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Minimal SO(10) GUT: Phenomenological and Cosmological Implications

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arXive:ph-0905???

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Outline

- Motivations for GUT
- Some shortcomings & problems
- Realistic SO(10) Model with Interesting Predictions :

  GUT breaking, -TD splitting –Proton lifetime

  Calculable thresholds, Predictive fermon pattern
- Baryogenesis via Leptogenesis
- Inflation in SO(10) GUT
- Summary

SUSY GUT →

- Charge Quantization,Unification of multiplets $\subset 16\, of\ SO(10)$
- Neutrino Masses (via ν_R of SO(10)); L-violation
- Successful Coupling Unification
- Stab. Hierarchy (Light Higgs) ← low SUSY scale
- Dark Matter Candidate (LSP)
- Baryogenesis via Leptogenesis
- Prediction: B-violation → proton decay

SUSY GUT puzzles:

- GUT Symmetry Breaking? (flat directions/goldstones)
- Doublet-Triplet Splitting
- How/why mu-Term ~ 100 GeV ? (harder in GUT)
- Proton Stability (especially d=5 decay)
- Fermion Masses & Mixings (flavor problem)
- SUSY FCNC (sflavor problem)
- Minimal & Economical System
  - Calculability of GUT Threshold Corrections
  - Perturbativity all the way up to M_{Planck}

All these issues are closely related and

Unless *Unified* solution is found,
none of the predictions can be trusted..

Realistic SUSY SO(10)

SO(10) ->
Solution of DT hierarchy via missing VEV

Dimopoulos, Wilczek'81
Babu, Barr'93
Barr, Raby'97

- `Higgs' System: $H(10) \supset h_u + h_d + T_H + \bar{T}_H \longleftrightarrow 16_3 16_3 H$

$$A(45) + C(16) + \bar{C}(16^*) \text{ for } SO(10) \xrightarrow{BR} SU(3) \times SU(2) \times U(1)$$

$$C'(16) + \bar{C}'(16^*) \quad \text{and} \quad Z(1) + S(1)$$

Insuring desirable sym. br. & NO flat dir./pseudo-Golds.

- Additional Symmetries: $U(1)_A \times Z_2$

Symmetry Breaking, All order DT splitting,
mu –term, Nucleon stability (predictions)
Realistic & simple fermion pattern

$U(1)_A \times Z_2$ Transformations:

	$A(45)$	$H(10)$	$H'(10)$	$C(16)$	$\bar{C}(\overline{16})$	Z	S	$C'(16)$	$\bar{C}'(\overline{16})$	$16_{1,2}$	16_3
$\mathcal{U}(1)$	0	1	-1	$\frac{k+4}{2k}$	$-\frac{1}{2}$	$\frac{2}{k}$	$\frac{2}{k}$	$\frac{k-4}{2k}$	$-\frac{k+8}{2k}$	-1	$-\frac{1}{2}$
Z_2	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$P_{1,2}$	$+$

`Scalar' Superpotential (fixed):

$$
\begin{aligned}
W(A) &= M_A \mathrm{tr} A^2 + \frac{\lambda_A}{M_*}\left(\mathrm{tr} A^2\right)^2 + \frac{\lambda'_A}{M_*}\mathrm{tr} A^4 \ , \\
W(A,C,C') &= C\left(\frac{a_1}{M_*} ZA + \frac{b_1}{M_*} C\bar{C} + c_1 S\right)\bar{C}' + C'\left(\frac{a_2}{M_*} ZA + \frac{b_2}{M_*} C\bar{C} + c_2 S\right)\bar{C} \\
W(DT) &= \lambda_1 HAH' + \lambda_{H'}\frac{(S^k, Z^k)}{M_*^{k-1}}(H')^2 + \lambda_2 H\bar{C}\bar{C} + \frac{\lambda_3}{M_*} AH'CC' \ .
\end{aligned}
$$

Missing VEV Solution:

$$\langle A\rangle = \mathrm{i}\sigma_2 \otimes \mathrm{Diag}\,(a,\ a,\ a,\ 0,\ 0)$$

(incl. FI-term) Fixed VEVs:

$$\langle A\rangle, \langle C\rangle, \langle \bar{C}\rangle, \langle Z\rangle, \langle S\rangle \neq 0$$

Light MSSM doublets & No other pseudo-goldstones

DT Splitting & GUT Spectrum

$$M_{D,T} = \begin{array}{c} \\ \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_C \\ \bar{5}_{C'} \end{array} \begin{array}{c} \begin{array}{cccc} 5_H & 5_{H'} & 5_{\bar{C}} & 5_{\bar{C}'} \end{array} \\ \left(\begin{array}{cccc} 0 & \eta_{D,T}\lambda_1 a & \lambda_2 c & 0 \\ -\eta_{D,T}\lambda_1 a & M_{H'} & 0 & 0 \\ 0 & 0 & 0 & \kappa_{D,T} Y_1 \\ 0 & Y_{D,T} & \kappa_{D,T} Y_2 & M_{C'} \end{array} \right) \end{array}$$

$\Rightarrow$ M(hu, hd)=0, M(triplets)-heavy

with $\eta_D = 0\ ,\quad \eta_T = 1\ ,\quad \kappa_D = 3\ ,\quad \kappa_T = 2\ .$

10H-plets:

$$M(\Psi^{10}) = \begin{array}{c} \\ \Psi_A^{10} \\ \Psi_C^{10} \\ \Psi_{C'}^{10} \end{array} \begin{array}{c} \begin{array}{ccc} \Psi_A^{\overline{10}} & \Psi_{\bar{C}}^{\overline{10}} & \Psi_{\bar{C}'}^{\overline{10}} \end{array} \\ \left(\begin{array}{ccc} M_\Psi & 0 & X_1 \\ 0 & 0 & \kappa_\Psi Y_1 \\ X_2 & \kappa_\Psi Y_2 & M'\eta \end{array} \right) \end{array}$$

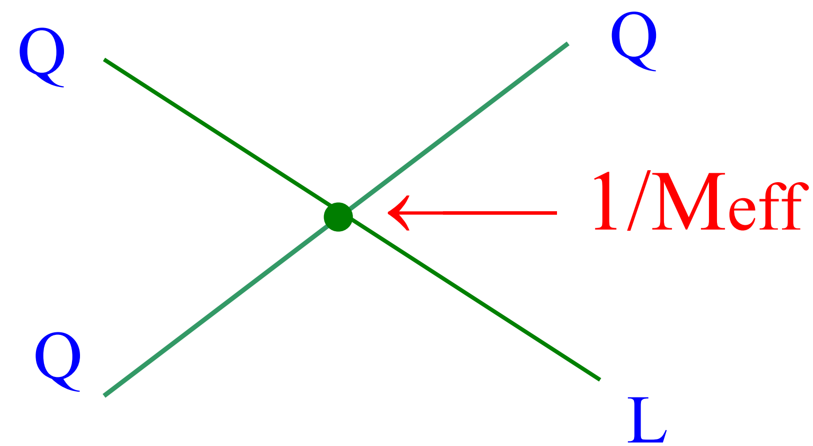
→**All hevy**

with $\Psi = (u^c, q, e^c)\ ,\quad \kappa_\Psi = (2, 1, 0)\ ,\quad M_\Psi$

Well defined spectrum:

$$\frac{M_{D_1} M_{D_2} M_{D_3}}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} = \frac{9}{4 M_{\text{eff}} \cos\gamma}\ , \qquad \text{with} \qquad \frac{1}{M_{\text{eff}}} = \left(M_T^{-1}\right)_{11} = \frac{M_{H'}}{\lambda_1^2 a^2}$$

Meff: In d=5 decay



In RG Equations →

$$\frac{M_{D_1} M_{D_2} M_{D_3}}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} = \frac{9}{4 M_{\rm eff} \cos\gamma}$$

$\cos\gamma \simeq \tan\beta/60$

$$\alpha_{\rm U}^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln\frac{\Lambda}{M_Z} + \Delta_{i,\ w}^{(2)} + \Delta_i^{\rm GUT}$$

Calculable Thresholds!

RG and Gauge Coupling Unification

$$\ln\frac{M_{\text{eff}}\cos\gamma}{M_Z}=\frac{5\pi}{6}\left(3(\alpha_2^{-1}+\Delta^{(2)}_{2,\,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,\,w}-\frac{1}{4\pi})-(\alpha_1^{-1}+\Delta^{(2)}_{1,\,w})\right)$$

$$-\ln\frac{4\kappa^{5/2}}{9}+\ln\frac{(4+p^2)^{3/2}(1+\tilde{p}^2)^2}{(4+\tilde{p}^2)^{3/2}(1+p^2)^2}+\ln\frac{p}{\hat{p}}\ ,$$

$$\ln\frac{(M_X^2M_\Sigma)^{1/3}}{M_Z}=\frac{\pi}{18}\left(5(\alpha_1^{-1}+\Delta^{(2)}_{1,\,w})-3(\alpha_2^{-1}+\Delta^{(2)}_{2,\,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,\,w}-\frac{1}{4\pi})\right)$$

$$+\frac{1}{6}\ln\kappa-\frac{1}{6}\ln\frac{(4+p^2)(1+\tilde{p}^2)^2}{(4+\tilde{p}^2)(1+p^2)^2}-\frac{1}{3}\ln\frac{p}{\hat{p}}\ .$$

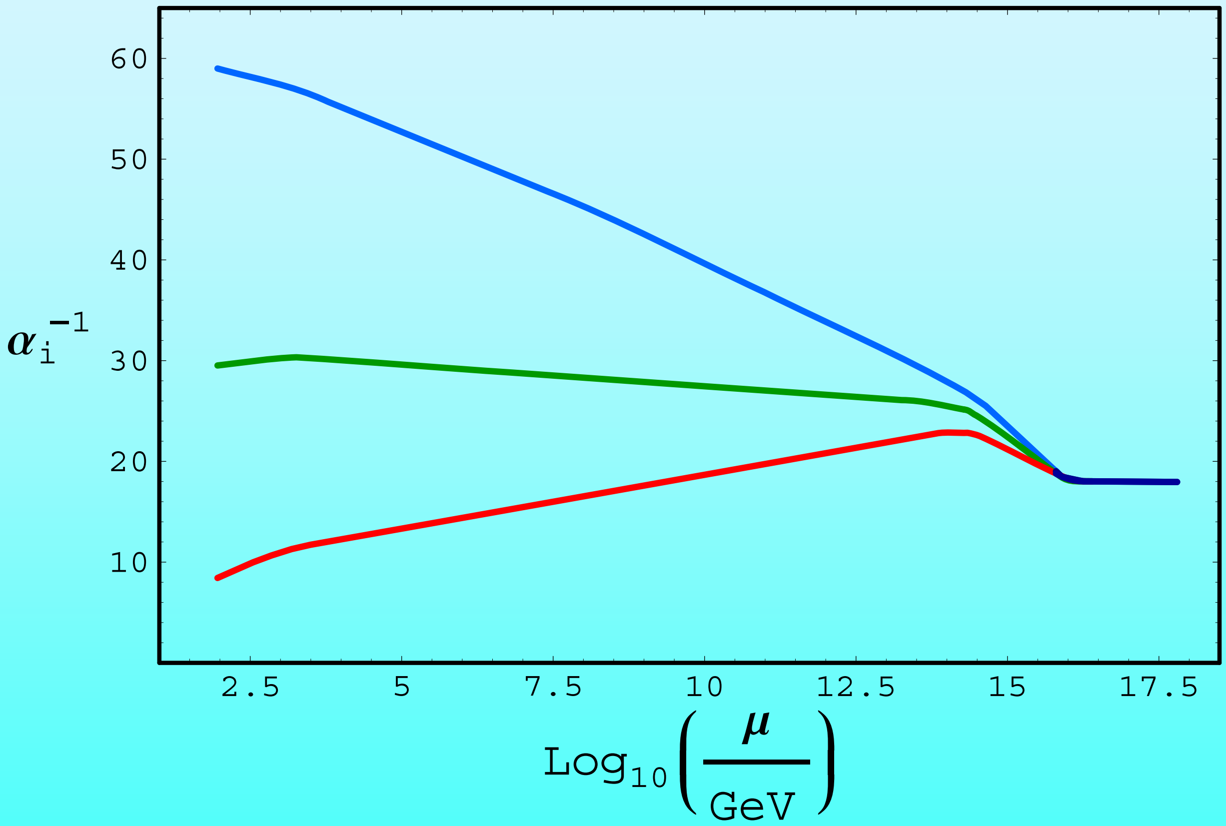
Model Gives: $p=\tilde{p}$; with $\hat{p}/p\sim 10^{-4}$, $\cos\gamma\approx 1/20$

$\alpha_3(M_Z)=0.1176$, $M_{\text{eff}}=\text{few}\times 10^{19}$ GeV

$(M_X^2M_\Sigma)^{1/3}\approx 10^{16}$ GeV

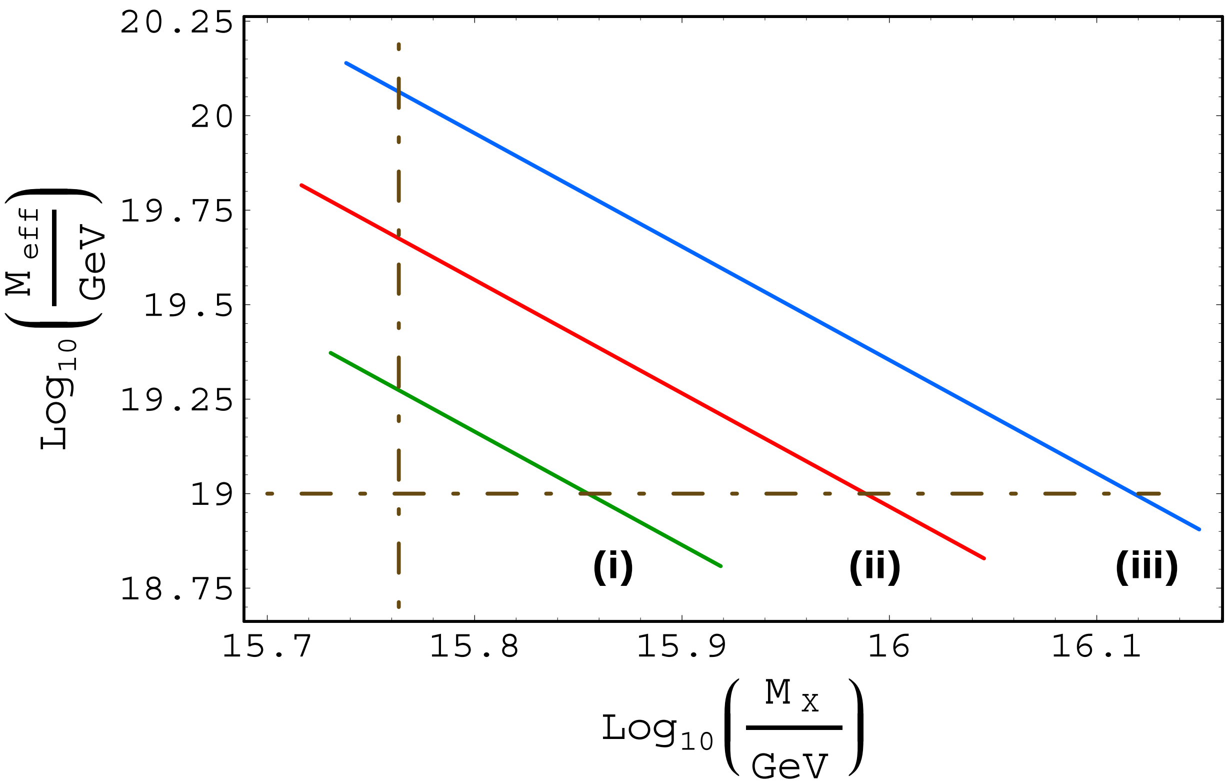
[Compare with SU(5) - Hisano, Murayama, Yanagida'93**]**

Gauge Coupling Unification



Correlation Between M_{eff} & M_X

$$M_{\rm eff} \simeq 10^{19}{\rm GeV}\cdot\left(\frac{10^{16}{\rm GeV}}{M_X}\right)^3\left(\frac{1/100}{r}\right)\left(\frac{0.63\times 3}{\eta_\gamma \tan\beta}\right)\left\{\frac{\exp[2\pi(\Delta^{(2)}_{2,\,w}-\Delta^{(2)}_{3,\,w}-\delta\alpha_3^{-1})]}{2.24\cdot 10^{-2}}\right\}$$



$$r \equiv M_\Sigma / M_X$$

$$(\text{i, ii, iii}) \rightarrow \alpha_3(M_Z) = (0.1156,\ 0.1176,\ 0.1196)$$

$$\tan\beta = 3, \qquad r = 1/200$$

→Correlation Between d=5 & d=6 Proton Decays and Upper Bounds on Lifetimes

Symmetries give/sugges reanges: $\frac{1}{10} \lesssim r(= M_\Sigma/M_X) \lesssim \frac{1}{200}$

$$M_{\text{eff}} \approx (1/28 - 24)M_* \approx (7 \cdot 10^{16} - 6 \cdot 10^{19})\ \text{GeV}$$

`Naturalness' Requirement: $m_{\tilde{q}} \leq 1.5\,\text{TeV}$

Empirical Bound: $M_{\tilde{w}} \geq 125\,\text{GeV}$

& Correlation Leads to:

$$\Gamma^{-1}_{d=6}(p \to e^+\pi^0)_{\text{max}} \lesssim\ 1.2 \cdot 10^{35}\ \text{yrs}$$

$$\Gamma_{d=5}^{-1}(p \to \bar{\nu}K^+) \leq 3 \times 10^{34}\ \text{yrs}$$

Potentially observable with improvement of exp. sensitivity by factor 10

Comment: For calculating d=5 Proton decay, well defined Yukawa sector is important

Yukawa Sector with Q4 Flavor Symmetry

Q_4 → Interesting (predictive) Textures,
Solves SUSY FCNC problem

Q_4: $\overrightarrow{16} \equiv (16_1, 16_2) \sim 2$, $16_3 \sim 1$ $\qquad \overrightarrow{X}, \overrightarrow{Y} \sim 2-\text{Flavons}$

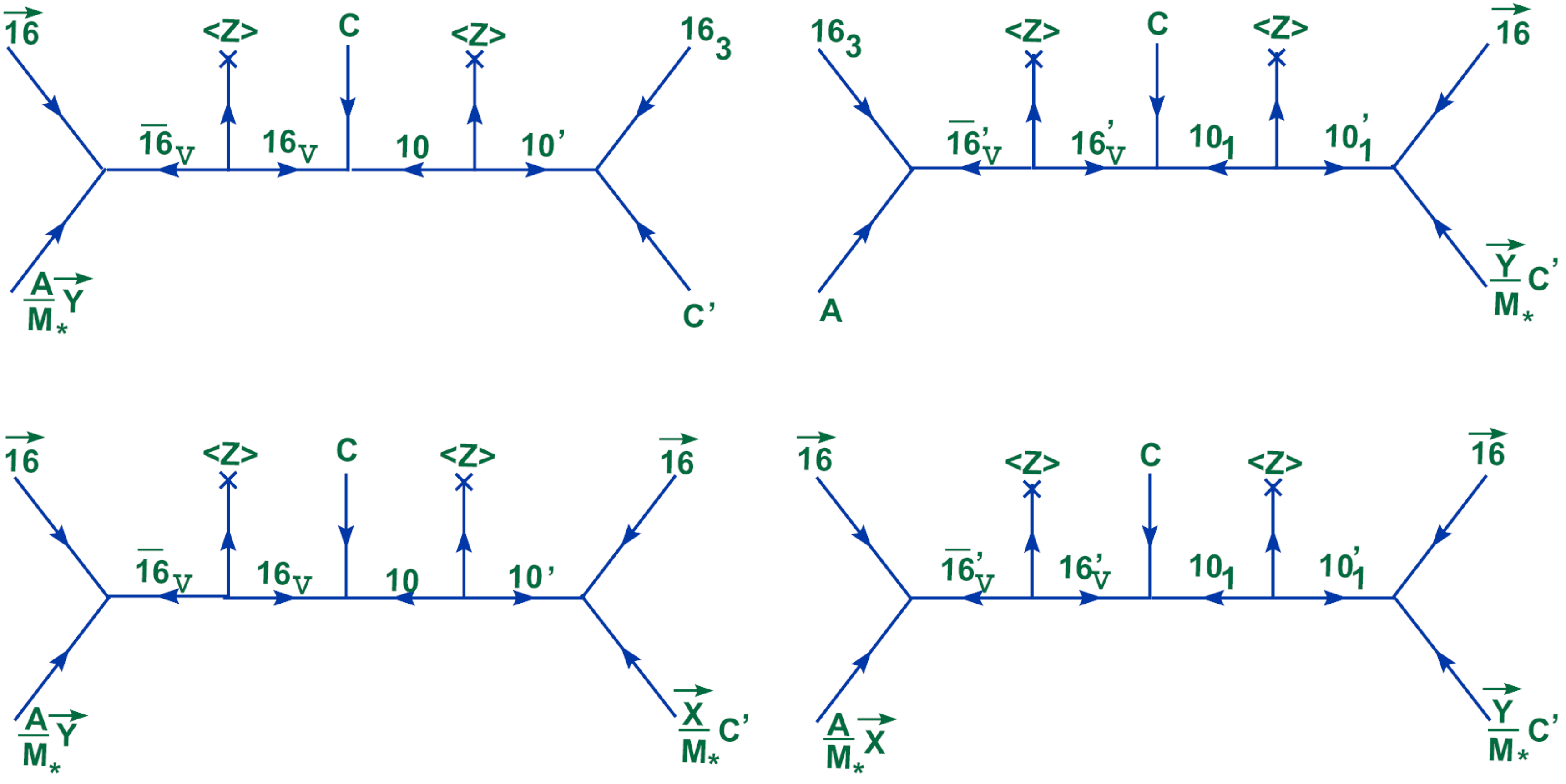
Effective Yukawa Interactions are fixed by symmetries → Predictive

$$W^{(D)}_{\rm Yukawa} = 16_3 16_3 H + \frac{\overrightarrow{X}}{M_*}\overrightarrow{16}16_3 H + \frac{SZ^2A}{M_*^4}\overrightarrow{16}\overrightarrow{16}H + \frac{Z^3C}{M_*^4}\overrightarrow{16}\overrightarrow{16}C' +$$

$$\frac{AC\overrightarrow{Y}}{M_*\langle Z\rangle^2}\left(\overrightarrow{16}\cdot 16_3 + 16_3\cdot\overrightarrow{16}\right)C' + \frac{AC}{M_*^2\langle Z\rangle^2}(\overrightarrow{X}\,\overrightarrow{16})(\overrightarrow{Y}\,\overrightarrow{16})C' \ .$$

Can be obtained through renorm. Interactions:

Generation of Effective Yukawa Couplings



Yukawa Matrices

$$Y_u = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \end{array} \begin{array}{c} \begin{array}{ccc} u_1^c & u_2^c & u_3^c \end{array} \\ \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \sigma \\ 0 & \sigma & 1 \end{pmatrix} \end{array} \lambda_t \qquad Y_d = \begin{array}{c} \\ d_1 \\ d_2 \\ d_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1^c & d_2^c & d_3^c \end{array} \\ \begin{pmatrix} 0 & \epsilon' + \eta' & 0 \\ -\epsilon' - \eta' & \xi_{22}^d & \sigma + \epsilon \\ 0 & \sigma + \overline{\epsilon} & 1 \end{pmatrix} \end{array} \lambda_b$$

$$Y_e = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{ccc} e_1^c & e_2^c & e_3^c \end{array} \\ \begin{pmatrix} 0 & -3\epsilon' - \eta' & 0 \\ 3\epsilon' + \eta & 3\xi_{22}^d & \sigma + 3\overline{\epsilon} \\ 0 & \sigma + 3\epsilon & 1 \end{pmatrix} \end{array} \lambda_b \qquad Y_\nu = \begin{array}{c} \\ \nu_1 \\ \nu_2 \\ \nu_3 \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1^c & \nu_2^c & \nu_3^c \end{array} \\ \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & \sigma \\ 0 & \sigma & 1 \end{pmatrix} \end{array} \lambda_t$$

$$\lambda_b = \lambda_t \cos\gamma$$

Input Parameters: with $\xi^d_{22} = 0$, 5 modulo+2 phases

$$\sigma = 0.051, \quad \epsilon = 0.038, \quad \bar{\epsilon} = 0.129\, e^{i\,0.63}$$

$$\epsilon' = 1.79 * 10^{-4}, \quad \eta' = 0.0043\, e^{i\,0.57}$$

Results: $m_b = m_\tau, \quad \frac{m_c}{m_t} = 2.6 * 10^{-3}, \quad \frac{m_s}{m_b} = 1.6 * 10^{-2}, \quad \frac{m_\mu}{m_\tau} = 0.059$

At GUT scale: $\frac{m_u}{m_t} = 1.2 * 10^{-5}, \quad \frac{m_d}{m_b} = 1.14 * 10^{-3}, \quad \frac{m_e}{m_\tau} = 2.8 * 10^{-4}$

At M(Z) scale: $|V_{us}| = 0.227, \quad |V_{cb}| = 0.04, \quad |V_{ub}| = 0.003$

CP violation: $\bar{\eta} = 0.3 \qquad \overline{\rho}+i\overline{\eta} = -V_{ud}V_{ub}^*/(V_{cd}V_{cb}^*)$

All in good agreement with experiments
[taking into account FCNC effects
in K, B-system coming from our SO(10)]

‘Majorana’ Couplings:

$$W^{(M)}_{\text{Yukawa}} = \frac{Z^{k-4}\overrightarrow{Y}^2}{M_*^{k-3}M^2_{N''}}\overrightarrow{16}^2\bar{C}^2 + \frac{Z^{k-2}\overrightarrow{Y}}{M_*^{k-2}M^2_{N'}}\overrightarrow{16}16_3\bar{C}^2 + \frac{Z^{k-1}S}{M_*^{k-1}M^2_N}16^2_3\bar{C}^2$$

$$M_{\nu^c} = \begin{array}{c} \\ \nu_1^c \\ \nu_2^c \\ \nu_3^c \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1^c & \nu_2^c & \nu_3^c \end{array} \\ \begin{pmatrix} n_1 q^2 & 0 & 0 \\ 0 & n_2 q^2 & q \\ 0 & q & 1 \end{pmatrix} \end{array} M_R$$

Input: $q = 0.025 * e^{i\,0.05}$, $n_1 = 4 * e^{-3\,i}$, $n_2 = 0.17 * e^{-0.6\,i}$

$$M_R = 2 * 10^{14}\ \text{GeV}$$

$$m_\nu = m_D \frac{1}{M_{\nu C}} m^T_D \rightarrow \Delta m^2_{atm} = 0.003\ \text{eV}^2,\quad \Delta m^2_{sol} = 8 * 10^{-5}\ \text{eV}^2$$

$\theta_{12} = 33^o$, $\theta_{23} = 38^o$, $\theta_{13} = 4^o$

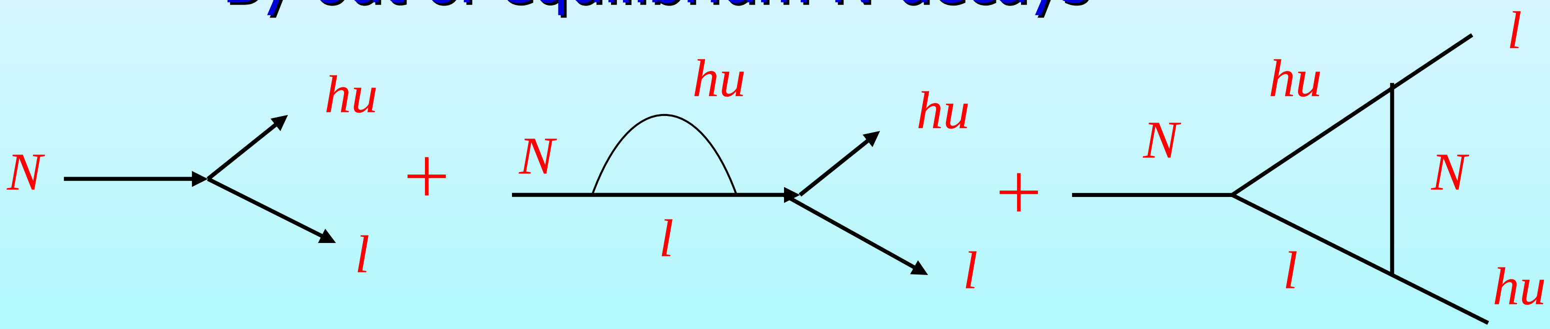
Good choice also for Leptogenesis..

Leptogenesis

RHNs → L, CP viol → **Leptogenesis** (Fikugita, Yanagida'86)

By out of equilibrium N-decays

N→hu l:



Hierarchical RHNs: $M_R = (10^{11},\ 5\cdot 10^{11},\ 2\cdot 10^{14})\,\mathrm{GeV}$

High T_R → SUSY gravitino problem
unless very light or heavy gravitino ☹

CP-assymetry:

$$\epsilon_1 \simeq -\frac{3}{8\pi}\frac{1}{[\mathbf{Y}_\nu \mathbf{Y}_\nu{}^\dagger]_{11}} \sum_j \mathrm{Im}\left\{[\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger]^2_{1j}\right\}\left(\frac{M_1}{M_j}\right)$$

Leads to observed baryon asymmetry: $\frac{n_b}{s} \approx 8\cdot 10^{-11}$

Inflation in SO(10)

Singlet sector: $W_{Sing} = \kappa \overline{P}(P^2 + \mu Z)$

P cancels D_A-term & triggers other VEVs

$$V(P) = g^2{}_A(-\xi + P^2 + \ldots)^2 + \kappa^2 |P^2 + \mu Z|^2 \qquad Z > Zc \rightarrow P = 0$$

D-term flatness trajectory: $|Z|^2 - |q||\varphi|^2 = \xi$

With flat W & F-term, $\varphi = \tilde{\nu}^C,\ H'u + H'd, \ldots$

$$Z = \sqrt{\xi}\,\mathrm{Cosh}\left(\frac{\rho}{\sqrt{\xi}}\right), \quad \varphi = \sqrt{\xi / q}\,\mathrm{Sinh}\left(\frac{\rho}{\sqrt{\xi}}\right),$$

$$Z, \varphi \propto \mathrm{Exp}\left(2\frac{\rho}{\sqrt{\xi}}\right), \quad |\partial Z|^2 + |\partial\varphi|^2 \propto \mathrm{Exp}\left(2\frac{\rho}{\sqrt{\xi}}\right)(\partial\rho)^2 = 1/2(\partial\eta)^2$$

For large fields:

$$\mathrm{Exp}\left(\frac{\rho}{\sqrt{\xi}}\right) \simeq \sqrt{2}\eta / \sqrt{\xi}$$

$$V_{infl} = (\kappa\mu)^2 \xi\,\mathrm{Cosh}^2\left(\frac{\rho}{\sqrt{\xi}}\right) \propto \mathrm{Exp}\left(2\frac{\rho}{\sqrt{\xi}}\right) \sim \frac{1}{2}(\kappa\mu)^2\eta^2 \ !$$

Inflation in SO(10) - results, implications:

Quadratic pot. $V_{infl} = \frac{1}{2}(\kappa\mu)^2\eta^2 \Rightarrow n_s = 0.967, \quad r = 0.13$

(Testable by PLANCK)

$\delta T / T \approx 6 \cdot 10^{-6}$ with $\kappa\mu/\mathrm{M_{Pl}} \sim 10^{-5}$

?-Large field inflation $(\eta_Q \sim \sqrt{N} M_{Pl})$ **Effects from SUGRA corrections?**

*** D_A=0, W=0, F(\phi)=0 conditions can help (under investigation)**

Some more implications:

- During inflation <A>#0 → SO(10) broken & no monopole problem
- If $\varphi = \tilde{\nu}^c$ inflation breaks L-number
  & can produce L-asymmetry via decays at the end of inflation
- Due to multifield system, phase transitions can happen can hibrid/small field inflation can be realized?
  If yes, then all SUGRA cors. Under controll

Summary

Presented minimal/economical SUSY SO(10)

- Minimal Higgs System, Simple GUT Breaking
  and Natural DT Splitting to all orders

- Stabile Proton with Predictive (& Testable)
  Correlation Between d=5 & d=6 Decay Modes

- Simple Fermion Pattern by flavor symmetry,
  and No SUSY FCNC problem

- Realistic Neutrino sector

  → Baryogenesis through Leptogenesis

- Good potential for successful inflation

Thank You