Threshold resummation for pair production of coloured heavy particles at hadron colliders

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Pair-production of heavy particles at hadron colliders

$$p_i(r_i) + p_j(r_j) \longrightarrow H(R)H^{'}(R') + X \qquad H, H' \equiv t,\ \tilde{q},\ \tilde{g},\ \ldots$$

Partonic cross section for pair-production of heavy particles at hadron colliders contains terms kinematically enhanced in the partonic threshold region $\hat{s} \sim 4\bar{m}^2$, $\bar{m} \equiv (m_H + m_{H'})/2$.

- **Coulomb singularities**: $\sim \alpha_s^n/\beta^n$, $\beta = \sqrt{1 - 4\bar{m}^2/\hat{s}} \Leftrightarrow$ Coulomb interactions of slowly-moving particles
- **Threshold logarithms**: $\sim \alpha_s^n \ln^{2n} \beta^2 \Leftrightarrow$ soft-gluon exchange

Small coupling but effectively “non-perturbative” dynamics $\Rightarrow$ Must be resummed to all orders when the partonic threshold region dominates the total cross section!

- Absolute normalisation of the total cross section
- Generally observed to reduce factorisation-scale dependence

Moment-space VS Momentum-space resummation

The theoretical basis for resummation is the factorisation of hard and soft dynamics in the threshold region (more generally for $Q^2 \sim \hat{s}$, even if $Q^2 \neq 4\bar{m}^2$)

$$\hat{\sigma} = H \otimes S$$

- Resummation traditionally performed in Mellin-moment space:

$$H \otimes S \Rightarrow H(N)S(N) \qquad \alpha_s^n \ln^{2n} \beta \Rightarrow \alpha_s^n \ln^{2n} N$$

- Threshold logs exponentiated by solving evolution equations for $H(N)$ and $S(N)$.
- Requires numerical inversion of the Mellin transform and prescription to deal with Landau poles in the integrand

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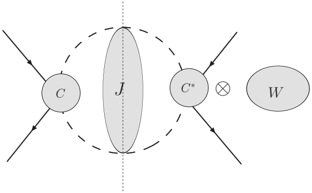
In this talk: apply formalism proposed by [Neubert and Becher '06] to resummation of the total cross section for $p_i p_j \longrightarrow HH' + X$.

- Based on effective-field theory description of the process (SCET+NRQCD)
- Threshold resummation performed directly in momentum space

Cross-section factorisation near threshold

Extra factorisation of the cross section near the true partonic threshold $\hat{s} \sim (m_H + m_{H'})^2$



$$\hat{\sigma}_{pp'}(\hat{s}, \mu) = \frac{1}{2\hat{s}N_{pp'}} \sum_{i,i',\ell,\ell'} \sum_{R_\alpha} C^{(\ell,i)}_{pp'} C^{(\ell',i')*}_{pp'} \times \int d\omega \, J^{(\ell,\ell')}_{R_\alpha}(E - \tfrac{\omega}{2}) \, W^{R_\alpha}_{ii'}(\omega, \mu)$$

- Hard coefficients $C^{(\ell,i)}_{pp'}$ encoding the short-distance structure of the production process
- Process-independent soft function $W^{R_\alpha}_{ii'}$ (expectation value of soft Wilson lines)

$$W^{R_\alpha}_{ii'}(\omega, \mu) = P^{R_\alpha}_{\{k\}} c^{(i)}_{\{a\}} c^{(i')*}_{\{b\}} \int \frac{dz_0}{4\pi} e^{i\omega z_0/2} \langle 0|\overline{\mathrm{T}}[S^\dagger_{n,ib_1} S^\dagger_{\bar{n},jb_2} S_{v,b_4,k_4} S_{v,b_3,k_3}](z)\mathrm{T}[S_{\bar{n},a_2j} S_{n,a_1i} S^\dagger_{v,k_1a_3} S^\dagger_{v,k_2a_4}](0)|0\rangle$$

- Potential function $J^{(\ell,\ell')}_{R_\alpha}$ encoding Coulomb interactions

Contrary to the conventional approach there is a set of soft functions $W^{R_\alpha}_{ii'}$!
(corresponding to irreducible representations of $R \otimes R' = \sum_\alpha R_\alpha$)

Resummation of threshold logarithms

Factorisation-scale independence of the total cross section translates into evolution equation for the soft function $W_{ij}^{R_\alpha}$

$$\frac{d}{d\ln\mu}\mathbf{W}^{R_\alpha}(\omega,\mu) = -\int_0^\omega d\omega\left(\frac{1}{\omega-\omega'}\right)_{[\mu]}\left[\Gamma_{R_\alpha}\mathbf{W}^{R_\alpha}(\omega',\mu)+\mathbf{W}^{R_\alpha}(\omega',\mu)\Gamma^\dagger_{R_\alpha}\right] -\gamma^S_{R_\alpha}\mathbf{W}^{R_\alpha}(\omega,\mu)-\mathbf{W}^{R_\alpha}(\omega,\mu)\gamma^{S\dagger}_{R_\alpha}$$

- Γ_{R_α} controls resummation of double logs, γ^{S,R_α} resums single logs.
- $\mathbf{W}^{R_\alpha}$ is matrix in colour space $\Rightarrow$ in general mixing of different colour structures. With suitable choice of colour basis can be diagonalised to all orders in α_s (at least for cases of phenomenological interest at Tevatron/LHC: $t\bar{t}$, squarks, gluinos, etc...)

Resummed expressions for $W_{ii'}^{R_\alpha}$ and $J_{R_\alpha}^{(\ell,\ell')}$

- Threshold logarithms are resummed by renormalising $\mathbf{W}^{R_\alpha}$ at a soft scale μ_s and evolving it to the common factorisation scale μ.

$$W_{ii}^{R_\alpha,\mathrm{res}}(\omega,\mu) = \exp[-4S_i^{R_\alpha}(\mu_s,\mu) + 2a_i^{S,R_\alpha}(\mu_s,\mu)] \times \tilde{s}_{ii}^{R_\alpha}(\partial_\eta,\mu_s)\left(\frac{\omega}{\mu_s}\right)^{2\eta}\frac{\theta(\omega)}{\omega}\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

μ_s **must be chosen such that the fixed-order perturbative expansions of** $W_{ii}^{R_\alpha}(\omega,\mu_s)$ **is well behaved.**

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- Resummation of Coulomb corrections is well known from quarkonia physics. $J_{R_\alpha}^{(\ell,\ell')}$ related to zero-distance Green function of $-\vec{\nabla}^2/(2m_{\text{red}}) - \alpha_s(-C_{R_\alpha})/r$:

$$J_{R_\alpha}^{(\ell,\ell')}(E) \propto -\frac{(2m_{\text{red}})^2}{4\pi}\text{Im}\left\{\sqrt{-\frac{E}{2m_{\text{red}}}} + \alpha_s(-C_{R_\alpha})\left[\frac{1}{2}\ln\left(-\frac{8\,m_{\text{red}}E}{\mu^2}\right) - \frac{1}{2} + \gamma_E + \psi\left(1 - \frac{\alpha_s(-C_{R_\alpha})}{2\sqrt{-E/(2m_{\text{red}})}}\right)\right]\right\}$$

Squark-antisquark production at the LHC

In the rest of this talk:

$$PP \longrightarrow \tilde{q}\bar{\tilde{q}} + X$$

Perform NLL resummation of soft-gluon corrections + Coulomb singularities:

- Two-loop cusp anomalous dimension Γ and QCD β-function
- One-loop soft anomalous dimension γ^S
- Tree-level fixed-order soft functions $W_{ii'}^{R_\alpha}$

The effective-theory resummed cross section is matched onto the full NLO result [Zerwas et al., '96]

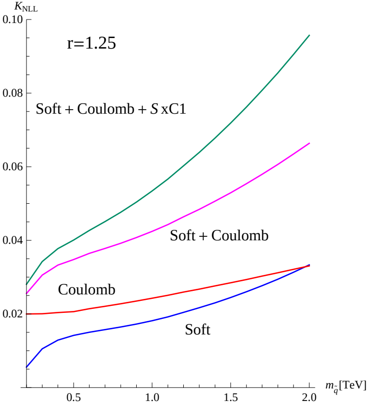
$$\hat{\sigma}_{pp'}^{\text{match}}(\hat{s},\mu_f) = \left[\hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s},\mu_f) - \hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s},\mu_f)|_{\text{NLO}}\right] + \hat{\sigma}_{pp'}^{\text{NLO}}(\hat{s},\mu_f)$$

Full NLO result computed using fitted scaling functions provided by [Langenfeld, Moch, '09]

NLL corrections to $\tilde{q}\bar{\tilde{q}}$ production [PRELIMINARY]

$$K_{\rm NLL} - 1 = \frac{\sigma^{\rm match}}{\sigma^{\rm NLO}} - 1$$

- Use MSTW2008 PDFs
- μ set to $m_{\tilde{q}}$
- $r \equiv m_{\tilde{g}}/m_{\tilde{q}} = 1.25$
- $\mu_s = \bar{\mu}_s$, where $\bar{\mu}_s$ chosen such that one-loop soft corrections are minimised



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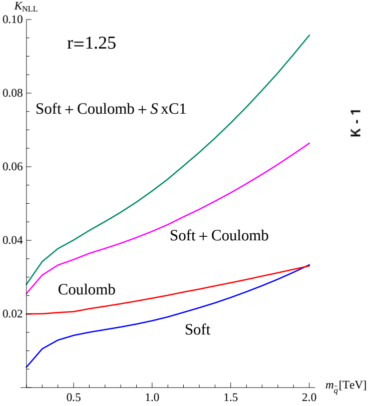
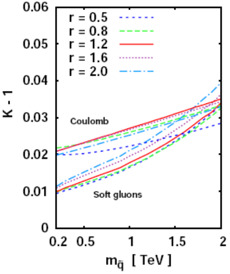


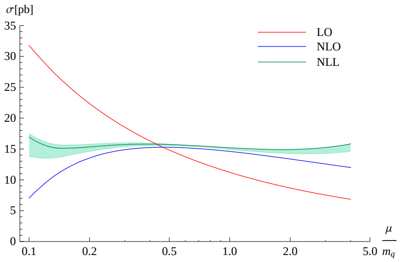


[Kulesza '08, *Talk given at IPPP Durham*]

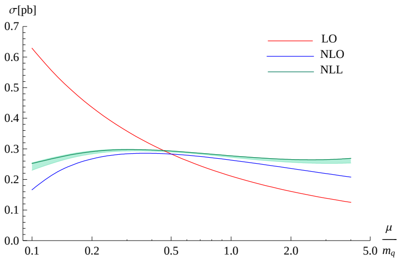
Factorisation-scale dependence

Resummation of threshold logarithms sensibly reduces the factorisation-scale dependence of the cross section

$m_{\tilde{q}} = 500\,\text{GeV}$



$m_{\tilde{q}} = 1\,\text{TeV}$



Green band obtained by varying $\bar{\mu}_s/2 < \mu_s < 2\bar{\mu}_s$

Conclusions and Outlook

- Momentum-space resummation based on effective-theory framework works well and is in good agreement with analogous results in moment space
- For squark-antisquark production resummation effects beyond NLO amount to $3-10\%$ in the range $0.2-2\,\mathrm{TeV}$
- Even for small squark masses resummation dramatically improves factorisation-scale dependence of the cross section
- Formalism can be applied to arbitrary final states (squark-squark, squark-gluino, gluino-gluino, etc...)

The formalism

Near threshold ($\beta \ll 1$) partonic cross section receives contributions from hard ($k^2 \sim \bar{m}^2$) and long-distance ($k^2 \lesssim \bar{m}^2\beta^2$) dynamical modes:

Long-distance modes

- **collinear**: $k_- \sim m$, $k_+ \sim m\beta^2$, $k_\perp \sim m\beta$
- **potential** : $k_0 \sim m\beta^2$, $|\vec{k}| \sim m\beta$ ($\Leftrightarrow$ Coulomb singularities)
- **soft**: $k_0 \sim |\vec{k}| \sim m\beta^2$

The full MSSM is matched on an effective Lagrangian from which hard modes are removed.

$$\mathcal{L}_{\rm MSSM} \longrightarrow \mathcal{L}_{\rm SCET} + \mathcal{L}_{\rm PNRQCD}$$

- $\mathcal{L}_{\mathbf{SCET}}$: describes interactions of collinear (ξ_c, A_c) and soft (A_s) modes

$$\mathcal{L}_c = \bar{\xi}_c \left(in\cdot D + i\not{D}_{\perp c} \frac{1}{i\bar{n}\cdot D_c} i\not{D}_{\perp c} \right) \frac{\not{\bar{n}}}{2}\, \xi_c - \frac{1}{2}\,\mathrm{tr}\left(F_c^{\mu\nu} F^c_{\mu\nu} \right)$$

- $\mathcal{L}_{\mathbf{PNRQCD}}$: contains interactions of potential ($\psi, \psi^{'}, A_p$) and soft (A_s) modes

$$\mathcal{L}_{\rm PNRQCD} = \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi' + \int d^3\vec{r} \left[\psi^\dagger \mathbf{T}^{(R)a} \psi \right](x+\vec{r}) \left(\frac{\alpha_s}{r} \right) \left[\psi'^\dagger \mathbf{T}^{(R')a} \psi' \right](x)$$

Structure of EFT amplitudes and soft-gluon decoupling

$$\mathcal{A}_{\{a\}}(pp' \to HH'X) = \sum_{\ell,i} c^{(i)}_{\{b\}}\; C^{(\ell,i)}_{pp'}(4\bar{m}^2,\mu)\; \langle H_{a_3}H'_{a_4}X|\mathcal{O}^{(\ell)}_{pp',\{b\}}(0)|p_{a_1}p'_{a_2}\rangle$$

- Tower of hard coefficients encoding the short-distance structure of the pair-production process
- $\mathcal{O}^{(0)}_{pp',\{b\}} \sim \phi_{\bar{c},b_2}\phi_{c,b_1}\psi'_{b_4}\psi_{b_3}$, with $\phi_c = \{W_c\xi_c, \mathcal{A}_c\}$.
- Matrix element evaluated using the EFT Lagrangian $\Rightarrow$ soft gluons interacting with everything and Coulomb interactions between the two heavy particles

At leading order in PNRQCD soft gluons can be removed from the effective Lagrangian via a field redefinition involving soft Wilson lines:

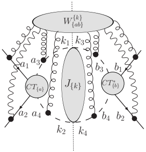
$$\begin{aligned}
\phi_c(x) &= S_n^{(r_i)}(x_-)\phi_c^{(0)}(x) \\
\psi(x) &= S_v^{(R)}(x_0)\psi_b^{(0)}(x) \\
S_n^{(r_i)\dagger}(in\cdot D)S_n^{(r_i)} &\Rightarrow in\cdot D_c \\
S_v^{(R)\dagger}(iD_s^0)S_v^{(R)} &\Rightarrow i\partial^0
\end{aligned}$$

Colour structure of the factorised cross section

After soft-gluon decoupling:

$$\hat{\sigma}_{pp'}(\hat{s},\mu) = \frac{1}{2\hat{s}N_{pp'}} \sum_{i,i',\ell,\ell'} \sum_{R_\alpha} C^{(\ell,i)}_{pp'} C^{(\ell',i')*}_{pp'} \int d\omega \, J^{(\ell,\ell')}_{R_\alpha}(E - \tfrac{\omega}{2}) \, W^{R_\alpha}_{ii'}(\omega,\mu)$$

- Hard coefficients $C^{(l,i)}_{pp'}$ is decomposed on a basis of colour operators $o^{(i)}_{\{c\}}$
- Potential function $J^{(\ell,\ell')}_{R_\alpha}$ is projected over irreducible representations of the final state: $R \otimes R' = \sum_\alpha R_\alpha$
- $W^{R_\alpha}_{ii'}$ is given by a set of matrices acting on the vector space spanned by $o^{(i)}_{\{c\}}$



$$W^{R_\alpha}_{ii'}(\omega,\mu) = P^{R_\alpha}_{\{k\}} c^{(i)}_{\{a\}} c^{(i')*}_{\{b\}} \int \frac{dz_0}{4\pi} e^{i\omega z_0/2} \langle 0|\bar{\mathrm{T}}[S^\dagger_{n,ib_1} S^\dagger_{\bar{n},jb_2} S_{v,b_4,k_4} S_{v,b_3,k_3}](z) \, \mathrm{T}[S_{\bar{n},a_2 j} S_{n,a_1 i} S^\dagger_{v,k_1 a_3} S^\dagger_{v,k_2 a_4}](0)|0\rangle$$

All-order colour structure of $W_{ii'}^{R_\alpha}$ for $3 \otimes \bar{3}$ (I)

Consider pair production of particle-antiparticle in the fundamental representation (ex. $t\bar{t}$, $\tilde{q}\bar{\tilde{q}}$)

$$3 \otimes \bar{3} = 1 \oplus 8$$

Projectors on the irreducible representations: $P^S_{\{k\}} = \frac{1}{N_C}\delta_{k_1k_2}\delta_{k_3k_4}$ $\quad P^8_{\{k\}} = 2T^A_{k_2k_1}T^A_{k_3k_4}$

- **Quark-antiquark channel:** $q\bar{q} \to H^{(3)}H^{(\bar{3})}$

$$c^{(1)}_{\{a\}} = \frac{1}{N_C}\delta_{a_1a_2}\delta_{a_3a_4} \qquad c^{(2)}_{\{a\}} = \frac{2}{\sqrt{N_C^2-1}}T^A_{a_2a_1}T^A_{a_3a_4}$$

$$\begin{aligned} W^S_{ii'}(z,\mu) &= \mathrm{diag}\left(W_{DY}, 0\right) \\ W^8_{ii'}(z,\mu) &= \mathrm{diag}\left(0, \frac{1}{(N_C^2-1)}\langle 0|\mathrm{Tr}[\overline{\mathrm{T}}[S_n^\dagger T^a S^{(8)}_{v,ac}S_{\bar{n}}](z)\mathrm{T}[S_{\bar{n}}^\dagger S^{(8),\dagger}_{v,cb}T^b S_n](0)]|0\rangle\right) \end{aligned}$$

- $W^{S/8}_{i\neq i'}(z,\mu) \propto \mathrm{Tr}[T^A] = 0$
- $W_{DY} = \frac{1}{N_C}\langle 0|\mathrm{Tr}\overline{\mathrm{T}}[S_n^\dagger S_{\bar{n}}](z)\mathrm{T}[S_{\bar{n}}^\dagger S_n](0)|0\rangle$.
- Conventional soft function $W_{ii'} = \sum_\alpha W^{R_\alpha}_{ii'} = \mathrm{diag}(W_{DY}, W^8_{22})$

- **Gluon-gluon channel:** $gg \to H^{(3)} H^{(\bar{3})}$

$$c^{(1)}_{\{a\}} = \frac{1}{N_C D_A}\delta_{a_1 a_2}\delta_{a_3 a_4} \quad c^{(2)}_{\{a\}} = \frac{1}{\sqrt{2 D_A B_F}} D^A_{a_2 a_1} T^A_{a_3 a_4} \quad c^{(3)}_{\{a\}} = \sqrt{\frac{2}{N_C D_A}} F^A_{a_2 a_1} T^A_{a_3 a_4}$$

$$W^S_{ii'}(z,\mu) = \mathrm{diag}\,(W_{DY}, 0, 0)$$

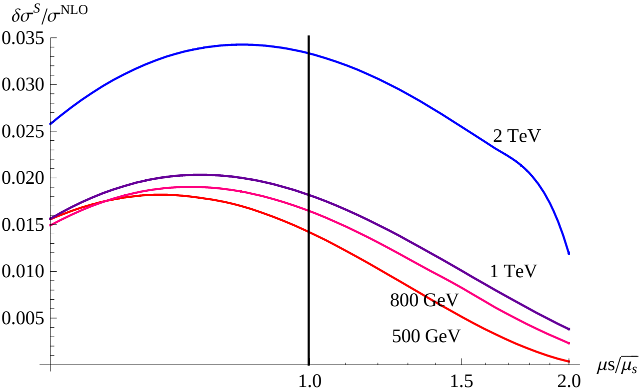
$$W^8_{ii'}(z,\mu) = \mathrm{diag}\left(0, \frac{1}{4 D_A B_F}\langle 0|\mathrm{Tr}[\overline{\mathrm{T}}[S^\dagger_n D^a S^{(8)}_{v,ac} S_{\bar{n}}](z)\mathrm{T}[S^\dagger_{\bar{n}} S^{(8),\dagger}_{v,cb} D^b S_n](0)]|0\rangle, \right.$$
$$\left. \frac{1}{N_C D_A}\langle 0|\mathrm{Tr}[\overline{\mathrm{T}}[S^\dagger_n F^a S^{(8)}_{v,ac} S_{\bar{n}}](z)\mathrm{T}[S^\dagger_{\bar{n}} S^{(8),\dagger}_{v,cb} F^b S_n](0)]|0\rangle \right)$$

$$W_{ii'}(z,\mu) = \mathrm{diag}\left(W_{DY}, W^8_{22}, W^8_{33}\right)$$

- At threshold soft function for $r_i + r_j \to R + R'$ is reduced to sum of soft functions for $r_i + r_j \to R_\alpha$, where $R \otimes R' = \sum_\alpha R_\alpha$
- Conventional soft function $W_{ii'} = \sum_\alpha W^{R_\alpha}_{ii'}$ for $t\bar{t}/\tilde{q}\bar{\tilde{q}}$ at threshold is diagonal to all order in α_s in the same colour basis that diagonalises the one-loop soft function
  (Extends recent results for the two-loop massive soft anomalous dimension in the threshold limit [Mitov et al. '09; Neubert et al. '09])

Soft-scale dependence

Resummation introduces extra scale μ_s $\Rightarrow$ How does the resummed result depend on μ_s?



In the traditional Mellin-space resummation this is not visible (soft scale implicitly set to $\mu_s = 2m_{\tilde{q}}/N$)