Fermion Mass Hierarchy from the Soft Wall

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Outline

- Warped extra dimensions may provide an alternative explanation for the hierarchy between gravity and ElectroWeak scales.

Randall and Sundrum (1999)
Arkani-Hamed et al. (1998)

- More recently, an alternative to the hard wall termination of RS models have been proposed and given the name of Soft Wall models.

- In these models one has a non compact extra dimension with UV (4d) boundary and the effective metric is not AdS. Instead it decays faster enough to make the extra dimension of finite length.

- The departure from the AdS behavior is associated with a smooth Vacuum Expectation Value acquired by some dilaton field.

Karch, Katz, Son and Stephanov (2006)
Falkowski and Pérez-Victoria (2008)
Batell, Gherghetta and Sword (2008)
Delgado and DD (2009)

- The former motivation for the soft wall models was the appearance of linear KK excitations ($m_n \sim n$) which may provide a description of mesons spectrum in QCD, $\mathrm{AdS}/\mathrm{QCD}$.

Csaki and Reece (2007)
Gursoy and Kiritsis (2008)
Gursoy, Kiritsis and Nitti (2008)
Batell and Gherghetta (2008)

Field Content

Field	$SU(2)_L \times U(1)_Y$	off-shell dynamical d.o.f.
Ψ_L^i	$2 \otimes 1$	8
$\Psi_R^{1,2}$	$1 \otimes 1$	8
H^i	$2 \otimes 1$	8
A_M	$3 \oplus 1$	4×4
g_{MN}	$1 \otimes 1$	0
ϕ	$1 \otimes 1$	0

The Action

$$
\begin{aligned}
S = \int_\Sigma \sqrt{g}\, e^{-\phi} \Big[& \frac{\mathrm{i}}{2} \bar{\Psi}_L^i \gamma^M \mathcal{D}_M^L \Psi_L^i + \frac{\mathrm{i}}{2} \bar{\Psi}_R^s \gamma^M \mathcal{D}_M^R \Psi_R^s + M\, \bar{\Psi}_L^i \Psi_L^i + M\, \bar{\Psi}_R^s \Psi_R^s \\
& + \lambda_1\, \epsilon_{ij} \bar{\Psi}_L^i \Psi_R^1\, H^{*\,j} + \lambda_2\, H^i \bar{\Psi}_L^i \Psi_R^2 + \mathrm{h.c.} \\
& + g^{MN}\, (D_M H)^\dagger\, D_N H - m_h^2 H^\dagger H + \frac{1}{4 g_5^2}\, g^{MR} g^{NS}\, \mathrm{tr}\left\{F_{MN} F_{RS}\right\} \Big] \\
- \int_{\partial\Sigma} \sqrt{g}\, e^{-\phi} & \left[\lambda_0 R^2 \left(H^2 - v_0^2 \right)^2 - e_5^5 \frac{1}{2} \left(\bar{\Psi}_L^i \Psi_L^i - \bar{\Psi}_R^s \Psi_R^s \right) \right] ,
\end{aligned}
\tag{1}
$$

$\Sigma = \mathbb{R}^4 \times [z_0, \infty)$, $z_0 > 0$, $\langle \phi \rangle = \mu^2 z^2$, $g_{MN} = f^2\, \eta_{MN}$, $f = \frac{R}{z}$ and $z_0 \sim R$

In addition ...

$\gamma^M = e^M_A \gamma^A$ with $\gamma^A = (\gamma^\mu, -\mathrm{i}\gamma^5)$, $\gamma^5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$ and $e^M_A = \frac{1}{f}\delta^M_A$.

$A_M = g_5 A^\alpha_M T_\alpha$, $[T_\alpha, T_\beta] = \mathrm{i} f^\gamma_{\alpha\beta} T_\gamma$,
D_M, $\mathcal{D}^R_M$, $\mathcal{D}^L_M$ gauge and Lorentz covariant derivatives.

$$m_h^2 = \frac{a\,(a-4)}{R^2} - 2a\frac{\mu^2}{R^2}\,z^2\,,$$

$\lambda_{1,2}$ and g_5 are the 5d Yukawa and gauge coupling constants, respectively. λ_0 and a are dimensionless constants while $[v_0] = E^{3/2}$.

The lowest modes of the above (dynamical) fields are to be identified with the Standard Model ones.

Equations of Motion for the Higgs VEV

$$H(x,z) = \begin{pmatrix} 0 \\ \mathfrak{H}(z) \end{pmatrix}, \qquad \mathfrak{H}(z) \in \mathbb{R}$$

$$\mathfrak{H}'' + \left(3\frac{f'}{f} - \phi'\right)\mathfrak{H}' - f^2\left(\frac{a(a-4)}{R^2} - 2a\frac{\mu^2}{R^2}z^2\right)\mathfrak{H} = 0\,,$$

$$\mathfrak{H}' - 2\lambda_0 R^2\left(\mathfrak{H}^2 - v_0^2\right)\mathfrak{H}\Big|_{z_0} = 0\,,$$

$$\mathfrak{H} = \begin{cases} 0 \\ \left(\frac{z}{z_0}\right)^a \sqrt{\frac{a}{2z_0\lambda_0 R^2} + v_0^2} \end{cases}$$

The scalar part of the action (1) can be written as

$$\int_{\partial\Sigma} \sqrt{g}\, e^{-\phi}\lambda_0 R^2\left(\mathfrak{H}^4 - v_0^4\right),$$

decreasing around $\mathfrak{H} = 0$ for $\lambda_0 < 0 \Longrightarrow$ the non trivial solution is the minimum of the action.

We choose $a = 2$. Indeed this is the lowest order polynomial behavior for which the induced potential is binding enough for the fermionic solutions to be normalizable.

Fermionic Equations of motion

$$\Psi^i_L \equiv \Psi_L \qquad \Psi^a_R \equiv \Psi_R\,, \qquad \lambda_{1,2} \equiv \lambda$$

$$\mathrm{i} e^M_A \gamma^A \mathcal{D}^R_M \Psi_L - \frac{1}{2} e^5_A \gamma^A \phi' \Psi_L + M\,\Psi_L + \lambda\,\mathfrak{H}(z)\,\Psi_R\,,$$

$$\mathrm{i} e^M_A \gamma^A \mathcal{D}^R_M \Psi_R - \frac{1}{2} e^5_A \gamma^A \phi' \Psi_R + M\,\Psi_R + \lambda\,\mathfrak{H}(z)\,\Psi_L = 0\,,$$

$$\left(\mathbf{1} \pm \gamma^5\right) \Psi_{L,R}\Big|_{z_0} = 0\,.$$

Smallest (Dirac) Eigenmass

$$m^2 R^2 \approx 2\,\frac{1}{\Gamma\left(|\alpha| - \frac{1}{2}\right)}\;(\zeta_0)^{|\alpha| + \frac{1}{2}}\;,$$

$$\alpha = M\,R\,, \qquad \zeta_0 = \frac{\lambda}{\sqrt{R}}\sqrt{\frac{1}{\lambda_0} + v_0^2 R^3}\,,$$

for $\zeta_0 \ll 1$ and $|\alpha| > 1/2$.

Higgs Wave Functions

$$H(x,z) = \mathfrak{H}(z) + \tilde{H}(x,z)\,,$$

$$\Box\tilde{H} - \partial_5^2\tilde{H} - \left(3\frac{f'}{f} - \phi'\right)\partial_5\tilde{H} + f^2\left(\frac{a\,(a-4)}{R^2} - 2a\frac{\mu^2}{R^2}\,z^2\right)\tilde{H} = 0\,,$$

$$\partial_5\tilde{H} - 2\lambda_0 k^2\left[\left(\left|\mathfrak{H} + \tilde{H}\right|^2 - v_0^2\right)\tilde{H} + \left|\tilde{H}\right|^2\mathfrak{H} + \left(\tilde{H} + \tilde{H}^*\right)\mathfrak{H}\right]_{z_0} = 0\,,$$

$$\Box\tilde{H} = -{m_H}^2\tilde{H}\,.$$

Smallest Eigenmass (Higgs Mass)

$${m_H}^2 \approx 2\mu^2\frac{1}{|\ln(\mu R)|}\,.$$

Then we have to take $\mu \sim$TeV.

Smallest Fermion Mass vs Effective Yukawa Couplings

We can estimate the order of magnitude of the lightest fermionic masses as

$$(m_{\mathfrak{f}} R)^2 \sim \frac{2}{\Gamma\left(|\alpha_{\mathfrak{f}}| - \frac{1}{2}\right)} \left[\sqrt{\frac{|\alpha_{\mathfrak{f}}| - \frac{3}{2}}{|\alpha_{\mathfrak{f}}| - \frac{1}{2}}} \frac{y_{\mathfrak{f}\,4d}}{2\left|\ln\left(\mu R\right)\right|} \right]^{\left|\alpha_{\mathfrak{f}}\right| + \frac{1}{2}} .$$

$\mathfrak{f}$ represents the fermion flavor.

$y_{\mathfrak{f}\,4d}$ is the effective 4d Yukawa coupling.

$R^{-1} \sim 10^{19}$GeV

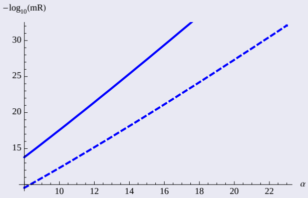

Figure: $y_{4d} \lesssim 1$ *(dashed line)*, $y_{4d} \sim 0.1$ *(solid line).*

$$\begin{cases} y_{4d} \lesssim 1 \begin{cases} m_t R \sim 10^{-17}, & |\alpha_t| \simeq 13 \\ m_\nu R \sim 10^{-28}, & |\alpha_\nu| \simeq 20 \end{cases} \\ y_{4d} \sim 0.1 \begin{cases} |\alpha_t| \simeq 10 \\ |\alpha_\nu| \simeq 15 \end{cases} \end{cases}$$

$R^{-1} \sim 10^4$GeV

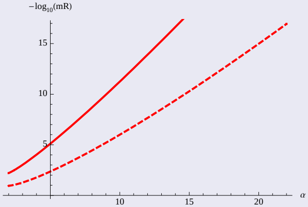

Figure: $y_{4d} \lesssim 1$ *(dashed line)*, $y_{4d} \sim 0.1$ *(solid line)*.

$$\begin{cases} y_{4d} \lesssim 1 \begin{cases} m_t R \sim 10^{-2}, & |\alpha_t| \simeq 5 \\ m_\nu R \sim 10^{-13}, & |\alpha_\nu| \simeq 18 \end{cases} \\ y_{4d} \sim 0.1 \begin{cases} |\alpha_t| \simeq 2 \\ |\alpha_\nu| \simeq 12 \end{cases} \end{cases}$$

- We have worked out a soft wall model for ElectroWeak physics where all the matter and gauge content propagate in the 5d bulk.

- By computing the effective 4d Yukawa couplings we find that the fermion physical masses depend as a power law of the former, the exponent being the corresponding 5d bulk masses.

- This non universal power law behavior allows us to reproduce the hierarchy of the Standard Model fermion masses (from top quark to the neutrinos) with a non hierarchical 5d bulk masses.

- A more realistic study including the CKM matrices and the ElectroWeak constraints, although their inclusion would not substantially change our results, as well as the underlying gravity model inducing the dilaton VEV and its stability should be address.