 683 "accidental" single scatter (such as those reported in 684 Ref. [36]). To understand the rate at which to expect 685 these events and their appearance in phase space, LUX 686 data were filtered to obtain two sets of data: events with 687 only one observed S1 and no S2, and events with only one 688 S2 and no S1. The S1-only and S2-only rates and spec-689 tra were input into a Monte Carlo generator, and random₇₂₆ 690 pairing of S1s and S2s provided a model to characterize₇₂₇ 691 these events. 692 It is possible to have energetic S1-only and S2-only₇₂₉ 693 events due to energy depositions in regions of poor $light_{730}$ 694

 events due to energy depositions in regions of poor light₇₃₀
 collection and charge extraction efficiencies; however, the₇₃₁
 most common S1-only and S2-only events consist of only₇₃₂
 a handful of photons or electrons, respectively. The acras
 cidental pairing of these pulses can produce a false event₇₃₄
 mimicking a lower-energy single scatter, falling in the
 region of phase space where the expected WIMP recoil₇₃₅ rate is the most probable. Using the S1-only and S2-only event rates, we are able to calculate an expectation for accidental coincidence events. However, the data quality cuts described in Sec. III reduce the expected rate of these events in the ROI considerably, and we expect less than a single accidental event for the exposure in this analysis.

V. STATISTICAL METHODOLOGY

In setting constraints on the coupling constant for a given mass-operator combination, we use hypothesis test inversion to determine a 2-sided frequentist confidence interval via the Neyman Construction [37]. This involves performing a series of hypothesis tests where the null hypothesis (H₀) is our model with the Parameter Of Interest (POI), μ , fixed at a given value, and the alternative hypothesis (H₁) is allowed to float to all real values:

$$\begin{aligned} H_0 : \mu &= \mu_0 \\ H_1 : \mu &\neq \mu_0 \end{aligned}$$
 (10)

Here, μ is simply the number of WIMP-nucleon scatters we expect to observe for a given model. The values of the POI corresponding to hypothesis tests whose p-value is greater than the significance $\alpha = 0.1$ form the 90% confidence interval on the POI for each signal model.

Our test statistic for these hypothesis tests is the Profile Likelihood Ratio (PLR). More specifically, we use the negative log likelihood, $q = -2 \ln(\lambda)$, where λ is the actual PLR:

722

723

724

$$\lambda(\vec{X}) = \frac{\mathcal{L}_P\left((\mu_0, \hat{\theta}) \middle| \vec{X}\right)}{\mathcal{L}_P\left((\hat{\mu}, \hat{\theta}) \middle| \vec{X}\right)}.$$
(11)

Here, P denotes that this likelihood has been modified by the presence of a profile. μ_0 is just the fixed POI, and the terms with hats are allowed to float to maximize the value of the profiled likelihood \mathcal{L}_P . The double hat $\hat{\theta}$ indicates that the values of the nuisance parameters, θ , that maximize the likelihood in the case of $\mu = \mu_0$ are not in general the same values that maximize it when μ is left to float. \vec{X} represents the data set used to compare against the model.

For this analysis we use the extended unbinned likeli-

736 hood as follows:

$$\cdot \prod_{\theta_i \in \vec{ heta}} \mathcal{P}_i\left(heta_i
ight)$$

789

Here n_{obs} is the number of events contained in the data set, \vec{X} ; $n_{\text{exp}} = n_{\text{sig}} + \sum_{b_i} n_{b_i} + n_{\text{wall}}$ is the number of events expected by the model with b_i indicat-790 737 738 739 ing one of our background models; and \vec{x}_i is a given 740 data point in the set \vec{X} . Each data point \vec{x}_i contains₇₉₁ 741 the set of 5 observables: $\{r, d, \phi, S1_c, \text{ and } S2_c\} \equiv \vec{\mathcal{O}}_{\scriptscriptstyle 792}$ 742 along with the analysis bin in which it was measured:793 743 {date bin(t), drift time bin(z)}. n_{sig} is the number of sig-794 744 nal events expected, and is used as a stand-in for our POI₇₉₅ 745 as we have not included any nuisance parameters that₇₉₆ 746 affect detector thresholds in this analysis, thus $n_{\rm sig}$ is a_{797} 747 function purely of $c_i^{(N)^2}$. n_{b_i} is similarly the number of r_{98} expected events from background source b_i , and the same r_{99} 748 749 is true of n_{wall} . $R_{\text{source}}, t_i, z_i$ is the fraction of the total₈₀₀ 750 number of expected events for that source that are ex_{-801} 751 pected to occur in the bin (date bin = t_i , drift time bin =₈₀₂ 752 z_i). Likewise, $\mathcal{P}_{\text{source}}, t_i, z_i \left(\vec{\mathcal{O}}_i \right)$ is the Probability Den-⁸⁰³ sity Function (PDF) modeled for the given source in the 753 754 given date bin and drift time bin. The final line in equa- $^{\rm 805}$ 755 tion 12 is the profile term. θ_i is a given nuisance parame-756 ter, and $\mathcal{P}_i(\theta_i)$ is the PDF describing the profile for that⁸⁰⁷ 757 nuisance parameter. In principle, the profiles of multiple 808 758 nuisance parameters could be correlated, but this was⁸⁰⁹ 759 determined to have minimal effect and was not imple- $^{\rm 810}$ 760 mented. The set of nuisance parameters $\vec{\theta}$ used in this⁸¹¹ 761 analysis is simply the number of expected events for each⁸¹² 762 different background source n_{b_i} . 763

We explicitly separate the wall model from the other⁸¹⁴ 764 backgrounds in Eqn. 12 because its implementation in 765 our software differs significantly from the others. As $^{\rm s16}$ 766 mentioned in Sec. IV, the spatial observables $\{r, d, \phi\}^{817}$ 767 were determined to be sufficiently independent of the cor-768 rected energy observables, $\{S1_c, S2_c\}$, once the detector⁸¹⁹ 769 was split up into its date bins and drift time bins. This⁸²⁰ 770 allowed for the implementation of the 5-dimensional PDF^{821} 771 to be split into the direct product 772 823

$$\mathcal{P}_{\text{source},t_i,z_i}\left(\vec{\mathcal{O}}_i\right) \equiv \mathcal{P}_{\text{source},t_i,z_i}\left(r_i, d_i, \phi_i\right) \tag{13}^{\texttt{824}} \\ \cdot \mathcal{P}_{\text{source},t_i,z_i}\left(S1_{c,i}, S2_{c,i}\right) \tag{326}$$

⁷⁷³ However, in the case of the wall model, this split is not₈₂₈ ⁷⁷⁴ feasible: the location of the wall as seen by the top PMT₈₂₉

array depends significantly on d and ϕ , while the reconstructed distance from the wall depends strongly on $S2_c$. Therefore, the PDFs for the wall model remain fully 5dimensional.

We found that our data sets do not lie in the asymptotic regime, and therefore unfortunately cannot make use of the asymptotic formulae that would greatly reduce the computation necessary for performing each hypothesis test [38]. Instead, we rely on comparing our test statistic to that of a collection of Monte Carlo psuedoexperiments simulated based on our models. Test statistic distributions are evaluated using a custom-built PLR framework utilizing RooFit [39] that has been optimized for the rapid computation of psuedoexperiments in our 5-dimensional regime.

VI. RESULTS

Figure 5 shows the final WS2014–16 data used in this analysis, with the events used in the PLR framework highlighting the behavior of the different background models. The max ROI is the region of $\{S1_c, S2_c\}$ space that includes at least 90% of the expected differential rate from each signal model. The initial constraints and final fits for each nuisance parameter are shown in Table I, where fit values are for the background-only scenario. We set a 2-sided frequentist confidence interval on the value of $c_i^{(N)^2}$ using the method discussed in Sec. V at a 90% confidence level ($\alpha = 0.1$). We do this for all operators, selecting values for the WIMP mass ranging from 10 GeV to 4 TeV. Upper-limits are shown in Fig. 6 and Fig. 7 for WIMP-neutron and WIMP-proton interactions, respectively. We explicitly note here that the $c_i^{(N)}$ have dimensionality of $[mass]^{-2}$ as the conventions of Ref. [14] use a dimensionless operator representation and normalize spinors to unity, which differs from the representation used in Ref. [13]. Consequently, results are scaled by a factor of m_v^2 in order to report dimensionless values similar to the results reported in Ref. [29] for convenience. Figures 6 and 7 additionally show the available comparisons with the upper-limits from the 1.4×10^4 kg·day exposure results using LUX WS2013 data [15].

Our data set shows consistency with our background models, resulting in p-values between 0.14 and 0.50 for the 28 operator/nucleon combinations at 50 GeV mass, with a median p-value of 0.28. Additionally, Kolmogorov-Smirnov tests for each of the five observables $-S1_c$, $S2_c$, r, d, and ϕ - compared to the background model PDFs return p-values: 0.39, 0.24, 0.60, 0.43, and 0.81, respectively. However, due to the presence of observed data in regions of phase space where our background models and signal models overlap, the upperlimits on the number of WIMP scatters are greater than the expectation for certain signal models. These data include low-energy accidental-like events, a handful of $\gamma - X$ -like events near the bottom of the fiducial volume, and ^{83m}Kr events from the lowest drift time bin. While

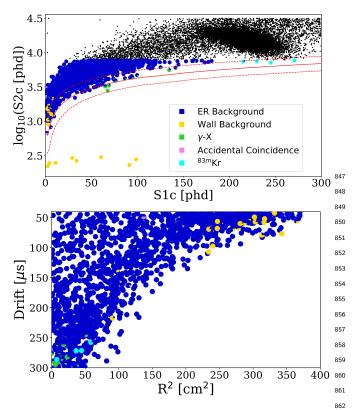


FIG. 5. The final unsalted WS2014–16 data used in this anal-⁸⁶³ ysis. Black markers indicate that the event was outside the⁸⁶⁴ final ROI used by the PLR. The remaining data are colored⁸⁶⁵ to indicate the values of a given model PDF at that point in⁸⁶⁶ phase space. Data can have multi-colored markers, indicat-⁸⁶⁷ ing that our expected background models overlap in certainses regions of phase space. Note that all 16 drift time bins are₈₆₉ merged in this plot, and the red solid and dashed lines rep-₈₇₀ resent the mean and 90% C.L. expected NR signal response₈₇₁ averaged over the 16 drift time bins. *Top*: Distribution of events in {S1, S2} space. *Bottom*: Spatial distribution of final events using radii as seen by the top PMT array and⁸⁷³ electron drift time. Note that the spatial distribution is not⁸⁷⁴ constant as a function of ϕ .

these events are included in our background model, un-⁸⁷⁸ 830 certainties in their expected rates leads to allowed signal⁸⁷⁹ 831 events from several EFT WIMP models. This results in⁸⁸⁰ 832 weaker sensitivity limits for several combinations of op-881 833 erator and mass when compared to the background-only⁸⁸² 834 expectation. Most operators remain within 2σ of our ex-835 pectation, with the exception of \mathcal{O}_{13} and \mathcal{O}_{15} , which dif-⁸⁸⁴ 836 fer from the expected sensitivity by as much as 2.8σ . This⁸⁸⁵ 837 is understood by referencing Fig. 1; the operator models⁸⁸⁶ 838 that produce our weakest limits are those with the lowest⁸⁸⁷ 839 expected differential rate at both the highest and lowest⁸⁸⁸ 840 energies in our ROI and are relatively flat at intermediate 841 energies (see \mathcal{O}_3 , \mathcal{O}_{13} , and \mathcal{O}_{15} and compare to Figures 6 842 and 7). When compared to our data (Figure 5), the⁸⁸⁹ 843 events that are not inconsistent with the signal models 844 follow this trend, where most of our observed background⁸⁹⁰ 845 leakage occurs at the intermediate energies of our ROI.891 846

TABLE I. The nuisance parameters used in the PLR framework, along with their initial constraints and fit values.

one, along with their initial constraints and it values.				
	Parameter	Constraint	Fit Value	
	Standard ER	1498.0 ± 187.5	$1495.1{\pm}51.2$	
	Wall-based Backgrounds	11.3 ± 2.8	12.8 ± 2.2	
	$\gamma - X = \frac{83 \text{mKr}}{100 \text{Kr}}$	$3.4{\pm}2.5$	5.2 ± 2.0	
	83m Kr	5.2 ± 1.5	$4.8 {\pm} 1.3$	
	Accidental Coincidence	$0.41^{+0.43}_{-0.41}$	$0.51{\pm}0.39$	

Figure 9 in Appendix A shows the data separated into the individual 16 drift and date bins, providing a qualitative characterization of the ER encroachment upon the signal region.

Despite the resultant limits being slightly poorer than our expectation, these are leading limits for several operators in various regions of parameter space using the $\{n,p\}$ basis, as opposed to {isoscalar, isovector}. Other reported $\{n,p\}$ limits in the literature were set using significantly less exposure than this analysis [15, 40]. Recent competitive analyses report their results using the {isoscalar, isovector} basis, such as XENON100, DEAP-3600, and PandaX-II [28–30], which prohibits direct comparison. However, we note that for xenon targets, the expected event rates for WIMP-n interactions are typically larger than that for WIMP-p interactions, but the isoscalar formulation splits the differences between these. Our WIMP-p limits are competitive (and sometimes more sensitive) than the isoscalar limits in Refs. [28, 29], indicating new exclusion of EFT WIMP parameter space, regardless of the chosen basis.

We also report the WS2014–16 inelastic scattering sensitivity limits excluding new EFT WIMP parameter space using the isoscalar basis. Figure 8 shows the inelastic EFT WIMP-nucleon isoscalar limits as a function of δ_m for a fixed WIMP mass of 1 TeV compared to the previous limits set by XENON100 [29]. At this mass we show similar limits to XENON100 despite using a larger exposure. This is due to overlap between our background models and signal models at higher energies; as we expect an increased amount of background in the signal region compared to traditional SI WIMP searches (due to $\gamma - X$ and 83m Kr contamination), our expected upper-limits increase, reducing the impact of the larger exposure. Despite this, our 1 TeV inelastic limits are competitive with XENON100, and in some cases more sensitive. This effect is more severe for higher energies where the overlap between background and signal models is the largest; smaller mass WIMP models are not significantly effected by this reduction of sensitivity, however, we show the 1 TeV examples for the purpose of comparing to the existing XENON100 limits.

877

VII. SUMMARY

We have expanded and improved the LUX background models to allow for characterization of data at energies

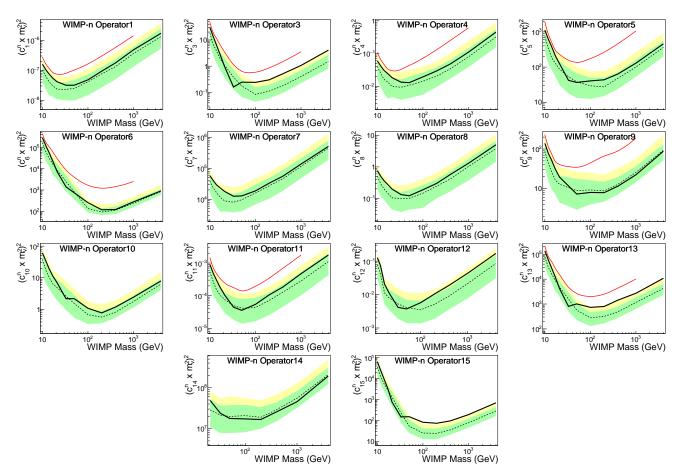


FIG. 6. The LUX WS2014–16 90% C.L. sensitivity limits for WIMP-neutron dimensionless couplings for each of the fourteen non-relativistic EFT operators. Solid black lines show the limit, while dashed black indicate the expectation, with green and yellow bands indicating the 1σ and 2σ sensitivity expectations, respectively. Each plot uses mass values of 10, 12, 14, 17, 21, 33, 50, 100, 200, 400, 1000, and 4000 GeV, except for Operators 12 and 14, which begin at 12 and 21 GeV, respectively. Red lines show the upper-limits from the WS2013 analysis [15].

much higher than a traditional WIMP search. These₉₁₃ 892 backgrounds include novel characterization of multiply-893 scattering $\gamma - X$ events disguised as single scatters, as 894 well as the inclusion of 83m Kr decays in our background 895 model. Utilization of the Noble Element Simulation 896 Technique allowed for efficient modeling of the ER and 897 NR LXe response, independently for each of the 16 time⁹¹⁵ 898 and drift time bins of WS2014-16 data. Additionally,916 899 NEST allows us to extrapolate the NR LXe response to⁹¹⁷ 900 higher energies than measured with in situ calibrations,⁹¹⁸ 901 after accounting for the uncertainties in all of the light⁹¹⁹ 902 and charge yield measurements combined from beyond⁹²⁰ 903 LUX. 904

We set exclusion limits for the 28 combinations of EFT_{923}^{922} operator and atomic nucleon in the $\{n, p\}$ basis. While we consider this basis to be more physically intuitive, it does not allow for direct comparison with recent EFT_{926}^{926} WIMP sensitivity limits in the {isoscalar,isovector} basis. 907

We also report the results of inelastic WIMP-nucleon⁹²⁸ scattering with respect to isoscalar nucleons at 1 TeV and⁹²⁹ compare to those reported by the XENON100 Collabo-⁹³⁰ ration [29].

ACKNOWLEDGMENTS

This work was partially supported by the U.S. Department of Energy (DOE) under Award No. DE-AC05-06OR23100, DE-AC02-05CH11231, DE-AC52-07NA27344. DE-FG01-91ER40618. DE-FG02-08ER41549, DE-FG02-11ER41738. DE-DE-FG02-91ER40688, FG02-91ER40674, DE-FG02-DE-NA0000979, 95ER40917. DE-SC0006605, DE-SC0010010, DE-SC0015535, and DE-SC0019066; the U.S. National Science Foundation under Grants No. PHY-0750671, PHY-0801536, PHY-1003660, PHY-1004661, PHY-1102470, PHY-1312561, PHY-1347449, PHY-1505868, and PHY-1636738; the Research Corporation Grant No. RA0350; the Center for Ultra-low Background Experiments in the Dakotas (CUBED); and the South Dakota School of Mines and Technology (SDSMT).

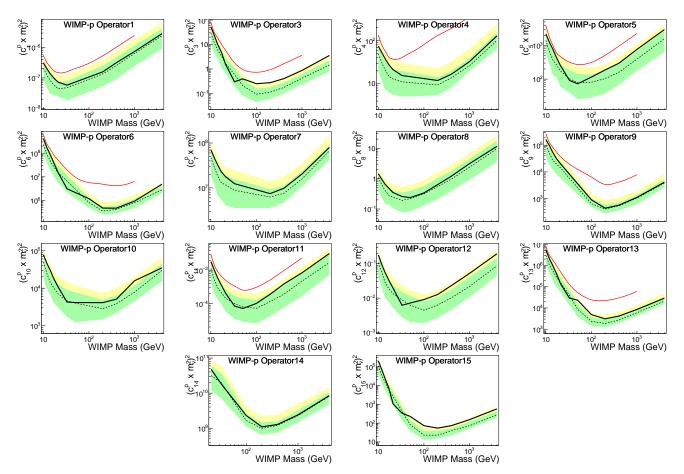


FIG. 7. The LUX WS2014–16 90% C.L. sensitivity limits for WIMP-proton dimensionless couplings for each of the fourteen non-relativistic EFT operators. Solid black lines show the limit, while dashed black indicate the expectation, with green and yellow bands indicating the 1σ and 2σ sensitivity expectations, respectively. Each plot uses mass values of 10, 12, 14, 21, 33, 50, 100, 200, 400, 1000, and 4000 GeV, with the exception of Operator 14, which begins at 21 GeV. Red lines show the upper-limits from the WS2013 analysis [15].

Laboratório de Instrumentação e Física Experimental₉₅₂ 931 de Partículas (LIP)-Coimbra acknowledges funding from₉₅₃ 932 Fundação para a Ciência e a Tecnologia (FCT) through₉₅₄ 933 the Project-Grant PTDC/FIS-NUC/1525/2014. Impe-955 934 rial College and Brown University thank the UK Royal₉₅₆ 935 Society for travel funds under the International Exchange957 936 Scheme (IE120804). The UK groups acknowledge in-958 937 stitutional support from Imperial College London, Uni-938 versity College London and Edinburgh University, and 939 from the Science & Technology Facilities Council for PhD 940 studentships R504737 (EL), M126369B (NM), P006795⁹⁵⁹ 941 (AN), T93036D (RT) and N50449X (UU). This work 942 was partially enabled by the University College London 943 (UCL) Cosmoparticle Initiative. The University of Edin-⁹⁶¹ 944 burgh is a charitable body, registered in Scotland, with⁹⁶² 945 Registration No. SC005336. 963 946 This research was conducted using computational re-

This research was conducted using computational resources and services at the Center for Computation and Visualization, Brown University, and also the Yale Science Research Software Core.

⁹⁵¹ We gratefully acknowledge the logistical and technical⁹⁶⁹

support and the access to laboratory infrastructure provided to us by SURF and its personnel at Lead, South Dakota. SURF was developed by the South Dakota Science and Technology Authority, with an important philanthropic donation from T. Denny Sanford. SURF is a federally sponsored research facility under Award Number DE-SC0020216.

Appendix A: Illustration of Data Separation by Time and drift time bin

We present in this appendix the full WS2014–16 data used in this analysis, separated into the 16 drift time bins: four temporal bins, each subdivided to correspond to a 65 μ s window of drift time. The livetimes for each temporal bin are: 43.9, 43.8, 85.8, and 137.7 days, respectively. Figure 9 illustrates the data compared to the relevant ER and NR simulated responses; bands represent 90% C.L. about the mean response. Despite the exclusion of exposures associated with the ^{83m}Kr calibration injections,

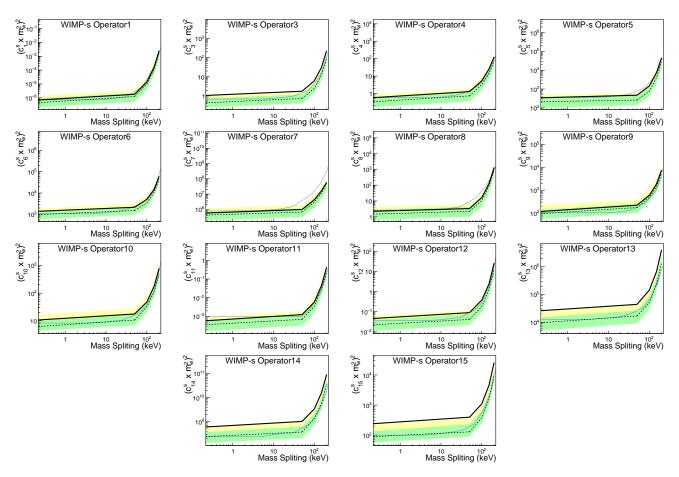


FIG. 8. The LUX WS2014–16 90% C.L. sensitivity limits for isoscalar WIMP-nucleon dimensionless couplings for each of the fourteen non-relativistic EFT operators and a fixed WIMP mass of 1 TeV. Solid black lines show the limit, while dashed black indicate the expectation, with green and yellow bands indicating the 1σ and 2σ sensitivity expectations, respectively. Each plot uses δ_m values of 0, 50, 100, 150, and 200 keV. Blue lines show limits from XENON100 [29].

⁹⁷⁰ many of these events can be seen in each drift time bin.

- 971 [1] V. Rubin and K. J. Ford, The Astrophysical Journal 159,990
 972 379 (1970).
- 973 [2] D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch,992
 974 S. W. Randall, C. Jones, and D. Zaritsky, Astrophys. J.993
 975 648, L109 (2006), arXiv:astro-ph/0608407 [astro-ph]. 994
- ⁹⁷⁶ [3] L. Anderson *et al.* (BOSS), Mon. Not. Roy. Astron. Soc.⁹⁹⁵
 ⁹⁷⁷ 441, 24 (2014), arXiv:1312.4877 [astro-ph.CO].
- [4] R. Adam *et al.* (Planck), Astron. Astrophys. **594**, A1997
 (2016), arXiv:1502.01582 [astro-ph.CO].
- [5] W. Hu and S. Dodelson, Ann. Rev. Astron. Astrophys.
 40, 171 (2002), arXiv:astro-ph/0110414 [astro-ph].
- [6] J. L. Feng, Ann. Rev. Astron. Astrophys. 48, 495 (2010);001
 arXiv:1003.0904 [astro-ph.CO]. 1002
- 984 [7] D. S. Akerib *et al.* (LUX), Phys. Rev. Lett. **112**, 091303003
 985 (2014), arXiv:1310.8214 [astro-ph.CO]. 1004
- 986 [8] D. S. Akerib *et al.* (LUX), Phys. Rev. Lett. **118**, 021303005
 987 (2017), arXiv:1608.07648 [astro-ph.CO]. 1006
- 988 [9] D. S. Akerib *et al.* (LUX), Phys. Rev. Lett. **118**, 021303007
 989 (2017), arXiv:1608.07648 [astro-ph.CO].

- [10] D. S. Akerib *et al.* (LUX), Phys. Rev. Lett. **118**, 251302 (2017), arXiv:1705.03380 [astro-ph.CO].
- [11] E. Aprile *et al.*, Physical Review Letters **121** (2018), 10.1103/physrevlett.121.111302.
- [12] X. Cui *et al.*, Physical Review Letters **119** (2017), 10.1103/physrevlett.119.181302.
- [13] A. L. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, and Y. Xu, JCAP **1302**, 004 (2013), arXiv:1203.3542 [hepph].
- [14] N. Anand, A. L. Fitzpatrick, and W. C. Haxton, Phys. Rev. C89, 065501 (2014), arXiv:1308.6288 [hep-ph].
- [15] D. S. Akerib and others., "An effective field theory analysis of the first lux dark matter search," (2020), arXiv:2003.11141 [astro-ph.CO].
- [16] D. Akerib *et al.*, Physical Review D **96** (2017), 10.1103/physrevd.96.112009.
- [17] D. Akerib *et al.*, Journal of Instrumentation **15**, T02007–T02007 (2020).

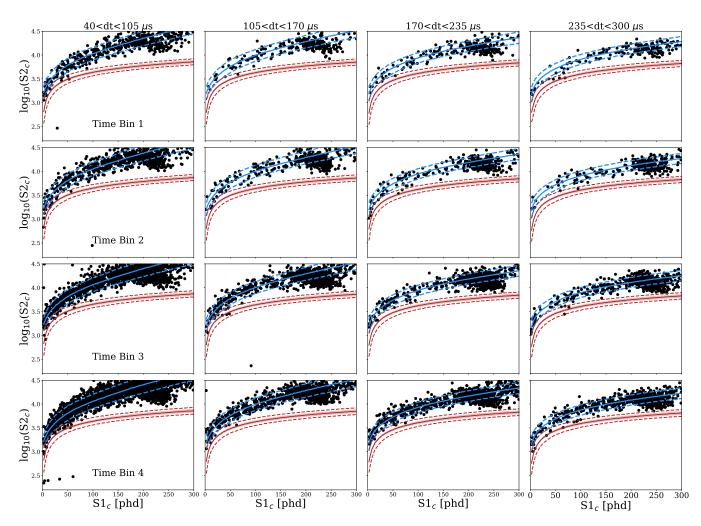


FIG. 9. The WS2014–16 data divided into the 16 drift time bins: four temporal bins of unequal livetime subdivided further into four spatial bins. Blue bands represent the mean and 90% C.L. ER response, while red illustrates the mean and 90% C.L. NR response. 20.8 live-days of data corresponding to significant 83m Kr contamination have been removed from this data set, however, remaining 83m Kr events can be seen encroaching upon the signal region at higher energies.

- 1008 [18] D. Akerib *et al.*, Physical Review D **93** (2016)₁₀₃₀ 1009 10.1103/physrevd.93.072009.
- 1010
 [19] D. Akerib et al., Physical Review D
 100
 (2019)1032

 1011
 10.1103/physrevd.100.022002.
 1033
- 1012 [20] D. S. Akerib *et al.* (LUX), (2016), arXiv:1608.0538h034 1013 [physics.ins-det]. 1035
- 1014[21] "Multiphysics software for optimizing designs,"10361015https://www.comsol.com/.
- 1016 [22] D. Akerib *et al.*, Journal of Instrumentation 12_{1038} 1017 P11022–P11022 (2017). 1039
- 1018
 [23] D. Akerib et al., Physical Review D
 102
 (2020)1040

 1019
 10.1103/physrevd.102.112002.
 1041
- 1020
 [24]
 D.
 Akerib
 et
 al.,
 Physical
 Review
 D
 98
 (2018)₁₀₄₂

 1021
 10.1103/physrevd.98.062005.
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
 1043
- 1022
 [25] D. Akerib et al., Physical Review D
 102
 (2020)]1044

 1023
 10.1103/physrevd.102.092004.
 1045
- [26] D. Akerib *et al.*, "Improving sensitivity to low-mass darko46
 matter in lux using a novel electrode background mitigat047
 tion technique," (2020), arXiv:2011.09602 [hep-ex]. 1048
- ¹⁰²⁷ [27] Y. Coadou, EPJ Web of Conferences 55, 02004 (2013). ¹⁰⁴⁹
- 1028 [28] P. Adhikari *et al.*, Physical Review D **102** (2020)₁₀₅₀ 1029 10.1103/physrevd.102.082001. 1051

- [29] E. Aprile *et al.*, Physical Review D **96** (2017), 10.1103/physrevd.96.042004.
- [30] J. Xia et al. (PandaX), arXiv:1807.01936 [hep-ex].
- [31] M. Szydagis, J. Balajthy, J. Brodsky, J. Cutter, J. Huang, E. Kozlova, B. Lenardo, A. Manalaysay, D. McKinsey, M. Mooney, J. Mueller, K. Ni, G. Rischbieter, M. Tripathi, C. Tunnell, V. Velan, and Z. Zhao, "Noble element simulation technique," (2020).
- [32] P. Sorensen and C. E. Dahl, Physical Review D 83 (2011), 10.1103/physrevd.83.063501.
- [33] G. Barello, S. Chang, and C. A. Newby, Physical Review D 90 (2014), 10.1103/physrevd.90.094027.
- [34] D. Akerib *et al.*, Physical Review Letters **116** (2016), 10.1103/physrevlett.116.161301.
- [35] D. Akerib *et al.*, Journal of Instrumentation **13**, P02001–P02001 (2018).
- [36] E. Aprile *et al.*, Physical Review D **99** (2019), 10.1103/physrevd.99.112009.
- [37] J. Neyman, Phil. Trans. R. Soc. Lond. A 236, 333 (1937).
- [38] G. Cowan, K. Cranmer, E. Gross, and O. Vitells, The European Physical Journal C **71** (2011), 10.1140/epjc/s10052-011-1554-0.

- 1052
 [39]
 W.
 Verkerke
 and
 D.
 Kirkby,
 (2003)1056

 1053
 10.1142/9781860948985_0039.
 1057
 1057
- [40] K. Schneck, B. Cabrera, D. Cerdeño, V. Mandic,
 H. Rogers, R. Agnese, A. Anderson, M. Asai, D. Bal-

akishiyeva, D. Barker,
,et~al.,Physical Review D $\mathbf{91}$ (2015), 10.1103/phys
revd.91.092004.