Testing Gaugino Mass Hypotheses at the LHC

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Motivation & Musings

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- **A1:** Short run: improving the toolkit
  - Boosted tops
  - M_{T2}, $M_{T\mathrm{Gen}}$, etc.
  - Jet reconstruction algorithms
  - Spin measurement techniques
  - Standard Model backgrounds
  - ...

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- **Q:** LHC era is (almost) here – *what should theorists be doing now?*
- **A1:** Short run: improving the toolkit
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  - ⋆ Jet reconstruction algorithms
  - ⋆ Spin measurement techniques
  - ⋆ Standard Model backgrounds
  - ⋆ ...
- **A2:** Medium run: think two to three years out.
  Post-discovery, what will be the big issues?
  - ⋆ Assume SUSY is established early on by standard methods – *what next?*
  - ⋆ Will want to connect observations to some underlying framework
  - ⋆ For me: an underlying *string* framework (hopefully)

Motivation & Musings

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- **A1:** Through fits to *simple benchmark models* **Allanach et al.**, **Ellis et al.**
  - For example, mSUGRA at first
  - Then perturbed sequentially using goodness-of-fit as our guide
  - Effectively a "directed walk through theory space"
  - **Pro:** Completely and utterly obvious thing to do
  - **Pro:** Quasi-model independent
  - **Con:** Computationally intensive
  - **Con:** May saturate its effectiveness early on in LHC era
  - **Con:** Suffers from the "LHC inverse problem"

Arkani-Hamed et al., **JHEP 0608** (2006) 070

Motivation & Musings

- **A2:** Via the method of *model footprints* **Kane et al.**

  - Pick models and map their parameter space onto signature space
  - **Pro:** *Guaranteed* to have consistent high-scale model – *assuming LHC data falls in at least one footprint! (See below)*
  - **Pro:** Requisite analysis can be done once and for all “offline,” ahead of the experiment
  - **Pro:** LHC inverse problem less of an issue
  - **Con:** Goodness-of-fit trickier to determine
  - **Con:** “Straw-man effect”: method best at discriminating between models, not so much establishing a model
  - **Con:** Maximally model-dependent: only as good as our imagination in dreaming up models (the problem with all top-down methods)

Motivation & Musings

- **A3:** Try to extract *broad characteristics* of the underlying theory
  - ⋆ Measurement of $m_{\tilde{N}_2} - m_{\tilde{N}_1}$
  - ⋆ Measurement of $m_{\tilde{N}_2}$ itself
  - ⋆ Extraction of the values of M_2, μ, $\tan\beta$,...
  - ⋆ Evidence of how the μ-term was generated in the first place

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  - ⋆ Evidence of how the μ-term was generated in the first place
  - ⋆ **Pro:** LHC inverse problem completely avoided
  - ⋆ **Pro:** Directly probe the things theorists most care about (*e.g.* gaugino mass unification) **Binetruy, Kane, Lykken and BDN**, **J. Phys. G32** (2006) 129
  - ⋆ **Pro/Con:** Never been tried before...
  - ⋆ **Con:** ...because it's outrageously difficult!
- Want to know this independent of everything else that's going on with the supersymmetry breaking Lagrangian (if possible)
- Big job: need a **tractable and concrete** starting point

“Mirage Pattern”

- Mirage pattern of gaugino masses – a one-parameter family:

$$M_1 \,:\, M_2 \,:\, M_3 \simeq (1 + 0.66\alpha) \,:\, (2 + 0.2\alpha) \,:\, (6 - 1.8\alpha)$$

- A logical first step
  - ⋆ Easy to understand and visualize
  - ⋆ Interpolates between mSUGRA ($\alpha = 0$) and AMSB limit ($\alpha \to \infty$)
  - ⋆ Motivated by a variety of constructions, including string theory (heterotic and Type II) as well as “deflected” AMSB
  - ⋆ Disadvantage: Only one-parameter family of models $\Rightarrow$ not fully general

- **All** values of α correspond to a unified pattern – the only issue is at which *energy scale* they unify

Choi & Nilles, **JHEP 0704** (2007) 006

  - ⋆ When $\alpha = 0$ gaugino masses unify at $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV
  - ⋆ Other α values give effective unification scale elsewhere (hence “mirage”)
  - ⋆ Example: $\alpha = 2$ gives $M_1 \simeq M_2 \simeq M_3$ at low-energy scale
  - ⋆ Scale dependent! Coefficients change with scale (here 1 TeV)

A Quick Derivation of the Mirage Pattern (I)

⇒ High scale: universal and anomaly-induced piece to gaugino masses

$$M_a(\Lambda_{\rm UV}) = M_a^{\rm univ}(\Lambda_{\rm UV}) + M_a^{\rm anom}(\Lambda_{\rm UV}) = M_u + g_a^2(\Lambda_{\rm UV})\frac{b_a}{16\pi^2}M_g$$

- Gauge couplings continue to unify at the $\Lambda_{\rm UV} = \Lambda_{\rm GUT}$ scale

$$g_1^2(\Lambda_{\rm UV}) = g_2^2(\Lambda_{\rm UV}) = g_3^2(\Lambda_{\rm UV}) = g_{\rm GUT}^2 \simeq \frac{1}{2}$$

- Anomaly piece is proportional to SM beta-function coefficients

$$b_a = -(3C_a - \sum_i C_a^i) \quad \Rightarrow \quad \{b_1, b_2, b_3\} = \left\{\frac{33}{5}, 1, -3\right\}$$

- If these are going to be competitive you need $M_g \gtrsim 30 M_u$

⇒ Now evolve to electroweak scale using one-loop RGEs

$$M_a(\Lambda_{\rm EW}) = M_u\left\{1 - g_a^2(\Lambda_{\rm EW})\frac{b_a}{8\pi^2}\ln\left(\frac{\Lambda_{\rm UV}}{\Lambda_{\rm EW}}\right)\left[1 - \frac{1}{2}\frac{M_g}{M_u\ln\left(\frac{\Lambda_{\rm UV}}{\Lambda_{\rm EW}}\right)}\right]\right\}$$

A Quick Derivation of the Mirage Pattern (II)

$\Rightarrow$ Introduce the parameter $\alpha = \frac{M_g}{M_u \ln(\Lambda_{\rm UV}/\Lambda_{\rm EW})}$

$$M_a\left(\Lambda_{\rm EW}\right) = M_u \left[1 - \left(1 - \frac{\alpha}{2}\right) g_a^2\left(\Lambda_{\rm EW}\right) \frac{b_a}{8\pi^2} \ln\left(\frac{\Lambda_{\rm UV}}{\Lambda_{\rm EW}}\right)\right]$$

- Some notable properties of this solution
  - ⋆ If you can engineer $M_g \sim 30 M_u$ then you obtain $\alpha \sim 1$
  - ⋆ When $\alpha = 2$ gaugino masses universal *at the electroweak scale*
  - ⋆ Take $\Lambda_{\rm EW} = 1000$ GeV, $\Lambda_{\rm UV} = \Lambda_{\rm GUT}$ and divide through by $M_1\left(\Lambda_{\rm EW}\right)|_{\alpha=0}$

$$M_1 \,:\, M_2 \,:\, M_3 \;=\; (1.0 + 0.66\alpha) \,:\, (1.93 + 0.19\alpha) \,:\, (5.87 - 1.76\alpha)$$

$\Rightarrow$ Finding the scale of "mirage unification": redefine $\alpha \equiv \frac{M_g}{M_u \ln\left(M_{\rm PL}/M_g\right)}$

$$M_a\left(\Lambda_{\rm EW}\right) = M_u \left\{1 - g_a^2\left(\Lambda_{\rm EW}\right) \frac{b_a}{8\pi^2} \left[\ln\left(\frac{\Lambda_{\rm UV}\left(M_g/M_{\rm PL}\right)^{\alpha/2}}{\Lambda_{\rm EW}}\right)\right]\right\}$$

- Effective unification scale is now at

$$\Lambda_{\rm mir} = \Lambda_{\rm GUT}\left(\frac{M_g}{M_{\rm PL}}\right)^{\alpha/2}$$

Two Benchmarks Models

- Model A

M.K. Gaillard and BDN, Int. J. Mod. Phys. A22 (2007) 1451

  - ⋆ Based on heterotic string theory
  - ⋆ Dilaton stabilized with non-perturbative corrections to the Kähler potential
  - ⋆ Stabilization mechanism causes $M_g \sim 30 M_u$
  - ⋆ Scalar masses generally universal
  - ⋆ Absolute prediction: $\alpha \gtrsim 0.12$

- Model B

Choi, Falkowsi, Nilles, Olechowski, NPB 718 (2005) 113
Falkowski, Lebedev, Mambrini, JHEP 0511 (2005) 034

  - ⋆ Based on Type II string theory
  - ⋆ Includes internal fluxes for moduli stabilization
  - ⋆ Large warping in compact space produces $M_g \sim 30 M_u$
  - ⋆ AMSB plays a large role in all soft terms
  - ⋆ Basic model predicts $\alpha \simeq 1$

Point	A	B
α	0.3	1.0
$\tan\beta$	10	10
$\Lambda_{\rm mir}$	2.0×10^{14}	1.5×10^{9}
M_1	198.7	851.6
M_2	172.1	553.3
M_3	154.6	339.1
A_t	193.0	1309
A_b	205.3	1084
A_τ	188.4	1248
$m^2_{Q_3}$	$(1507)^2$	$(430.9)^2$
$m^2_{U_3}$	$(1504)^2$	$(610.3)^2$
$m^2_{D_3}$	$(1505)^2$	$(352.2)^2$
$m^2_{L_3}$	$(1503)^2$	$(381.6)^2$
$m^2_{E_3}$	$(1502)^2$	$(407.9)^2$
$m^2_{Q_{1,2}}$	$(1508)^2$	$(208.4)^2$
$m^2_{U_{1,2}}$	$(1506)^2$	$(302.7)^2$
$m^2_{D_{1,2}}$	$(1505)^2$	$(347.0)^2$
$m^2_{L_{1,2}}$	$(1503)^2$	$(379.8)^2$
$m^2_{E_{1,2}}$	$(1502)^2$	$(404.5)^2$
$m^2_{H_u}$	$(1500)^2$	$(752.0)^2$
$m^2_{H_d}$	$(1503)^2$	$(388.7)^2$

All values in GeV

Physical Spectra for Benchmark Models

Parameter	Point A	Point B		Parameter	Point A	Point B
$m_{\widetilde{N}_1}$	85.5	338.7		$m_{\tilde{t}_1}$	844.7	379.9
$m_{\widetilde{N}_2}$	147.9	440.2		$m_{\tilde{t}_2}$	1232	739.1
$m_{\widetilde{N}_3}$	485.3	622.8		$m_{\tilde{c}_L}$, $m_{\tilde{u}_L}$	1518	811.7
$m_{\widetilde{N}_4}$	494.0	634.3		$m_{\tilde{c}_R}$, $m_{\tilde{u}_R}$	1520	793.3
$m_{\widetilde{C}_1^\pm}$	147.7	440.1		$m_{\tilde{b}_1}$	1224	676.8
$m_{\widetilde{C}_2^\pm}$	494.9	635.0		$m_{\tilde{b}_2}$	1507	782.4
$m_{\tilde{g}}$	510.0	818.0		$m_{\tilde{s}_L}$, $m_{\tilde{d}_L}$	1520	815.4
μ	476.1	625.2		$m_{\tilde{s}_R}$, $m_{\tilde{d}_R}$	1520	793.5
m_h	115.2	119.5		$m_{\tilde{\tau}_1}$	1487	500.4
m_A	1557	807.4		$m_{\tilde{\tau}_2}$	1495	540.4
m_{H^0}	1557	806.8		$m_{\tilde{\mu}_L}$, $m_{\tilde{e}_L}$	1500	545.1
$m_{H^\pm}$	1559	811.1		$m_{\tilde{\mu}_R}$, $m_{\tilde{e}_R}$	1501	514.6

Low Energy Physical Masses for Benchmark Points

Testing for the Mirage Pattern

$\Rightarrow$ Our goal is to ask how well we can determine α at the LHC using only **actual observations**

- Most importantly, can we demonstrate $\alpha \neq 0$?
- Want to do this independent of any particular model
- Not going to assume reconstruction any sparticle masses
- We *will* assume we know all other inputs for the Monte Carlo comparison to data – unrealistic but this is a first step

$\Rightarrow$ Basic idea: use an ensemble of signatures wisely chosen to perform a fit of Monte Carlo to "data"

- We break the problem into a "base model" specified by the parameters

$$\left\{ \begin{array}{c} \tan\beta,\ m_{H_u}^2,\ m_{H_d}^2 \\ M_3,\ A_t,\ A_b,\ A_\tau \\ m_{Q_{1,2}}, m_{U_{1,2}}, m_{D_{1,2}}, m_{L_{1,2}}, m_{E_{1,2}} \\ m_{Q_3}, m_{U_3}, m_{D_3}, m_{L_3}, m_{E_3} \end{array} \right\}$$

and a value of α which determines the three gaugino masses (with overall scale set by M_3)

Simulation Methodology: Overview

- Given a model we construct a ***model line*** by varying α while keeping the base model fixed

- For each point we generate data using `PYTHIA` + `PGS4` and construct our signatures

- Analysis is performed using a modification of `ROOT` generated by Baris Altunkaynak at Northeastern

  `http://www.atsweb.neu.edu/ialtunkaynak/heptools.html#parvicursor`

- How do we determine the value of α? We compare Monte Carlo predictions for our signatures against the “data”

- For example, we can ask whether we can distinguish the prediction for the case $\alpha = 0$ from the data we simulate at $\alpha \neq 0$

Interlude: On "Distinguishability"

$\Rightarrow$ We want to distinguish models A and B using the n (counting) signatures S_i

- Define a measure in *signature space* analogous to a chi-squared variable

$$(\Delta S_{AB})^2 = \frac{1}{n} \sum_i \left[\frac{S_i^A - S_i^B}{\delta S_i^{AB}} \right]^2$$

- Convert to effective cross-sections via $\bar{\sigma}_i = S_i / L$ and assuming errors are purely statistical ($\sqrt{N}$)

$$(\Delta S_{AB})^2 = \frac{1}{n} \sum_i \left[\frac{\bar{\sigma}_i^A - \bar{\sigma}_i^B}{\sqrt{\bar{\sigma}_i^A / L_A + \bar{\sigma}_i^B / L_B}} \right]^2$$

- We always include the Standard Model background so that $\bar{\sigma}_i = \bar{\sigma}_i^{\mathrm{SUSY}} + \bar{\sigma}^{\mathrm{SM}}$

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- So how big should $(\Delta S_{AB})^2$ be to say models A and B are distinguished from one another?

- LHC Inverse criterion: this number needs to be at *least* as big as the value induced by quantum fluctuations **Arkani-Hamed et al.**, **JHEP 0608** (2006) 070

Towards a Universal Definition of “Distinguishable”

- Effect of fluctuations estimated by comparing the same single model to itself many times and computing $(\Delta S_{AA})^2\big|_{95}$
- But this really depends on the model point and (especially) the signature list you choose to consider

Towards a Universal Definition of "Distinguishable"

- Effect of fluctuations estimated by comparing the same single model to itself many times and computing $(\Delta S_{AA})^2\big|_{95}$
- But this really depends on the model point and (especially) the signature list you choose to consider
- We can obtain an analytic answer valid for any model pair and any signature list provided
  - ⋆ Fluctuations for each signature are assumed to be **uncorrelated**
  - ⋆ We assume that our extracted $\bar{\sigma}_i$ are very close to the true cross-section values σ_i
  - ⋆ We assume assume the count rates form normal distributions
- Under these assumptions $(\Delta S_{AB})^2$ is a randomly-distributed variable with a probability distribution of

$$P(\Delta S^2) = n\,\chi^2_{n,\lambda}(n\Delta S^2)$$

where $\chi^2_{n,\lambda}$ is the **non-central chi-squared distribution** for n degrees of freedom

Non-central Chi-Square Distribution

$\Rightarrow$ $(\Delta S_{AB})^2$ distributed according to a non-central chi-square distribution

- The non-centrality parameter λ is given by

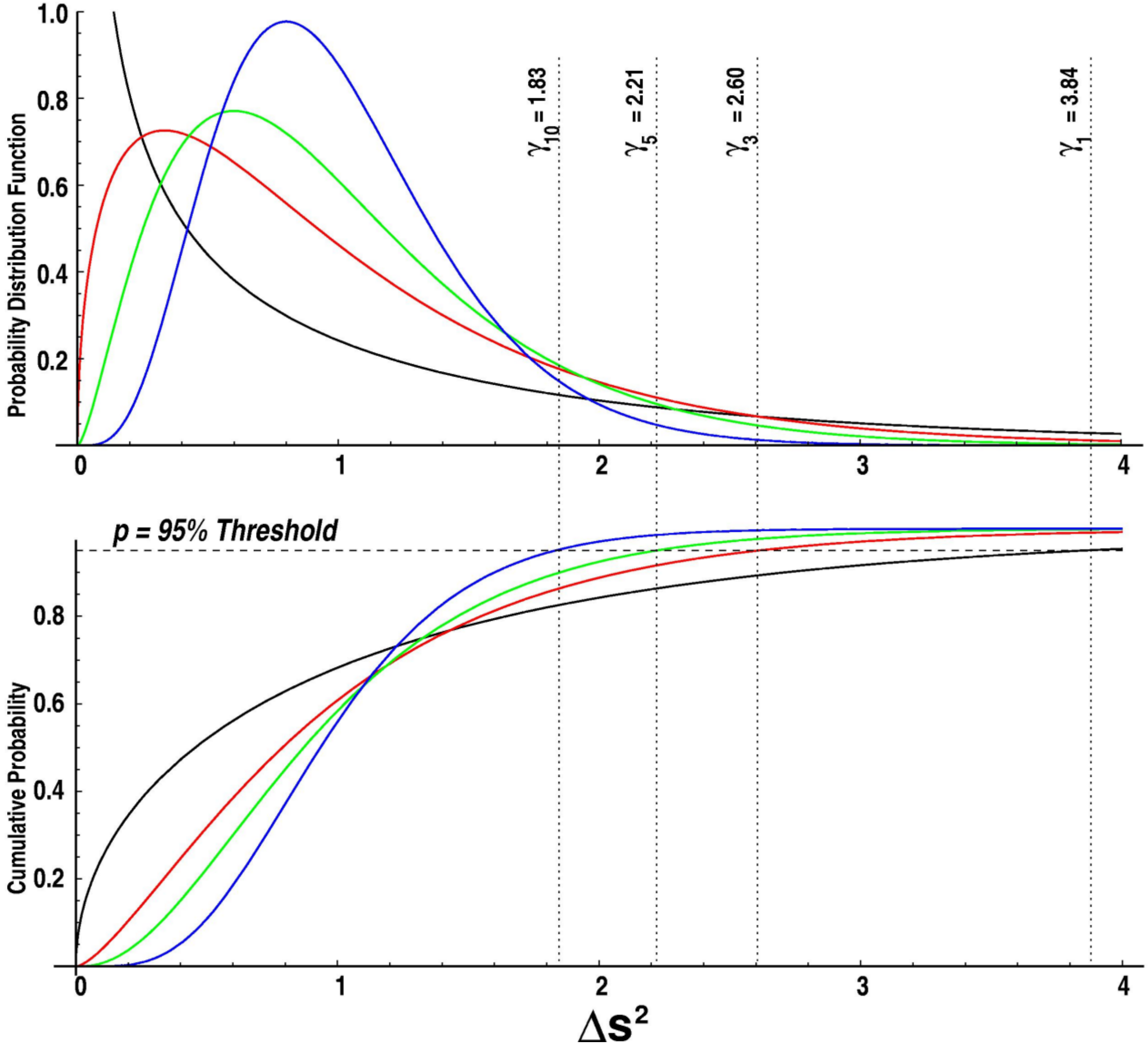
$$\lambda = \sum_i \frac{(\sigma_i^A - \sigma_i^B)^2}{\sigma_i^A/L_A + \sigma_i^B/L_B}$$

- Taking $\lambda = 0$ gives distribution for $(\Delta S_{AA})^2$
- Can now solve analytically for $(\Delta S_{AA})^2\big|_p \equiv \gamma_n(p)$ for any confidence level p as a function of the number of signatures n
- Having $(\Delta S_{AB})^2 > (\Delta S_{AA})^2\big|_{95}$ may be thought of as a *necessary* condition, but it is not *sufficient* to distinguish models A and B

$\Rightarrow$ For two models that truly are different we expect $\lambda \neq 0$

$\Rightarrow$ We want to quantify the probability that two truly distinct models undergo a fluctuation such that their measured $(\Delta S_{AB})^2$ is a very low value

Non-central Chi-Square Distribution & $\gamma_n(p)$

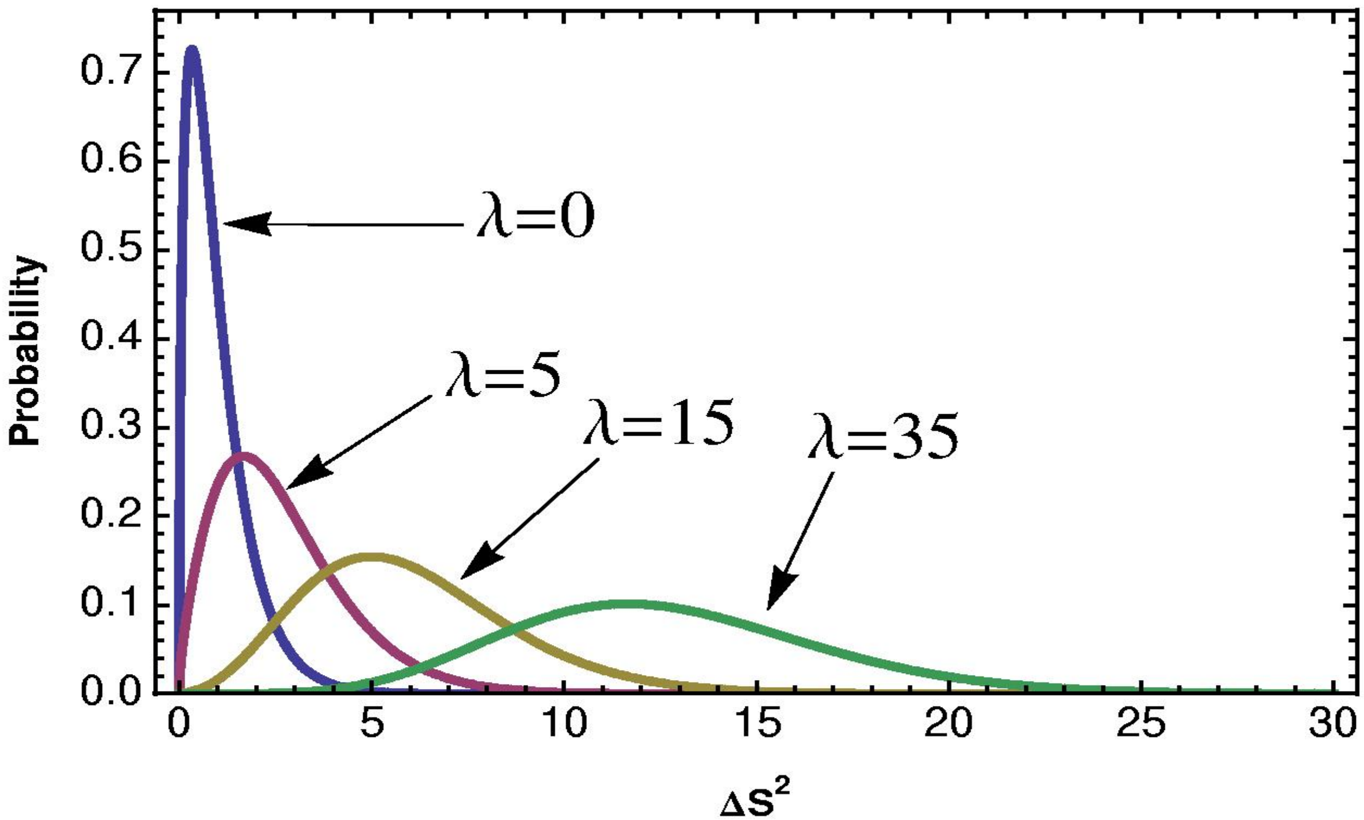


Our Distinguishability Criterion

$\Rightarrow$ Want the probability for $(\Delta S_{AB})^2$ to fluctuate below $\gamma_n(p)$ to be less than 5%

$$P = \int_{\gamma_n(p)}^{\infty} n\, \chi^2_{n,\lambda}(n\Delta S^2_{AB})\, d(\Delta S^2_{AB}) = \int_{n\gamma_n(p)}^{\infty} \chi^2_{n,\lambda}(y)\, dy \geq 0.95$$

- Value of this integral decreases monotonically as λ increases
- When $P = 0.95$ we have found the minimum value $\lambda_{\text{min}}(n)$ for the non-centrality parameter



Converting $\lambda_{\min}$ to Signatures

- Any combination of n-parameters yielding $\lambda > \lambda_{\min}(n)$ will be effective in demonstrating that the two models are indeed distinct, 95% of the time, with a confidence level of 95%

- The value of λ is proportional to integrated luminosity

$$L_{\min} = \frac{\lambda_{\min}(n)}{R_{AB}} \quad \text{with} \quad R_{AB} = \sum_i (R_{AB})_i = \sum_i \frac{(\sigma_i^A - \sigma_i^B)^2}{\sigma_i^A + \sigma_i^B}$$

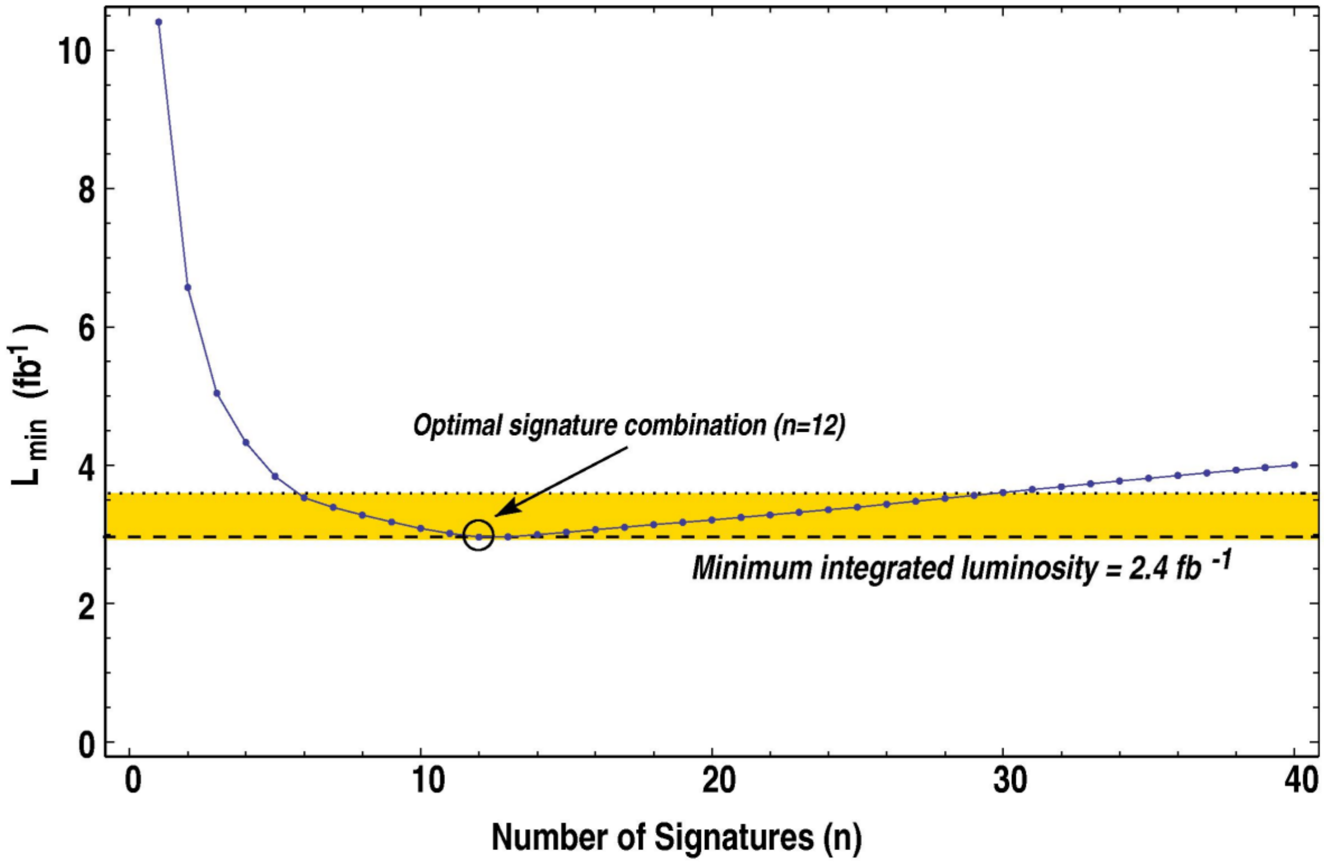
$\Rightarrow$ All the physics of the specific signature list is contained in R_{AB}!

- This just says given *any* signature list there is always some minimal luminosity that will distinguish the models

- Now the goal is clear: choose your signature list so as to maximize R_{AB}, with as few signatures as possible so as to minimize $\lambda_{\min}(n)$

Choosing an Optimal Signature List

- Given a model pair A and B compute the absolute quantity $(R_{AB})_i$ for all of the possible signatures you can imagine
- Now order them from highest R_i value (smallest $L_{\rm min}$) to smallest R_i value (largest $L_{\rm min}$) – what fraction of the list should you employ?



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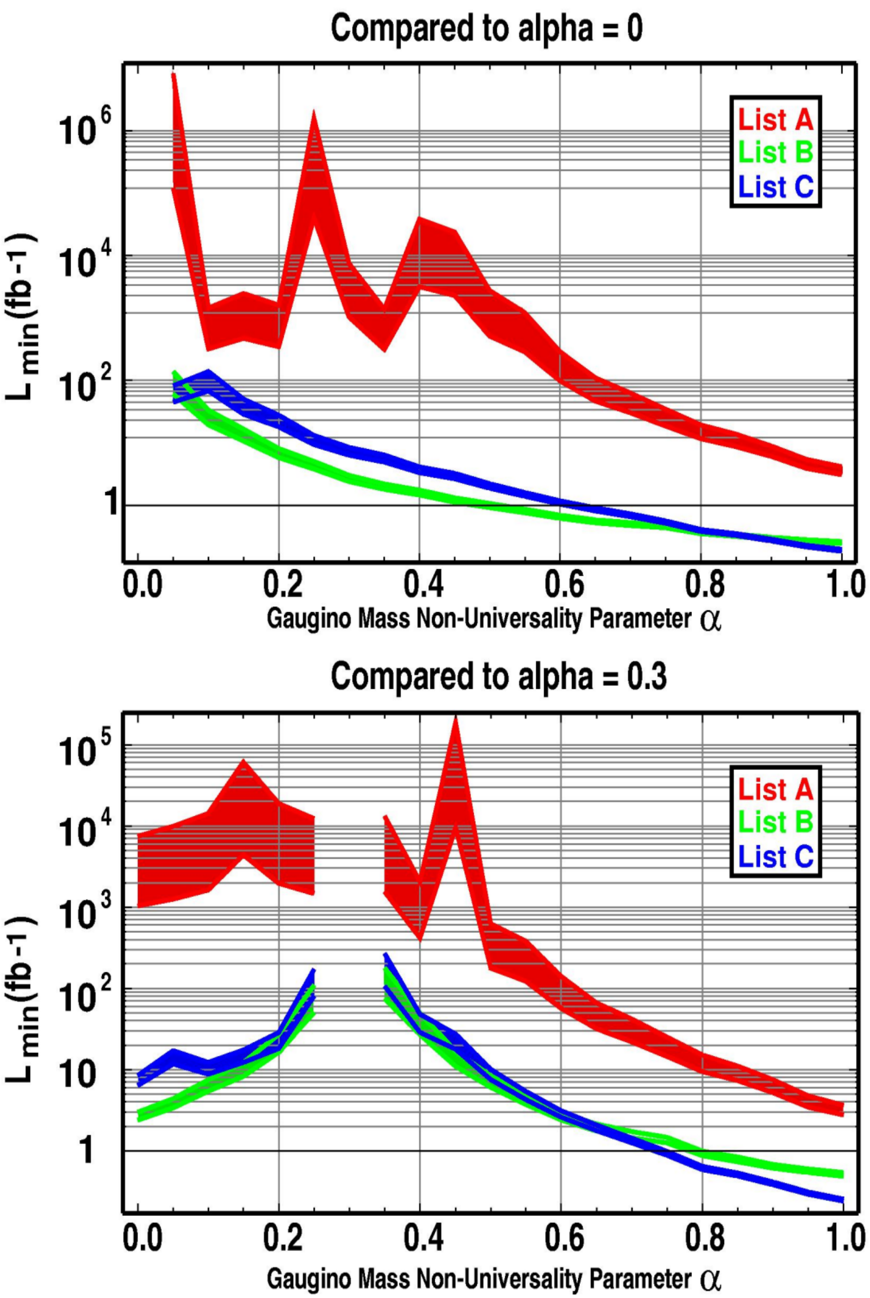
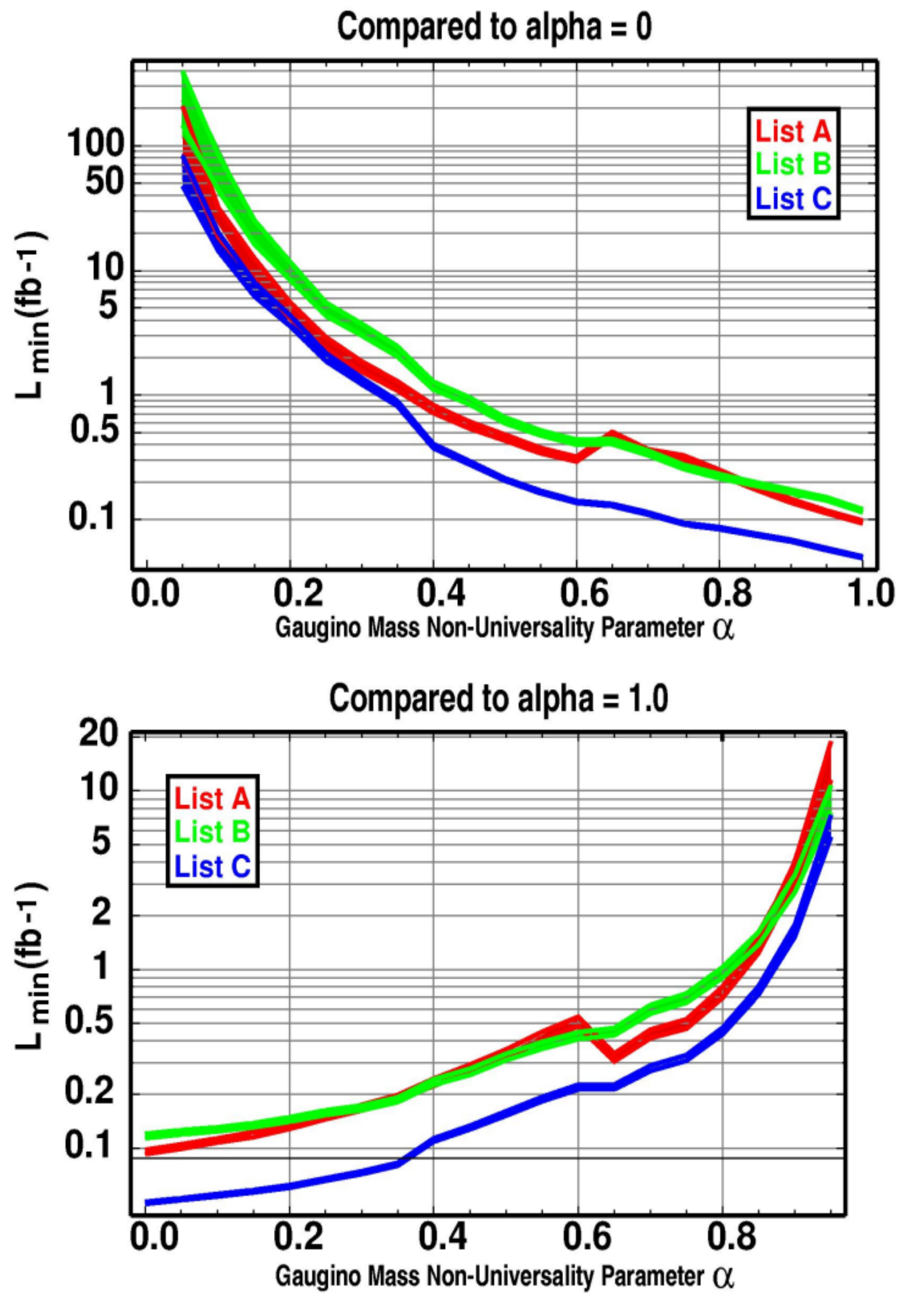
- No cheating! Can't use your best signature N times... (correlations)
- Kitchen sink method is not ideal!

  $\Rightarrow$ Take a big hit since $\lambda(n)$ eventually grows faster than $\sum_i R_i$
- For any *particular pair* of models you can optimize this choice
- But once you average over a large ensemble of models the list will now only be (at best) **close to optimal** for any model

Benchmark Results



Model A



Model B

(Intermediate) Conclusions

- LHC v2.0 will be about **synthesis** of multiple measurements

  $\Rightarrow$ *Theorists can and will play a major role here*

- Bigger is not necessarily better when using LHC observations!

  $\Rightarrow$ *Experimentalists know this well – theorists sometimes less so*

- Rather than fit to models can we fit to characteristics?

  $\Rightarrow$ *Yes, at least in this (artificial) first step*