Dynamics of Charged Hadronic Thresholds

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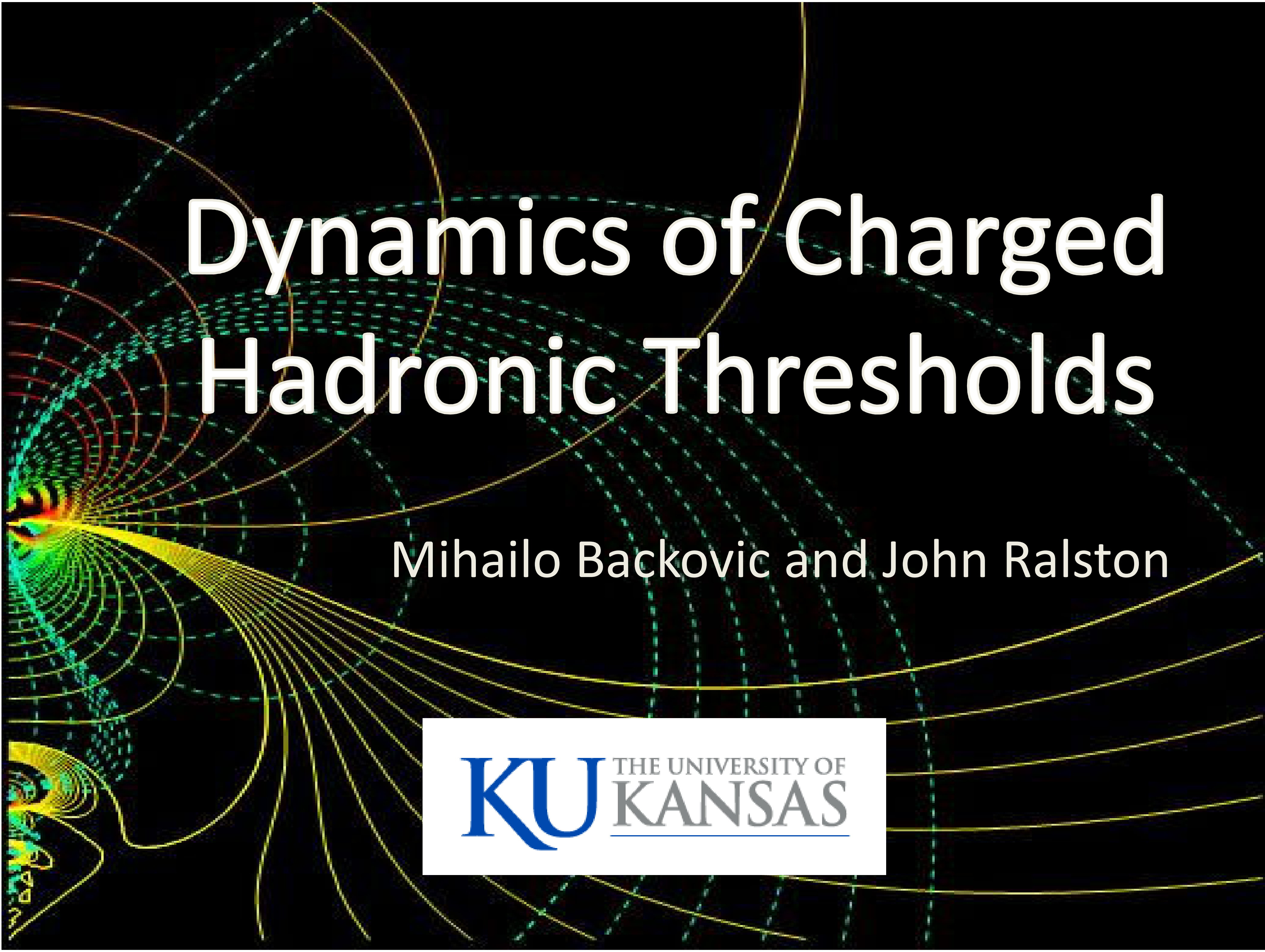


Outline

- Threshold Effects Matter!
- Sommerfeld Factor.
- New Approach.
- Applications – $\bar{p}p$ Threshold.

Threshold Effects Matter!

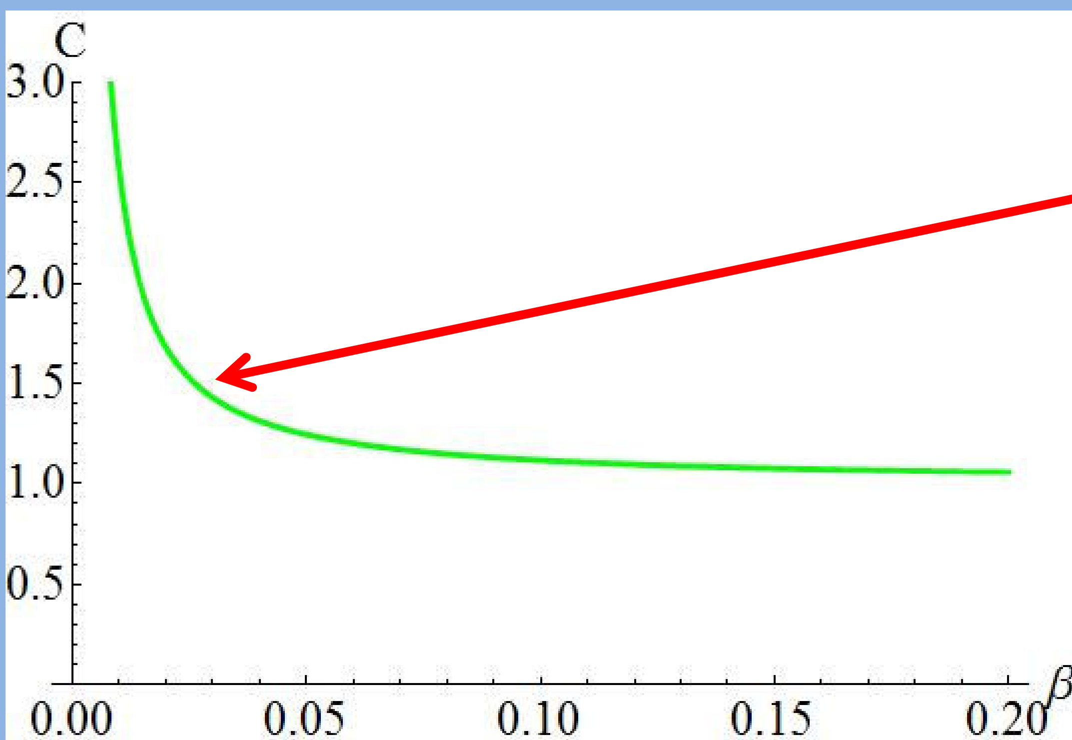
- Cross sections tend to misbehave at threshold.
- Threshold enhancements.
- Important in many areas – hadronic f.f. , LHC physics, Dark Matter physics...
- Let’s talk about thresholds of charged particles.

Sommerfeld Factor

- Common “patch” for threshold enhancements in charged systems

$$C(W_{\mathcal{B}\overline{\mathcal{B}}}) = \begin{cases} 1 & \text{for neutral } \mathcal{B} \\ \dfrac{\pi\alpha/\beta}{1 - e^{-\pi\alpha/\beta}} & \text{for charged } \mathcal{B} \end{cases}, \qquad \beta = \sqrt{1 - \frac{4M_{\mathcal{B}}^2}{W_{\mathcal{B}\overline{\mathcal{B}}}^2}}.$$

v/c



Falls off too sharply!
Significant only to the order of α/β.
(Few MeV for proton).

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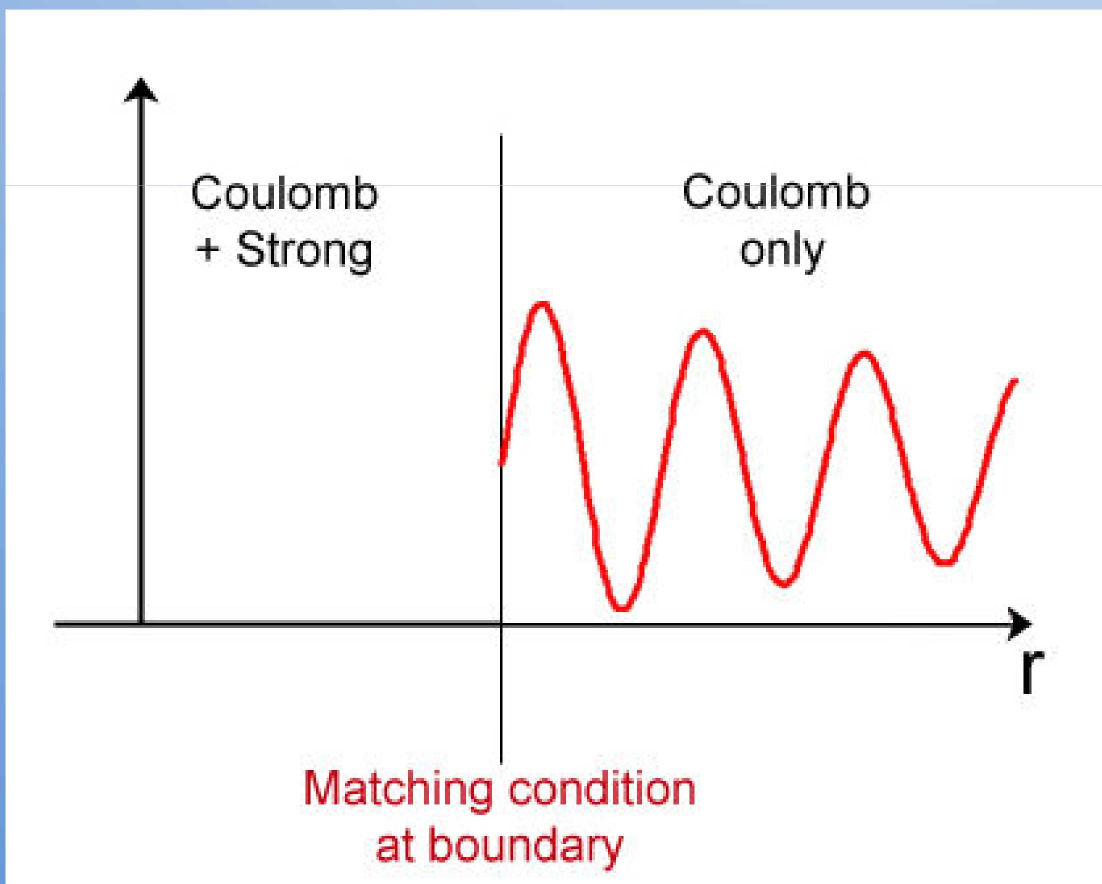
New Approach

- **<u>Strong interactions matter at threshold!</u>**
- Non-relativistic wavefunction approach (*<u>consider only s wave</u>*):



Wavefunction in the outside region will be some linear combination of Coulomb wave functions

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Coulomb + Strong amplitude

- Holstein, 2008

$$\Psi(r) = cos\delta_0 F_0(r) + sin\delta_0 G_0(r)$$

Set up a matching condition for the value of the wavefunction and the first derivative at some R to calculate the expression for phase shift → **nonrelativistic amplitude:**

$$f_s = \frac{1}{-\frac{1}{a_c} + 2m\alpha\left[\psi(1-i\eta) + \frac{1}{2i\eta} - log(i\eta)\right]}$$

$$\eta = \frac{\alpha}{\beta}, \psi(x) = \frac{d}{dx}\Gamma(x)$$

$$a_c \equiv a_c(R, \Psi(R), \Psi'(R))$$

$$\sigma_0 = Arg(\Gamma(1-i\eta))$$

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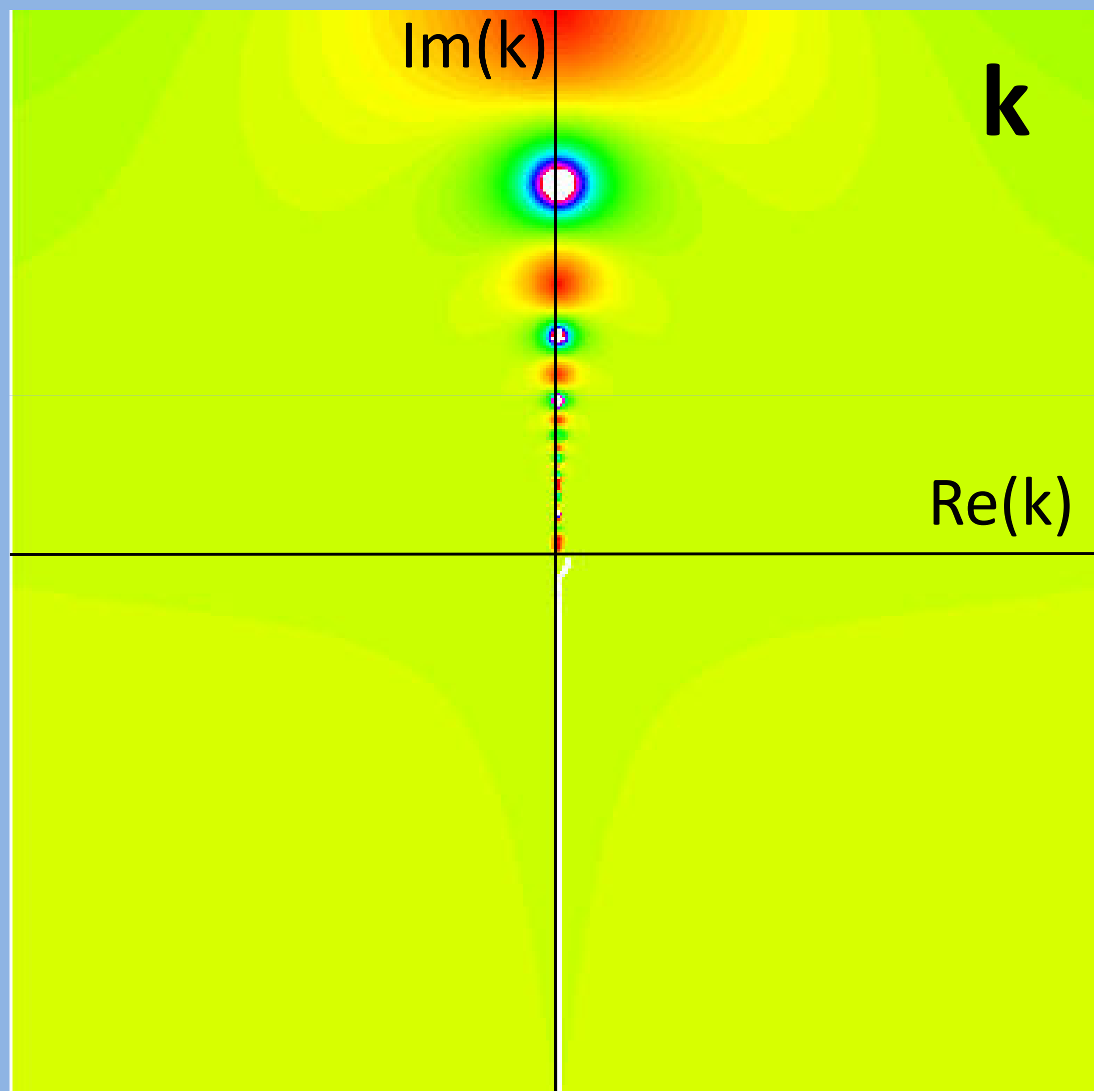
Analytic Structure of pure Coulomb Amplitude

|f| in the complex k plane



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Analytic Structure of pure Coulomb Amplitude

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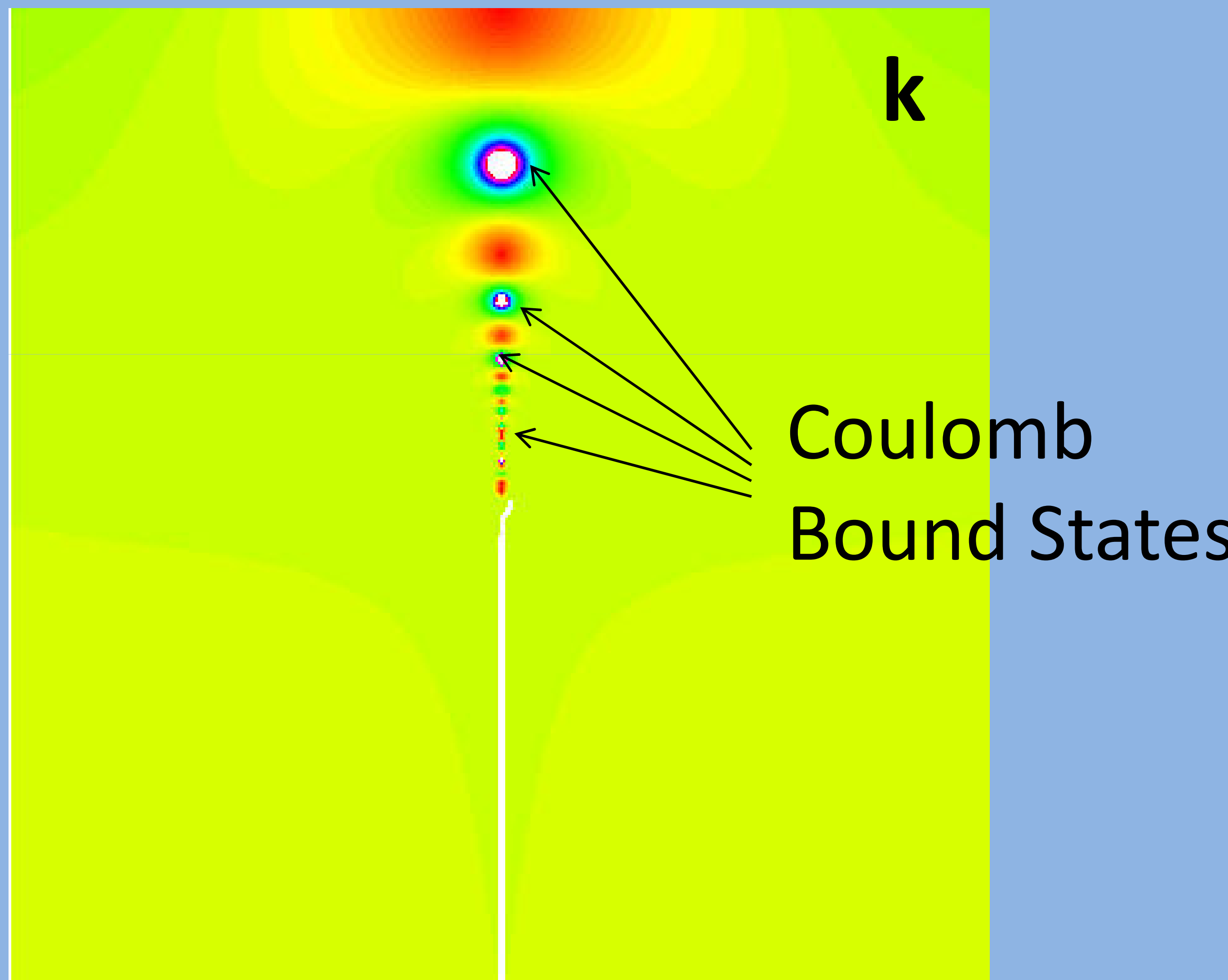

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Analytic Structure of pure Coulomb Amplitude

|f| in the complex k plane

Next Step: vary the scattering length

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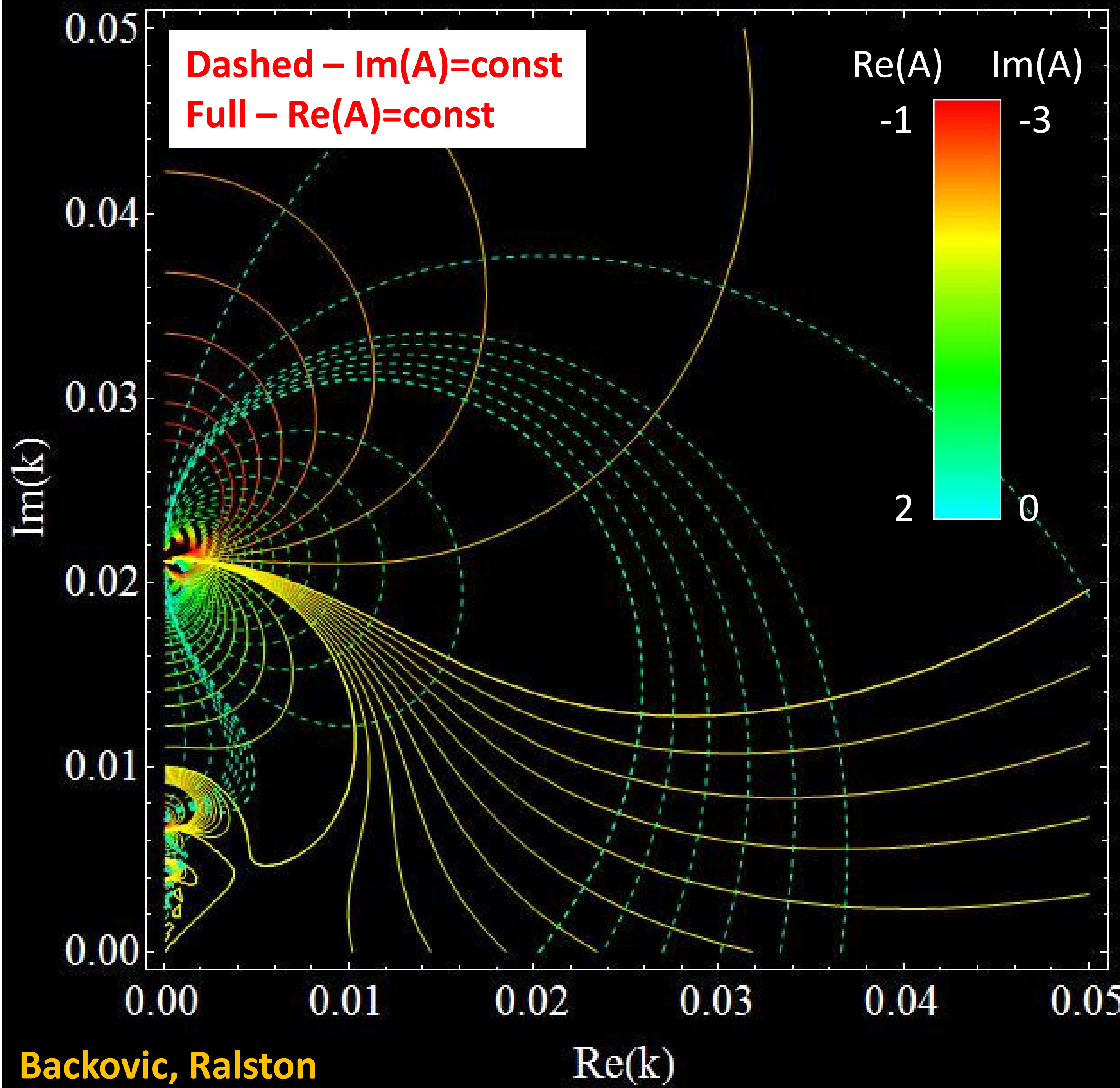
Analytic Structure of the Amplitude



$$A = -\frac{1}{2m\alpha a_c}$$

Because A is complex, coulomb poles move in the complex k plane .(due to strong interactions)

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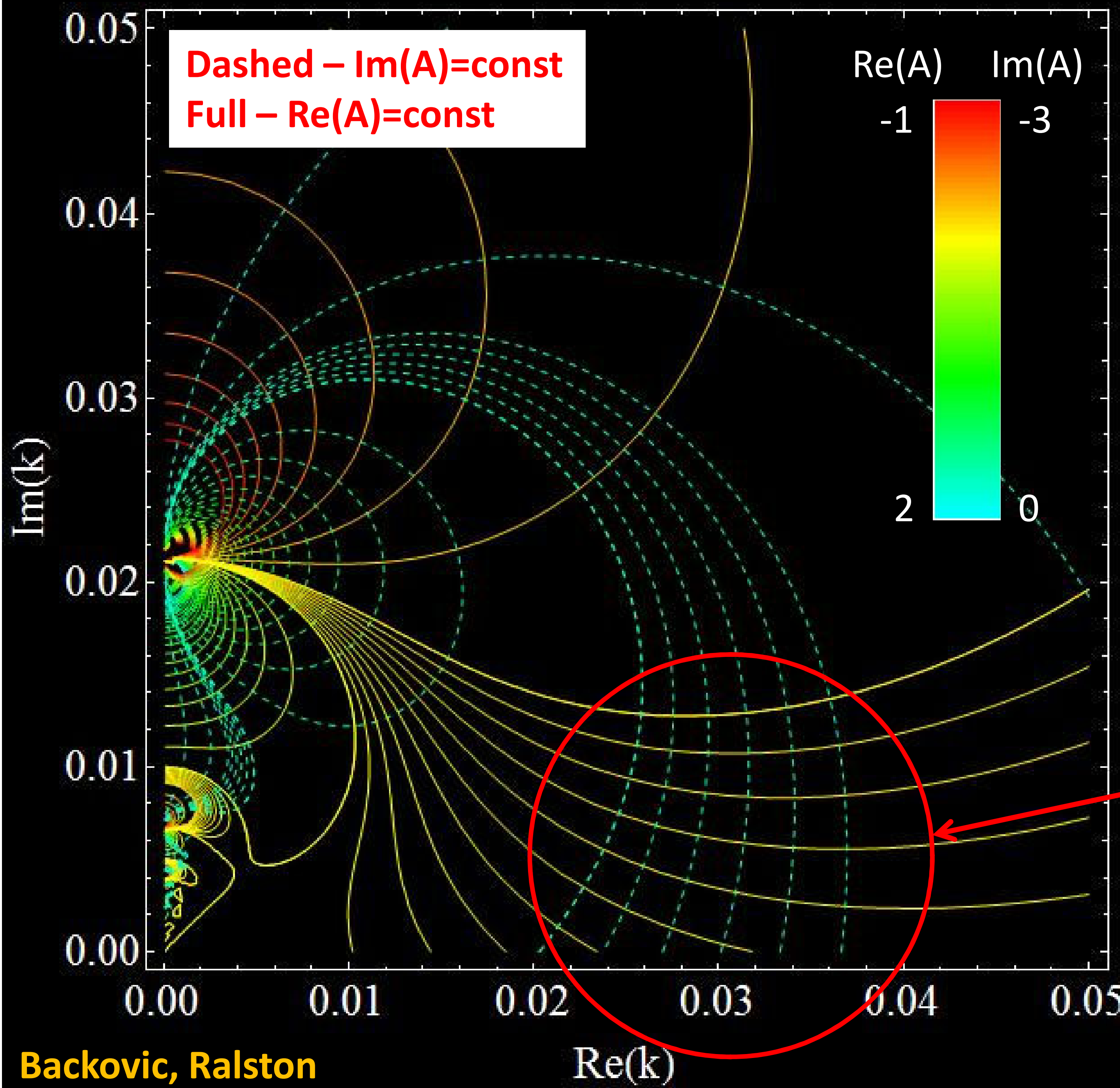
Analytic Structure of the Amplitude



$$A = -\frac{1}{2m\alpha a_c}$$

Coulomb poles move in the complex k plane due to strong interactions

General Mechanism of Threshold Enhancements

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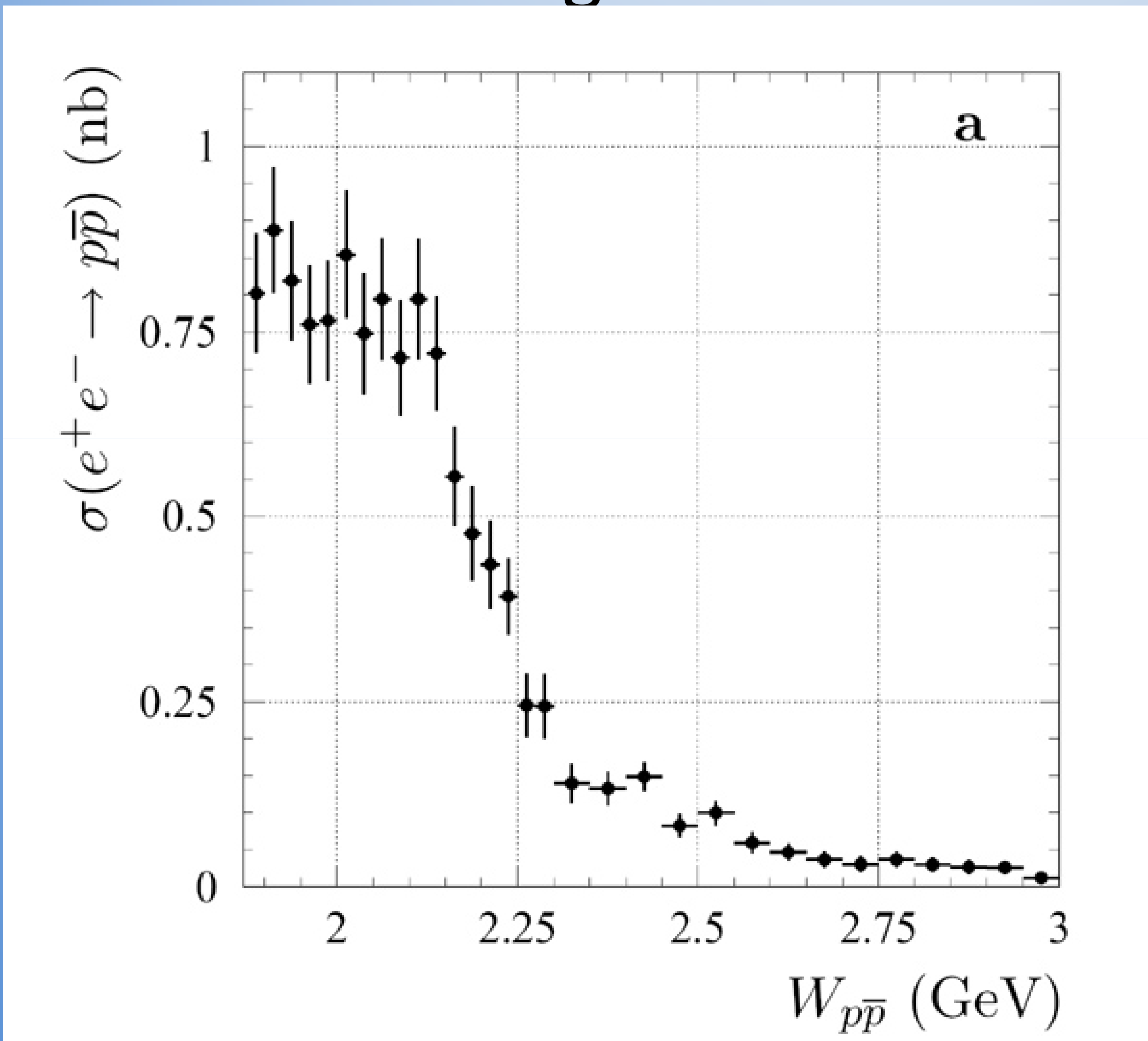
Application - $\bar{p}p$ threshold

Best data coming from BABAR:



Cross section enhanced at threshold.

Unexpected because phase space goes as velocity.

200 MeV plateau – Sommerfeld factor not sufficient

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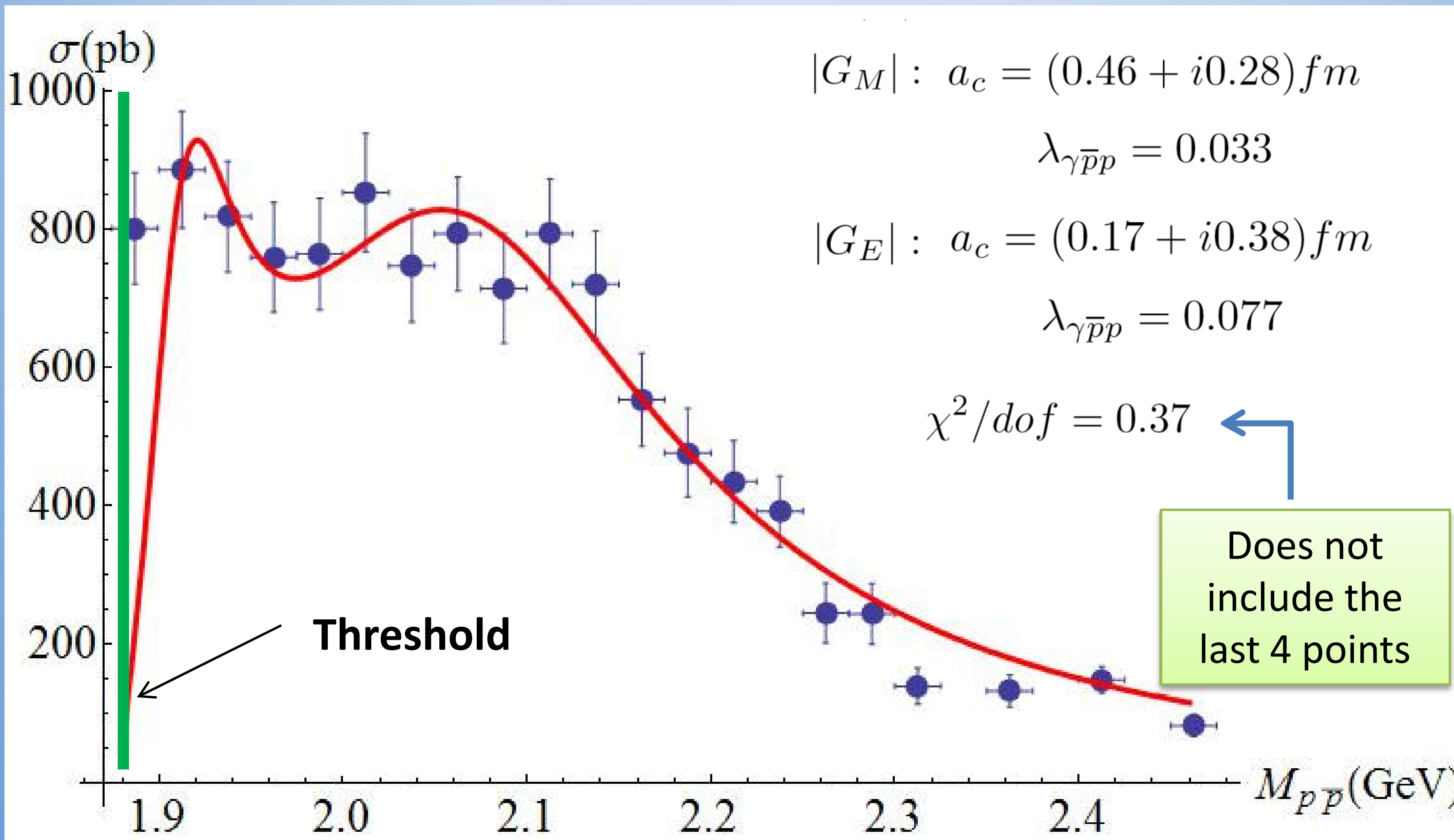
Application - p̄p threshold



$$\sigma_{p\bar{p}}(m) = \frac{4\pi\alpha^2\beta C}{3m^2}\left[|G_M(m)|^2 + \frac{2m_p^2}{m^2}|G_E(m)|^2\right]$$

Gs modeled using f from prev. slides

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Conclusions & Future Outlook

- Hadronic thresholds show surprising features.
- Problem more complex then commonly believed.
- Strong interactions matter at threshold!
- No threshold is safe!

Thank you!