Can the Higgs Boson be the Inflaton?

Hint: Remember Hinchliffe's rule.

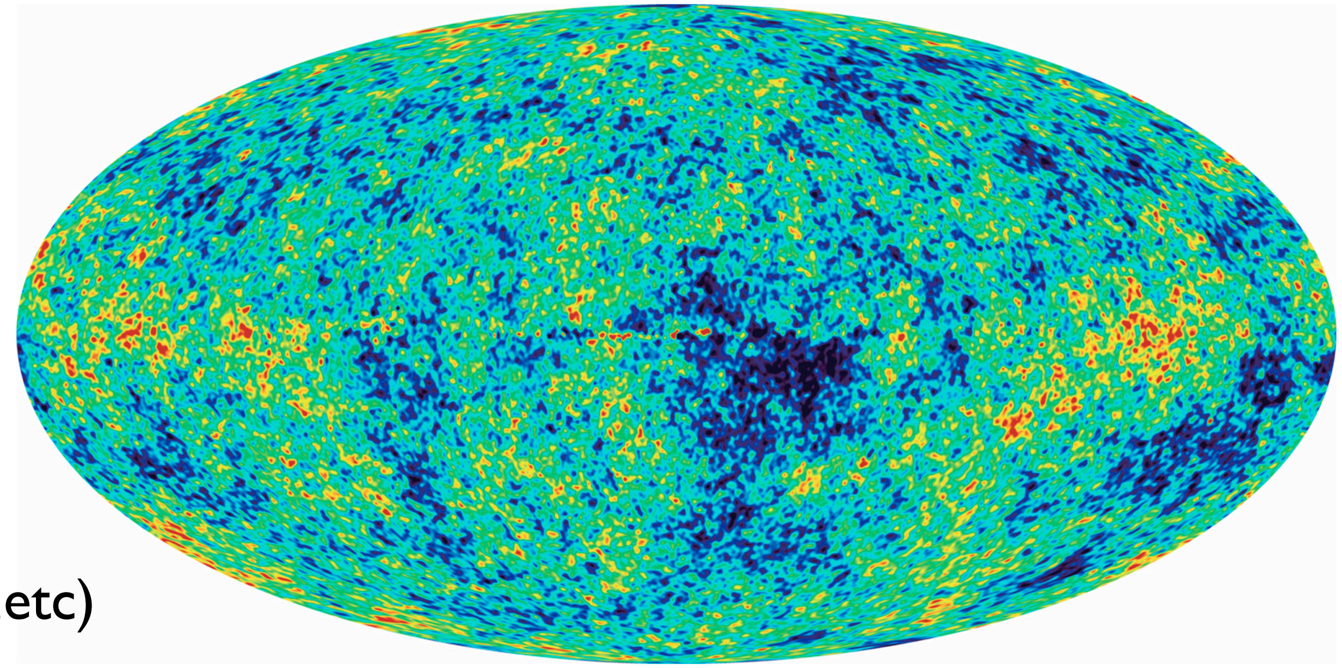
Michael Trott, PI

Based on arXiv:0902.4465 with C.P Burgess and Hyun Min Lee

Inflation basics

We can be somewhat confident that inflation occurred as it is consistent with CMB measurements and it efficiently addresses the:

- flatness,
- homogeneity, isotropy,
- horizon and
- undesired relic problems.
  (string modulii, monopoles, gravitinos,etc)



WMAP 05

Some mechanism constructed to produce a slow roll inflationary scheme, generally with a scalar coupled to gravity.

Recall the slow roll parameters: $\epsilon(\phi) = \frac{M_{pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2$ $\quad \eta(\phi) = M_{pl}^2\left(\frac{V''(\phi)}{V(\phi)}\right)$ $\quad \zeta(\phi) = M_{pl}^4\frac{(V''')(V')}{V^2}$

So long as $|\eta(\phi)| \ll 1$ and $\epsilon(\phi) \ll 1$

Scale factor of the universe accelerating $\frac{\ddot{a}}{a} > 0$ ie Inflation occurs

Also recall the defns: $H \equiv \frac{\dot{a}}{a} = \frac{\sqrt{V(\phi)/3}}{M_{pl}}$ $\quad M_{pl} = (8\pi G)^{-1/2} = 2.4 \times 10^{18}\,\mathrm{GeV}$

General EFT construction

Consider the general Lagrangian of scalers θ^i coupled to the metric:

$$-\frac{L_{eff}}{\sqrt{-g}} = v^4 V(\theta) + \frac{M_p^2}{2}\, g^{\mu\nu}\Big[W(\theta)\, R_{\mu\nu} + G_{ij}(\theta)\, \partial_\mu\theta^i \partial_\nu\theta^j\Big]$$

$$+A(\theta)(\partial\theta)^4 + B(\theta)\, R^2 + C(\theta)\, R\, (\partial\theta)^2 + \frac{E(\theta)}{M^2}\, (\partial\theta)^6 + \frac{F(\theta)}{M^2}\, R^3 + \cdots$$

All possible invariants involving one Riemann tensor and two derivatives acting on θ^i

M that makes up the dimensions is characteristic of whatever underlying microscopic physics has been integrated out, generally $M \ll M_p = (8\,\pi\, G)^{-1/2}$

All possible 3 Riemann tensor invariants, or two Riemann tensors and two covariant derivatives

General EFT construction

Consider the general Lagrangian of scalers θ^i coupled to the metric:

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$$+ A(\theta)(\partial\theta)^4 + B(\theta)\, R^2 + C(\theta)\, R\, (\partial\theta)^2 + \frac{E(\theta)}{M^2} (\partial\theta)^6 + \frac{F(\theta)}{M^2} R^3 + \cdots$$

Expand around a classical background solution

$$\theta^i(x) = \vartheta^i(x) + \frac{\phi^i(x)}{M_p} \quad \text{and} \quad g_{\mu\nu}(x) = \hat{g}_{\mu\nu}(x) + \frac{h_{\mu\nu}(x)}{M_p}\,,$$

The EFT is an expansion in $\frac{R}{M^2} \simeq \left(\frac{H}{M}\right)^2 \qquad \frac{(\partial\,\theta)^2}{M^2} \simeq \left(\frac{\mu_\phi}{M}\right)^2$

$\boxed{\text{LESSON 1: For these theories to have any hope of making sense } \frac{H}{M} \ll 1}$ "Adiabatic constraints"

Power Counting

Power counting is simple for $L_{int} = M^2\, M_{pl}^2 \sum_n \frac{c_n}{M^{d_n}} O_n \left(\frac{\phi}{M_{pl}}, \frac{h_{\mu\nu}}{M_{pl}} \right)$

- Number of loops in a connected graph $L = 1 + I - \sum_n V_n$
- Conservation of ends in a connected graph $2\, I + \xi = \sum_n N_n V_n$
- For a graph with V_n vertices get a factor $M_{pl}^{2-2L-\xi} \prod_n \left[c_n\, M^{2-d_n} \right]^{V_n}$

just topology

For an arbitrary graph regulated with dim reg:

$$A_\xi(E) \simeq M_{pl}^{2-2L-\xi} \prod_n \left[c_n\, M^{2-d_n} \right]^{V_n} \int \cdots \int \left(\frac{d^d p}{(2\,\pi)^d} \right)^L \frac{\prod_n p^{d_n V_n}}{(P^2 - m_i^2)^I}$$

$$\simeq E^2 M^2 \left(\frac{1}{M_{pl}} \right)^\xi \left(\frac{E}{4\pi M_{pl}} \right)^{2L} \prod_{d_n=2} (c_n)^{V_n} \prod_{d_n=0} \left[\lambda_n \left(\frac{v^4}{E^2 M_{pl}^2} \right) \right]^{V_n} \prod_{d_n \geq 4} \left[g_n \left(\frac{E}{M_{pl}} \right)^2 \left(\frac{E}{M} \right)^{d_n - 4} \right]^{V_n}$$

This is a powerfull eqn for studying scalar field inflation models.

Constraints

Power counting formula

$$A_\xi(E) \simeq E^2 M^2 \left(\frac{1}{M_{pl}}\right)^\xi \left(\frac{E}{4\pi M_{pl}}\right)^{2L} \prod_{d_n=2} (c_n)^{V_n} \prod_{d_n=0} \left[\lambda_n \left(\frac{v^4}{E^2 M_{pl}^2}\right)\right]^{V_n} \prod_{d_n \geq 4} \left[g_n \left(\frac{E}{M_{pl}}\right)^2 \left(\frac{E}{M}\right)^{d_n-4}\right]^{V_n}$$

Power counting constraints:

$$\frac{E}{4\pi M_{pl}} \ll 1$$

$$g_n \left(\frac{E}{M_{pl}}\right)^2 \left(\frac{E}{M}\right)^{d_n-4} \ll 1$$

LESSON 2: Power counting constraints for a sane expansion.

A particular model in an EFT will also have a cut off on E that above which unitarity will be violated.

This constraint on E will supply an upper bound on M for unitarity to be preserved.

LESSON 3: Unitarity violation constraints set an upper bound on E and thus on M.

Genealogy of Higgs Inflation

Non-minimal coupling models proposed/developed

Spokoiny Phys Lett B 147B 39 (1984)

Salopek, Bond, Bardeen Phys Rev D 40 1753 (1989)

Fakir, Unruh Phys Rev D 41 1783 (1990)

A second look and updated constraints

Kaiser Phys Rev D 52 4295 (1995) astro-ph/9408044

Komatsu, Futamase Phys Rev D 59 064029 (1999) atro-ph/9901127

Higgs inflation (re)proposed and (re)developed

Bezrukov, Shaposhnikov Phys Lett B 659, 703 (2008) arXiv:0710.3755

Barvinsky, Kamenshchik, Starobinsky JCAP 0811, 021 (2008) arXiv:0809.2104

Garcia-Bellido, Figueroa, Rubio arXiv:0812.4624

De Simone, Hertzberg, Wilczek arXiv:0812.4946

Bezrukov, Shaposhnikov arXiv:0812.4950

Higgs Inflation Basics

How does Higgs inflation work?

1) Minimal coupling case ($\xi = 0$) doesn't work.

$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) - \lambda_H \left(H^\dagger H - \frac{v^2}{2} \right)^2$$

Gauge rotation $H^T = \frac{1}{\sqrt{2}}(0, v + \phi)$

$$\mathcal{L}_\phi = (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}(2\,\lambda_H\, v^2)\,\phi^2 - (\lambda_H\, v)\,\phi^3 - (\frac{\lambda_H}{4})\phi^4$$

As $\langle \phi \rangle \gg v$ during inflation this term is totally dominant

End of inflation is defined as when the slow roll conditions is violated $\epsilon \sim 8\,\frac{M_{pl}^2}{\phi^2} \sim 1$ $\Rightarrow$ $\phi \sim \sqrt{8} M_{pl}$

Interested in the slow roll parameters when cosmologically interesting scales are leaving the horizon. The earlier field value is related to the number of e-folds:

$$N = \frac{1}{M_{pl}^2} \int_{\phi_{end}}^{\phi_0} d\,\phi\, \frac{V}{V'} \simeq 60 \quad \Rightarrow \quad \phi_0 \sim \sqrt{8N} M_{pl}$$

Slow roll conditions and amplitude of density pert for these field values:

$$\epsilon \sim \frac{1}{N} \qquad \eta \sim \frac{3}{2\,N} \qquad \text{and} \qquad \Delta_R^2 = \frac{V}{24\,\pi^2\,M_{pl}^4\,\epsilon} \qquad \text{at } k^\star = 0.002\,\mathrm{Mpc}^{-1}$$

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$\epsilon \sim \frac{1}{N}$ $\quad \eta \sim \frac{3}{2\,N}$ and $\Delta_R^2 = \frac{V}{24\,\pi^2\, M_{pl}^4\,\epsilon}$ at $k^\star = 0.002\,\mathrm{Mpc}^{-1}$

WMAP gives $\Delta_R^2 = (2.445\ \pm 0.096) \times 10^{-9}$ trivially one finds:

$$\frac{2\,\lambda_H\, N^3}{3\,\pi^2} = (2.445 \pm 0.096) \times 10^{-9} \Rightarrow \underline{\lambda_H \sim 10^{-13},\ m_H \sim 10^{-4}\,\mathrm{GeV}}$$

NOT the SM Higgs

Conclusion: VERY FLAT potentials are hard to come by for scalar field inflation models, usually a lot of tuning is involved.

Higgs Inflation Basics

How does Higgs inflation work?

2) Non-Minimal coupling case: the Lagrangian is

$$S = \int d^4 x \sqrt{-g} \left[(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}(2\,\lambda_H\, v^2)\,\phi^2 - (\lambda_H\, v)\,\phi^3 - (\frac{\lambda_H}{4})\phi^4 - \underline{\frac{\xi}{2}\,\phi^2\, R} - \frac{M_{pl}^2}{2} R \right]$$

Not too offensive a term.

To minimize suffering everyone uses the Weyl rescaling to the Einstein frame:

$$\hat{g}_{\mu\,\nu} = \Omega^2 g_{\mu\,\nu} \quad \text{where} \quad \Omega^2 = 1 + \frac{\xi\,\phi^2}{M_{pl}^2} \quad \text{and} \quad V_E(\phi) = \frac{V(\phi)}{(1 + \xi\frac{\phi^2}{M_{pl}^2})^2}$$

The Higgs field has a non-canonical kinetic term so perform a field redefinition:

$$\frac{d\,\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\,\xi^2\,\phi^2/M_{pl}^2}{\Omega^4}} \quad \xrightarrow[\xi \gg 1]{\phi \gg M_{pl}/\sqrt{\xi}} \quad \frac{d\,\chi}{d\phi} = \frac{\sqrt{6}M_{pl}}{\phi}$$

$$\phi \simeq \frac{M_{pl}}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6}\,M_{pl}}\right)$$

The Einstein frame potential in terms of the canonical field makes the exponential flatness manifest:

$$V_E(\chi) \simeq \frac{\lambda\, M_{pl}^4}{\xi^2}\,[1 + A\exp(-a\,\chi/M_{pl})]^{-2}$$
$$\simeq \frac{\lambda\, M_{pl}^4}{\xi^2}\,[1 - 2A\exp(-a\,\chi/M_{pl})]$$

EXPONENTIALLY FLAT
potential “naturally” comes about!

Higgs Inflation Basics

How does Higgs inflation work?

2) Non-Minimal coupling case: the Lagrangian is

$$S = \int d^4x \sqrt{\hat{g}} \left[(\partial_\mu \chi)(\partial^\mu \chi) - \frac{M_{pl}^2}{2} \hat{R} - V_E(\chi) \right]$$ where $$V_E(\chi) \simeq \frac{\lambda_H M_{pl}^4}{\xi^2} [1 - 2A \exp(-a\chi/M_{pl})]$$

The slow roll parameters for can be worked out and one can compare to experiment (WMAP)

Higgs inflation	WMAP05/BAO/SN
$n_s \simeq 0.968$	$n_s = 0.960 \pm 0.013$
$r \simeq 3.0 \times 10^{-3}$	$r < 0.22$
α negligible	$-0.032^{+0.021}_{-0.013}$

Further WMAP05/BAO/SN gives $\Delta_R^2 = (2.445 \pm 0.096) \times 10^{-9}$

and $$\Delta_R^2 = \frac{V}{24\,\pi^2\, M_{pl}^4\, \epsilon} \Rightarrow \xi \simeq 5 \times 10^4 \left(\frac{m_h}{\sqrt{2}\, v} \right)$$

Condition for this to work classically

Higgs Inflation Basics

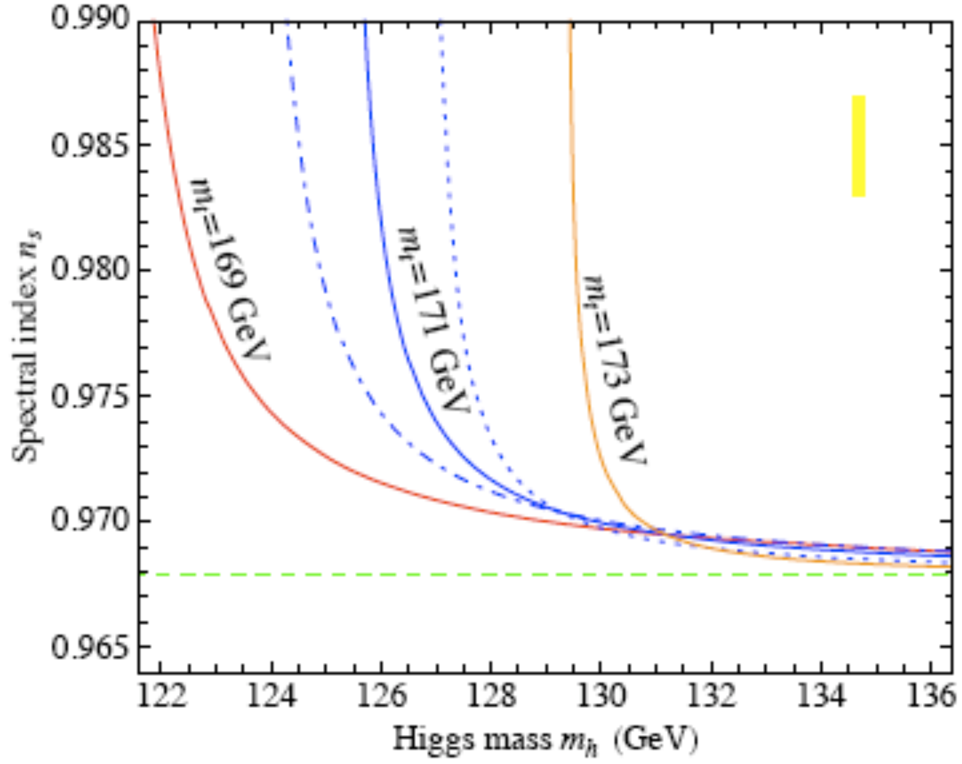
How does Higgs inflation work?

3) Non-Minimal coupling case quantum corrections

Inflation is happening at energies much higher than the SM energies, we should run our parameters.

In Higgs Inflation the running gives a relationship between the following parameters:

Real science, predictions!



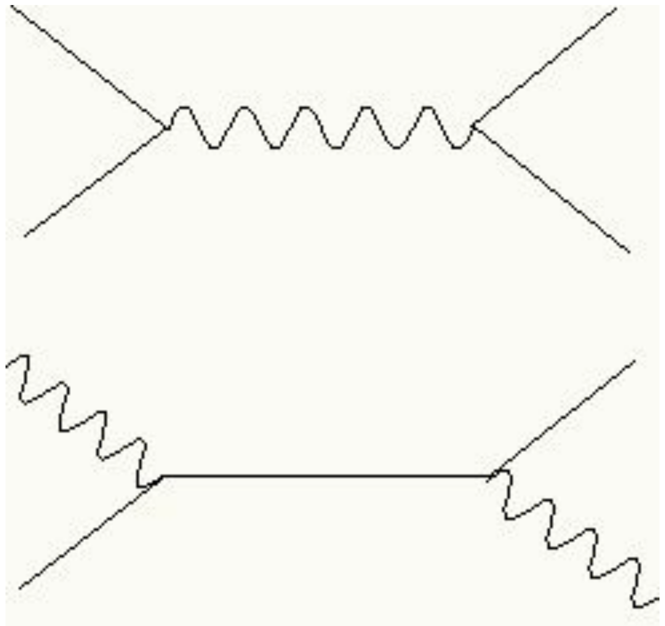
Wilczek,deSimone,Hertzberg
0812.4946

Similar results for running of spectral index and the tensor to scalar ratio.

Power Counting vs Higgs Inflation

For Higgs inflation the Hubble scale is $H \equiv \frac{\dot{a}}{a} = \frac{\sqrt{V(\phi)/3}}{M_{pl}}$ ⇒ $H \simeq \sqrt{\lambda_H}\frac{M_{pl}}{\xi}$

As $\xi \simeq 5 \times 10^4 \left(\frac{m_h}{\sqrt{2}\,v}\right)$ largest dependence on ξ is probably the origin of scattering that violates unitarity.



$h\,h \to h\,h$

$g\,h \to g\,h$

The power counting gives $A_4(E) \simeq \left(\frac{E}{M_{pl}}\right)^2 \left(\frac{E}{4\pi M_{pl}}\right)^{2L} \prod_n \xi^{V_n}$

Also it gives $\sum_n (N_n - 2)V_n = 2 + 2L$

Strongest dependence on ξ comes from multiple 3 field insertions

$$A_4(E) \simeq \left(\frac{\xi E}{M_{pl}}\right)^2 \left(\frac{\xi E}{4\pi M_{pl}}\right)^{2L}$$

Insisting on unitarity ie $\sigma \propto 1/E^2$ we find

Unitarity gives: $E < E_{max} \simeq \frac{M_{pl}}{\xi}$ $\quad M < \frac{M_{pl}}{\xi}$

People got the cut off wrong when they didn't power count!

Power Counting vs Higgs Inflation

So we have a small window of validity of this as an EFT

$$1 \gg \frac{H}{M} \gg \sqrt{\lambda_H}$$ all approximations valid, unitarity + adiabaticity

$$1 \gg \frac{H}{M} \gg 0.02$$ including the 2 loop running to the high scale

It gets worse:

- Higgs inflation relies on $V \propto (H^\dagger H)^2$ and a non minimal coupling $(H^\dagger H)\,R$

  True in Higgs inflation when $H^\dagger H \gg \frac{M_{pl}^2}{\xi} \gg v^2$

  Can't forbid by internal symmetries: $g(H^\dagger H)\,(\chi^\dagger \chi)$

  The window is sensitive to massive particles integrated out.

  $$\frac{g^2(H^\dagger H)}{(4\pi M_\chi)^2} \ll \xi \qquad \frac{g^3(H^\dagger H)}{(4\pi M_\chi)^2} \ll \lambda_H$$

- The cut off is $\frac{M_{pl}}{\xi}$ but the exponentially flat potential was for $\phi \simeq \frac{M_{pl}}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6}\,M_{pl}}\right)$

  Recall $V_E(\chi) \simeq \frac{\lambda_H M_{pl}^4}{\xi^2}\left[1 - 2A\exp(-a\chi/M_{pL}) + \cdots\right]$

  So we don't know the potential for the field values where inflation is said to occur

Conclusions

On the plus side:

- Higgs inflation, or something like it, is very promising in that an exponentially flat potential is somewhat "natural".
- The Higgs inflation proposal (or something like it) has the potential to relate WMAP data to the properties of the scalars measured in accelerators.

On the minus side:

- The large non minimal coupling constants lead to the theories of this type having ***cut offs far below the planck scale and below the slow roll region of the potential***.
- The tuning on the cut off scale can be found by ***power counting which finds the most problematic unitarity constraint***.

$$1 \gg \frac{H}{M} \gg \sqrt{\lambda_H}$$

Higgs inflation region of validity

- Very small tuned windows of validity as an EFT, that require no other new physics, and you don't actually know the potential during inflation.