Kaluza-Klein Masses and Couplings: Radiative Corrections to Tree-Level Relations

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Motivation

Large extra dimensions are an exciting possibility for physics beyond the Standard Model. They are phenomenologically viable, and they lead to interesting predictions.

At a collider, the most direct experimental signature of extra dimensions would be towers of Kaluza-Klein (KK) modes.

For simple compactifications, tree-level relations for KK masses and couplings are straightforward to calculate. However, these are subject to radiative corrections. The renormalized KK parameters are what would be observed at a collider such as the LHC or ILC.

A number of studies have examined radiative effects due to excited modes acting on zero modes. But few have examined effects of excited modes acting on themselves.

Deformations of KK Spectra

Suppose we compactify a single, flat extra dimension on a circle of radius R. Then the KK masses and couplings at *tree level* take the form:

$$m_n^2 = m^2 + \frac{n^2}{R^2}$$

$$\lambda_{n,n',\dots} = \lambda$$

How are these relations deformed under radiative corrections?

Possible Outcomes of Mass Renormalizations

1. The squared masses m_n^2 may receive corrections which are independent of mode number. In this case, the dispersion relation $m_n^2 = n^2/R^2 + m^2$ is stable under radiative corrections. In this case, the corrections may be absorbed into the bare (zero mode) mass term m^2.

2. The squared masses may receive corrections which are proportional to n^2. In this case, the corrections may be absorbed into an effective value of $1/R^2$. In this case, the *apparent* geometry of the extra dimension is renormalized.

3. The squared masses may receive corrections which have a nontrivial dependence on n. Such corrections cannot be absorbed into the bare mass or the radius. The *apparent* geometry of the extra dimension is "broken", and the experimental signature of the dimension is altered.

Renormalization of KK theories is surprisingly challenging.

- Higher dimensional Lorentz invariance is a local symmetry of the Lagrangian. However, the compactification breaks the symmetry globally. What are the appropriate symmetries to preserve in a calculation (4D or 5D?), and how are they preserved? What about gauge invariance?

- Higher dimensional theories are non-renormalizable. How do we make finite, regulator-independent predictions?

Before one can calculate radiative corrections, one needs a regulator of UV divergences which preserves all relevant symmetries. Previously, such a regulator did not exist for general calculations in KK theories.

We developed two new regulators specifically for KK theories. We have used them to determine radiative effects on excited modes.

Criteria for a Good Regulator in a KK Theory

A bad regulator will introduce unphysical artifacts.

A good regulator must control divergences, while introducing no artificial violations of higher dimensional symmetries.

- *e.g.,* higher dimensional Lorentz invariance
- *e.g.,* higher dimensional gauge invariance

Why Preserve Higher Dimensional Symmetries?

After all, the compactification breaks higher dimensional Lorentz invariance.

Key point: But this is a global breaking (*i.e.,* at long distances).

However, the regulator controls effects in the UV (*i.e.,* at short distances), where the higher dimensional symmetries are unbroken.

Regularization artifacts must not get blended with physical effects of compactification.

We developed regulators specifically to respect higher dimensional symmetries in KK theories, thus eliminating artifacts and avoiding such blending.

The Extended Hard Cutoff (EHC)

Extended Dimensional Regularization (EDR)

We used these regulators to calculate deformations of the spectra of masses and couplings of KK modes.

We considered certain toy models. . . .

Toy Models: φ^4 Theory and Yukawa Theory on an Extra Dimension Compactified to a Circle

Found cases in which the tree-level mass spectrum $m_n^2 = n^2/R^2 + m^2$ is broken under radiative corrections. But also found cases in which the mode-number dependence is stable.

Splittings between couplings for different KK modes are generated, even though they are uniform at tree level.

A γ^5-interaction is generated in Yukawa theory. This does not violate parity.

Lifetimes of the KK scalars in Yukawa theory increase with mode number.

5D φ^4 Theory
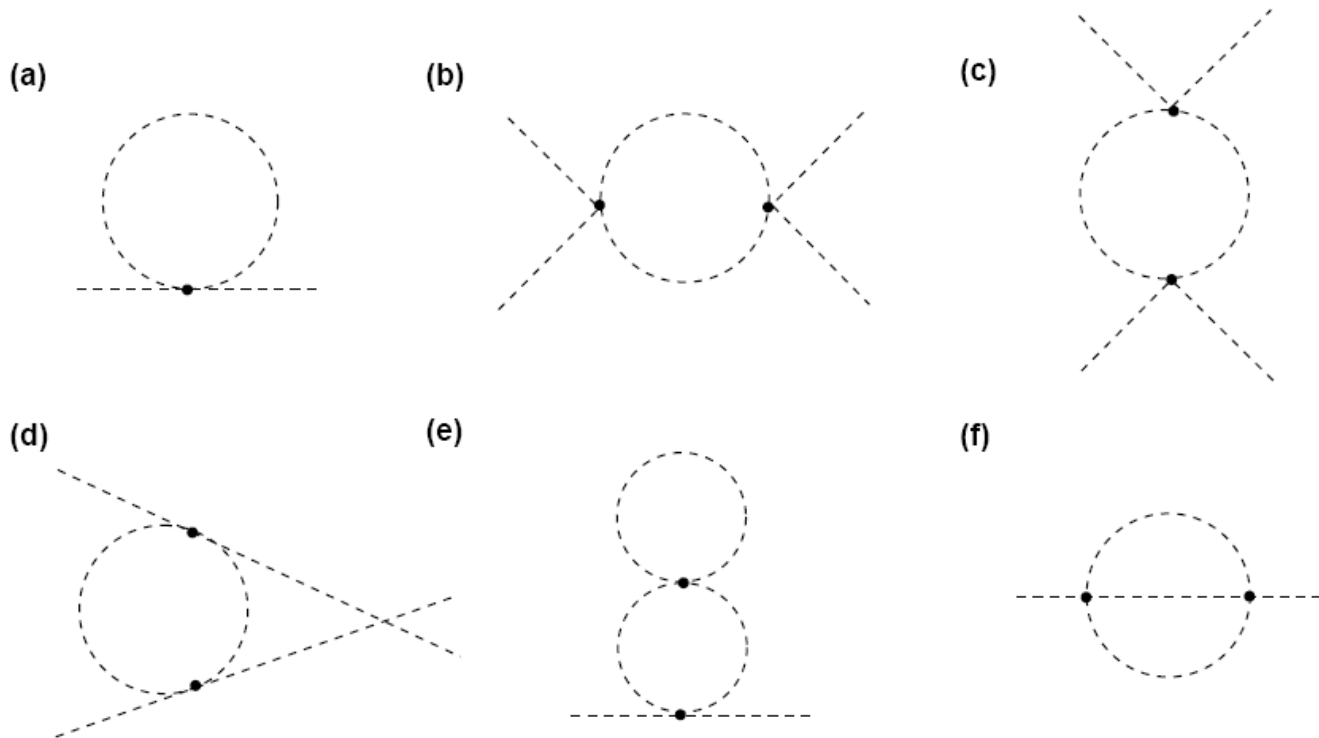
One-loop mass corrections are the same for all mode numbers, *i.e.*, the tree-level dispersion relation is stable. The couplings split nontrivially, however.

Coupling Corrections

Radiative corrections induce coupling splittings.

$$\lambda_{n,n',n'',n'''} = \delta_{n+n'-n''-n''',0} \left[\lambda + \Delta(\lambda_{n,n',n'',n'''} - \lambda_{0,0,0,0}) \right]$$

↑ ↑ ↑

Renormalized Coupling Between the Modes n, n', n'' and n'''

Renormalized Zero-Mode Coupling

Correction to the Difference Between $\lambda_{n,n',n'',n'''}$ and $\lambda_{0,0,0,0}$

Corrections to Coupling Differences
(Finite, Regulator-Independent Sums)

$$\Delta(\lambda_{n,n',n'',n'''} - \lambda_{0,0,0,0}) = \frac{\lambda^2}{4\pi} \left[\xi_{n+n'}(s) + \xi_{n-n''}(t) + \xi_{n-n'''}(u) \right] ,$$

Zero-Mode Coupling ↑   where

$$\xi_n(s) = \sum_{r=-\infty}^{\infty} \frac{1}{|n|} \sum_{j=0}^{|n|-1} \int_0^1 dv \left[\alpha_n^{(\xi)}(r,v,j;s) - \alpha_0^{(\xi)}(r,v;s) \right] ,$$

$$\alpha_n^{(\xi)} = \frac{1}{4\pi} \log(\rho^2 + \mathcal{M}^2(y;s)R^2) , \qquad \rho = r - v \text{ for } n \neq 0,$$

$$\mathcal{M}^2(y;s) = y(y-1)s + m_\phi^2 \quad \text{and} \quad y = \frac{v+j}{|n|} \qquad \text{for } n \neq 0.$$

Leads to Small Enhanced Productions of KK Modes at Colliders

$s = \mu^2 + 4m_\varphi^2,$

$t = u = -\mu^2/2$
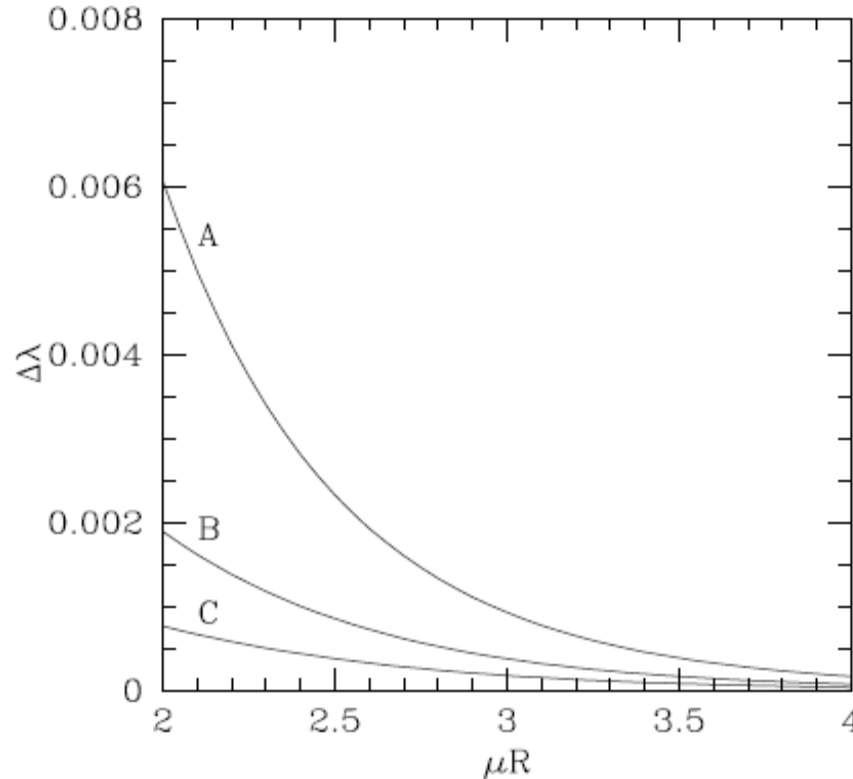
Curve A: $m_\varphi^2 R^2 = 0$

Curve B: $m_\varphi^2 R^2 = 0.25$

Curve C: $m_\varphi^2 R^2 = 0.5$

$\Delta\lambda = \Delta(\lambda_{0,0,1,-1} - \lambda_{0,0,0,0})/\chi_\lambda,$

$\chi_\lambda = \lambda^2/(4\pi).$

$\Delta\lambda$ is plotted against the energy scale of the experiment, μ.

5D Yukawa Theory

$$S = \int d^4x \left[\tfrac{1}{2} \sum_n \partial_\mu \phi_n^* \partial^\mu \phi_n + \sum_n \overline{\psi}_n i\gamma^\mu \partial_\mu \psi_n - \tfrac{1}{2} \sum_n m_{\phi n}^2 \phi_n^* \phi_n \right.$$

$$\left. - \sum_n \overline{\psi}_n M_n \psi_n - \sum_r \sum_{r'} \phi_{r'-r} \, \overline{\psi}_{r'} \, \hat{g}_{r,r'} \, \psi_r + \cdots \right],$$

where $\quad M_n = m_{\psi n}^{(D)} - i m_{\psi n}^{(A)} \gamma^5$

<div style="color:red; text-align:center">

↑   ↑

Dirac   Axial

Component   Component

↓   ↓

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$$\hat{g}_{r,r'} = g_{r,r'} + i \, g_{r,r'}^{(A)} \gamma^5$$

Tree-Level Relations

$$m_n^2 = m^2 + \frac{n^2}{R^2} \, ,$$

$$m_{\psi n}^{(D)} \;=\; m_\psi \, , \qquad m_{\psi n}^{(A)} \;=\; n/R \, ,$$

$$g_{r,r'} \;=\; g \;\equiv\; \frac{G}{\sqrt{2\pi R}} \, ,$$

and
$$g_{r,r'}^{(A)} \;=\; 0 \, .$$

Radiative Corrections: Results

- Distort the tree-level dispersion relations for m_n^2, $m_{\psi n}^{(D)}$ and $m_{\psi n}^{(A)}$. Renormalized squared masses cannot be written in the form

  $n^2/R^2 + m^2$ (Case 3).

- Distortions to mass spectra only occur when there is a nonzero bare mass.

- Induce splittings between the couplings.

- Induce a non-zero value for the γ^5-interaction $g^{(A)}$. *Nevertheless,* higher dimensional parity is preserved, because the $g^{(A)}$-terms are odd with respect to mode number.

Boson Mass Corrections

$$m_{\phi n}^2 = \frac{n^2}{R^2} + m_{\phi 0}^2 + \frac{1}{R^2}\Delta\left[m_{\phi n}^2 R^2 - m_{\phi 0}^2 R^2\right]$$

↑

Renormalized Squared Mass of the n'th Excited Mode

↑

Renormalized Squared Mass of the Zero Mode

↑

Correction to the Difference Between $m_n^2 R^2$ and $m_0^2 R^2$

Corrections to Squared-Mass Differences

$$\Delta \left[m_{\phi n}^2 R^2 - m_{\phi 0}^2 R^2 \right] = \frac{g^2}{4\pi} \sum_{r=-\infty}^{\infty} \frac{1}{|n|} \sum_{j=0}^{|n|-1} \int_0^1 dv \left[\alpha_n^{(\phi)}(r,v,j) - \alpha_0^{(\phi)}(r,v) \right] ,$$

$\uparrow$

Zero-Mode Coupling

where

$$\alpha_n^{(\phi)}(r,v,j) = \frac{1}{\pi}[(\rho^2 + (1-2y)|n|\rho)\log(\rho^2)$$
$$-(\rho^2 + (1-2y)|n|\rho + 3\mathcal{M}_\phi^2(y;m_\phi^2)R^2)\log(\rho^2 + \mathcal{M}_\phi^2(y;m_\phi^2)R^2)] ,$$

and
$$\mathcal{M}_\phi^2(y;\mu^2) = m_\psi^2 + y(y-1)\mu^2 .$$
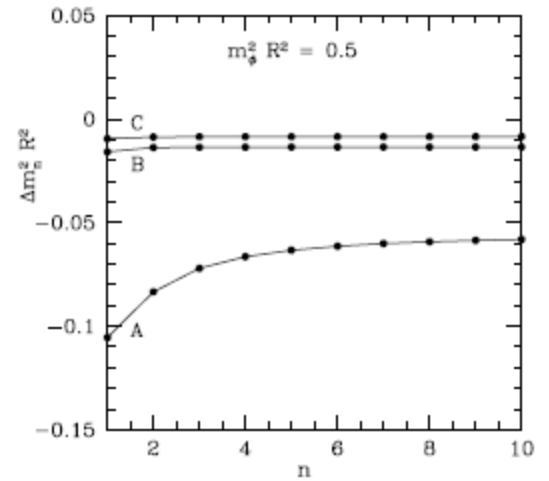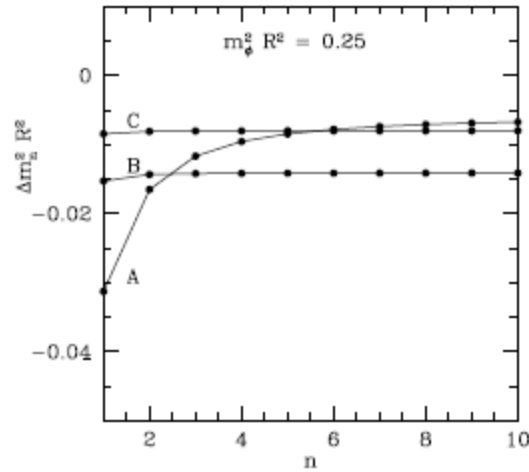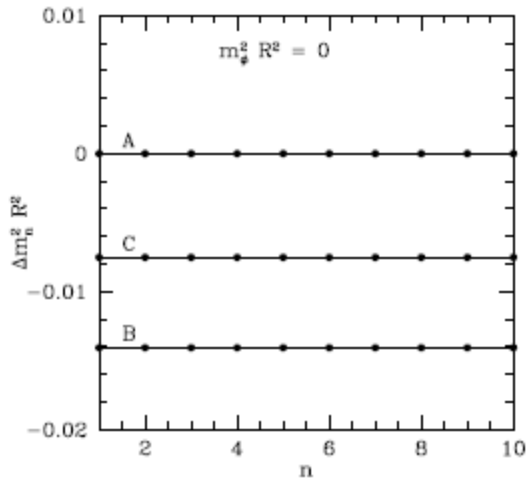
$$\Delta m_n^2 R^2 = \Delta(m_n^2 R^2 - m_0^2 R^2)/\chi_g,$$

$$\chi_g = g^2/(4\pi).$$

When $m_\varphi = 0$, the KK spectrum deforms by a constant splitting.

When $m_\varphi \neq 0$, the spectrum deforms via a function of mode number. Nontrivial dependence on the bare masses.
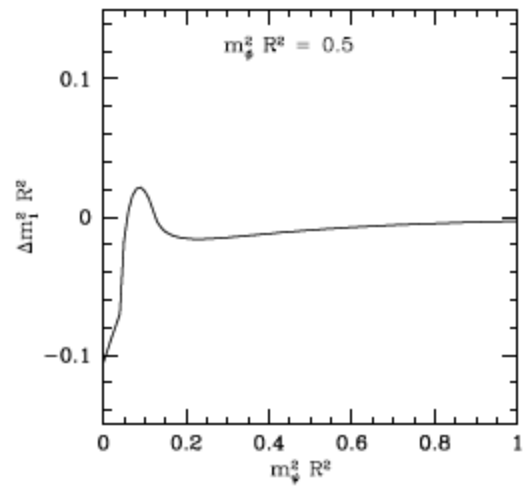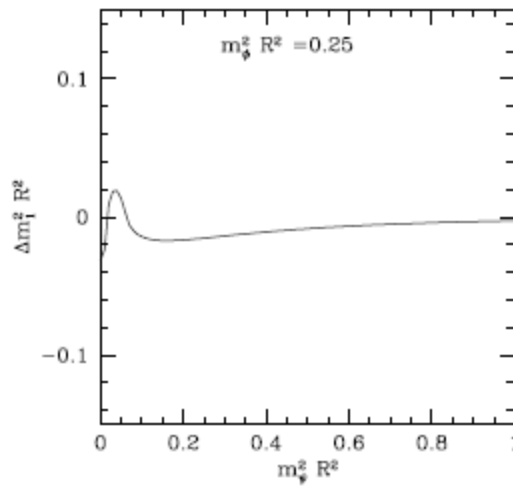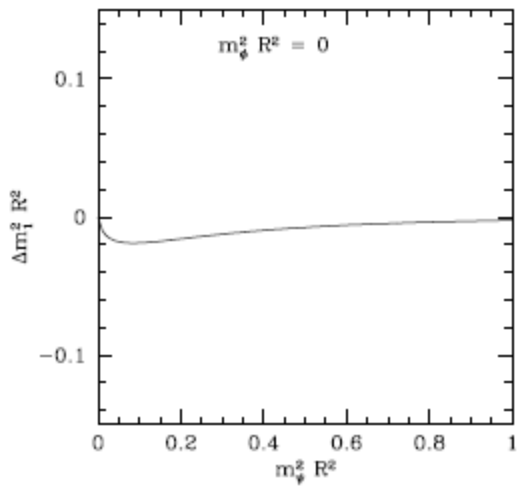
Non-monotonic behavior due to competition between the corrections to m_1^2 and m_0^2.

Kink at decay threshold.

Fermion Mass Corrections

Dirac Mass:

$$m_{\psi n}^{(D)} = m_{\psi 0}^{(D)} + \frac{1}{R}\Delta\left[m_{\psi n}^{(D)}R - m_{\psi 0}^{(D)}R\right]$$

Axial Mass:

$$m_{\psi n}^{(A)} = \frac{n}{R} + \frac{1}{R}\Delta(m_{\psi n}^{(A)}R)$$

Dirac Component

$$\Delta \left[m_{\psi n}^{(D)} R - m_{\psi 0}^{(D)} R \right] = \frac{g^2}{4\pi} \sum_{r=-\infty}^{\infty} \frac{1}{|n|} \sum_{j=0}^{|n|-1} \int_0^1 dv \left[\alpha_n^{(\psi D)}(r, v, j) - \alpha_0^{(\psi D)}(r, v) \right],$$

where

$$\alpha_n^{(\psi D)} = \frac{m_\psi R}{4\pi} (1 + y) \log(\rho^2 + \mathcal{M}_\psi^2(y) R^2),$$

and

$$\mathcal{M}_\psi^2(y) = (y - 1)^2 m_\psi^2 + y m_\phi^2.$$

Axial Component

$$\Delta(m^{(A)}_{\psi n} R) = \frac{g^2}{4\pi} \sum_{r=-\infty}^{\infty} \frac{1}{|n|} \sum_{j=0}^{|n|-1} \int_0^1 dv \, \alpha^{(\psi A)}_n(r, v, j) \,,$$

where $\qquad \alpha^{(\psi A)}_n = \frac{sign(n)}{4\pi} \rho \left[\log(\rho^2 + \mathcal{M}^2_\psi(y) R^2) - \log(\rho^2) \right] \,.$
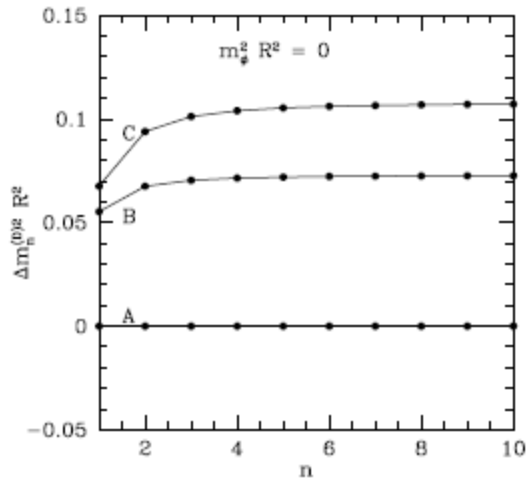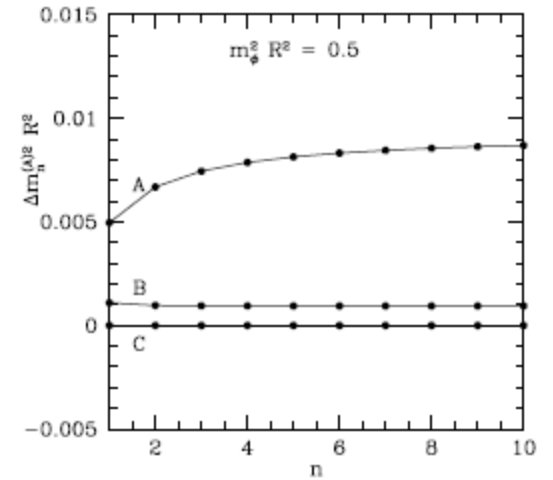
Net Corrections to Squared Masses
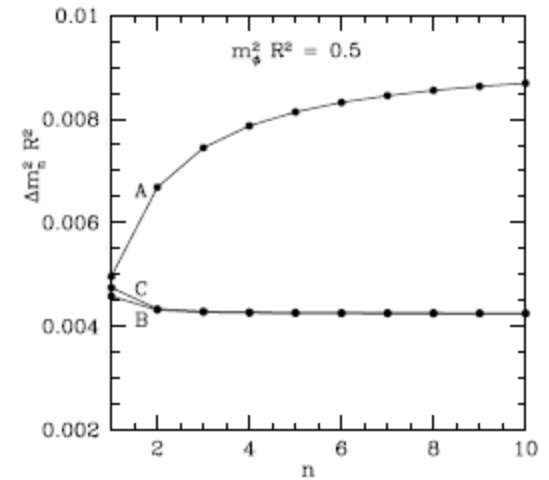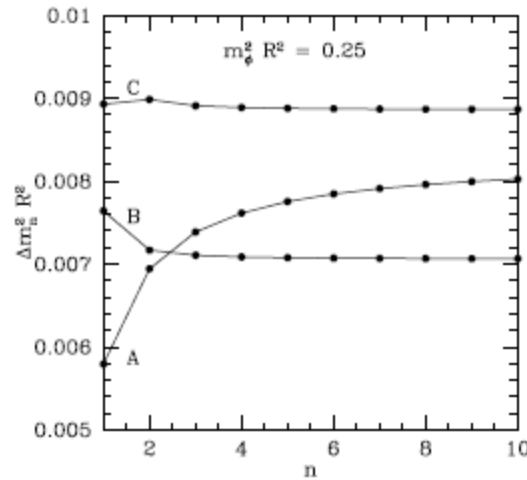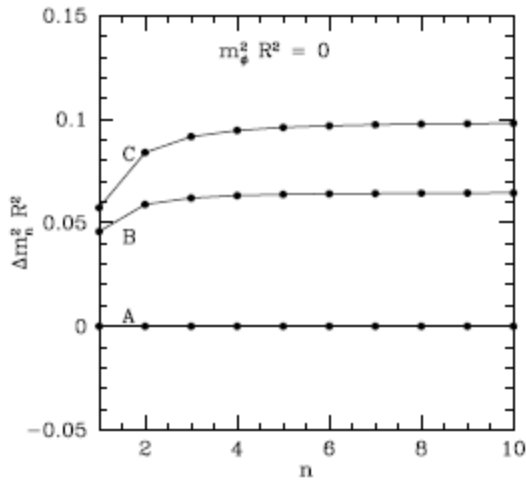
The KK spectrum is deformed.
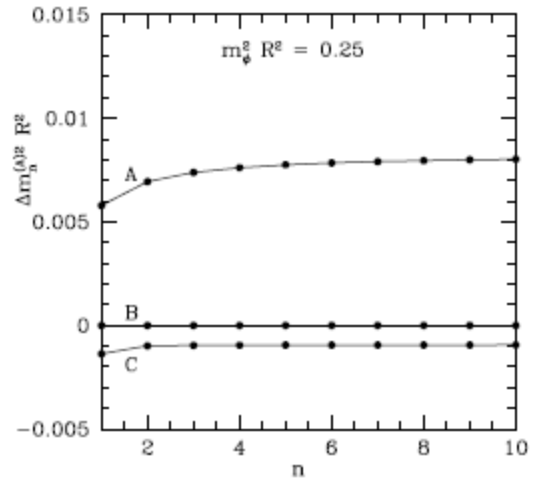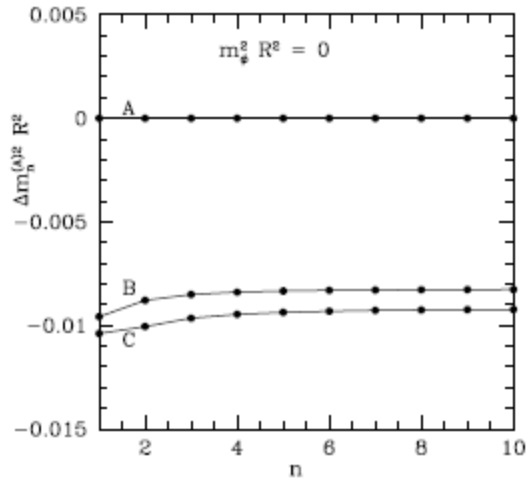
Competition between corrections to $m^{(D)}_n{}^2$ and $m^{(A)}_n{}^2$ leads to non-trivial dependencies on the bare masses.
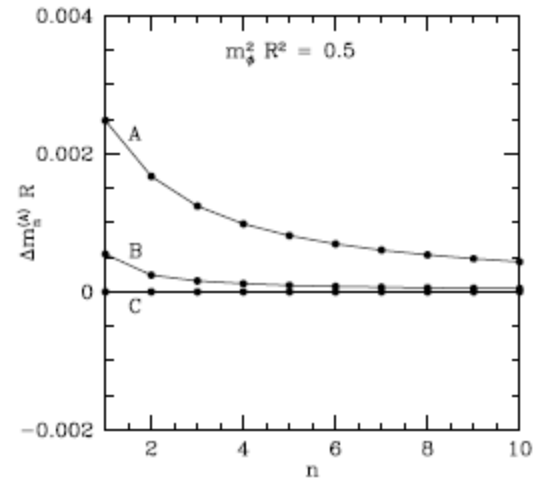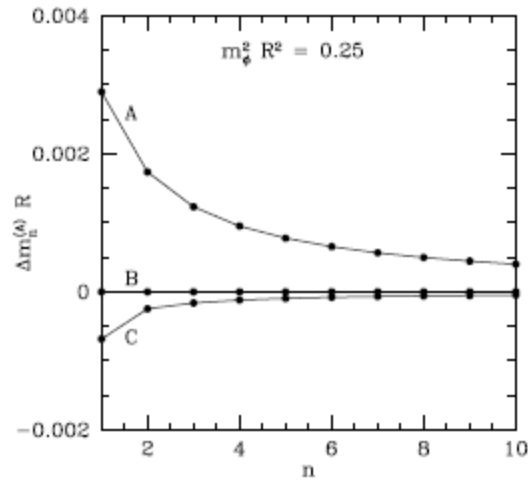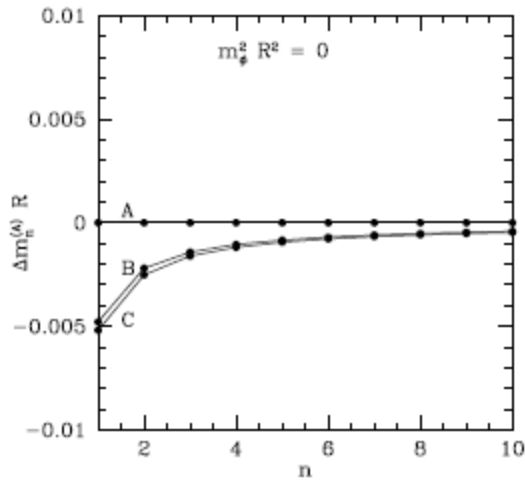
Corrections to $m^{(D)}_n{}^2$
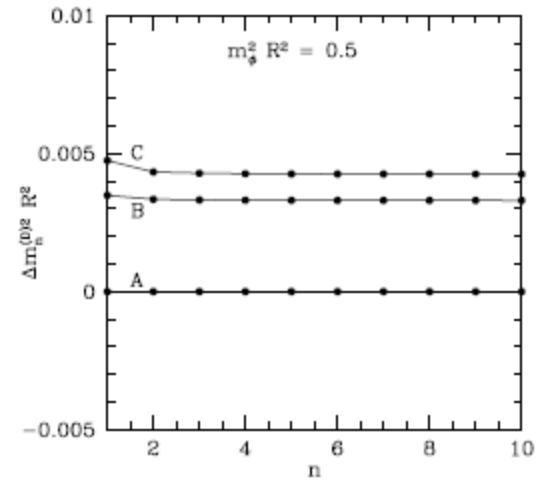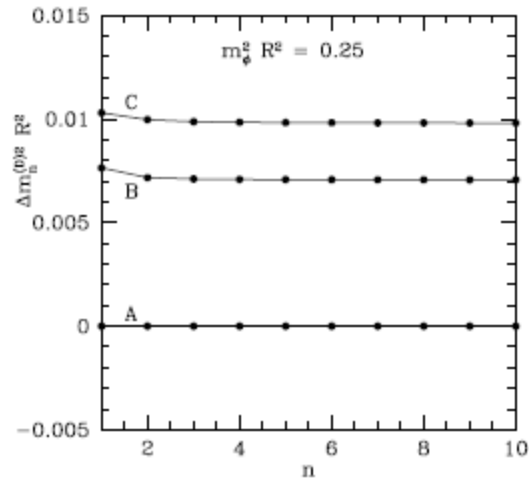
Corrections increase with m_ψ.

Corrections to m$^{(A)}_n{}^2$

Corrections decrease with m_ψ.

Corrections to $m^{(A)}_n$

The axial mass corrections are identically zero when m_ψ and m_φ are equal.
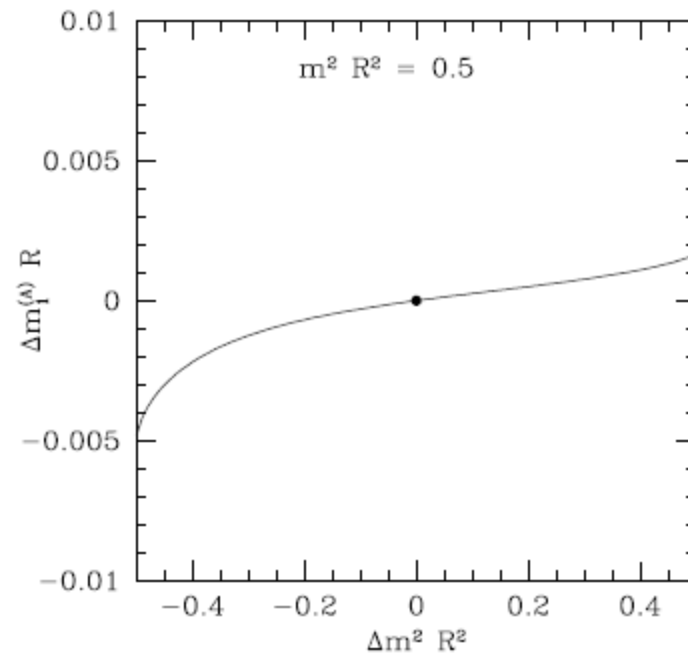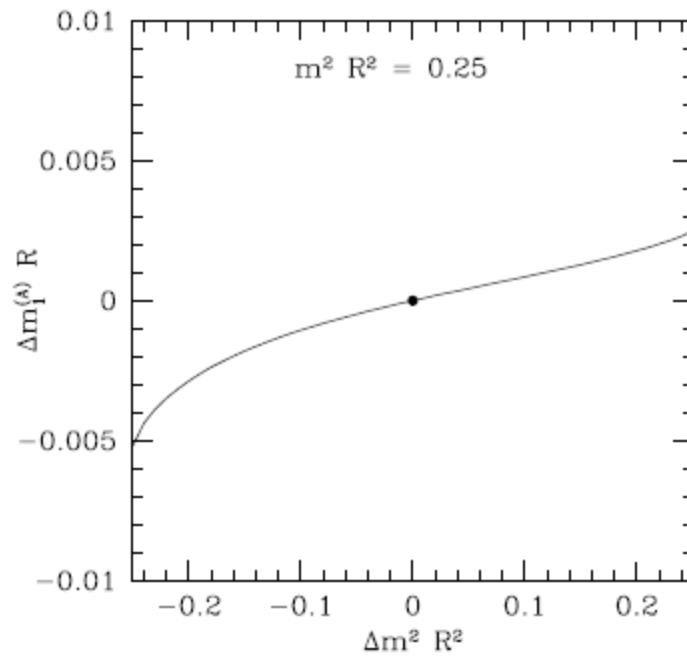
$$m_\psi^2 = m^2 - \Delta m^2, \qquad m_\varphi^2 = m^2 + \Delta m^2$$

The axial mass correction vanishes when m_ψ and m_φ are equal!

Related to SUSY?

Decays

The calculations which yield mass shifts for the scalar modes also yield decay rates. The lifetimes of these particles actually increase with KK mode number!

Why?

This is a manifestation of time dilation. The larger the KK mode number of a particle, the greater its momentum is along the extra dimension.

From the 4D EFT point of view, the decay products of a KK state are restricted by mode-number conservation. A very heavy KK mode cannot decay into a pair of light particles. This allows lifetimes to be long.

Moreover, a very heavy KK mode can only decay

- into very many light modes (phase space suppression), or

- into small numbers of heavy states, which must sequentially decay into lighter states

Long lifetimes for heavy KK states are natural.

Conclusions

- If large extra dimensions exist, then we must understand the phenomenology of such dimensions.

- The existence of Kaluza-Klein towers is the most direct experimental signature of extra dimensions.

- Radiative corrections could alter the phenomenology of extra dimensions. However, effects on excited modes have not received much attention.

- We developed regulators which enable calculations of radiative corrections on excited modes. Such effects have not received much attention.

- Calculations performed with our regulators confirm that radiative corrections can indeed lead to nontrivial phenomenology.

New Directions

We currently are applying our methods to high-precision calculations in higher dimensional theories (work in collaboration with Michael Ramsey-Musolf).

A calculation currently under way:

$\Gamma(\pi^+ \to e^+ \nu_e) / \Gamma(\pi^+ \to \mu^+ \nu_\mu)$

This is a highly sensitive probe to new physics. Effects from strong interactions cancel in the ratio.