The Thermal Abundance of Semi-Relativistic Relics

Mitsuru Kakizaki
(LAPTH, Universite de Savoie, CNRS)

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In collaboration with

- Manuel Drees (Bonn Univ.)
- Suchita Kulkarni (Bonn Univ.)

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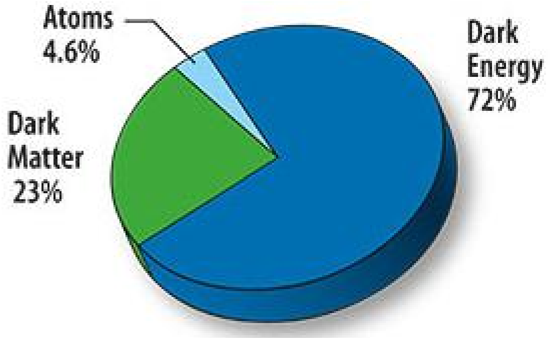
1. Motivation

- The abundance of thermal relics (e.g. DM) is determined by the Boltzmann equation:

$$\dot{n}_\chi + 3Hn_\chi = -\langle\sigma_{\rm eff} v\rangle(n_\chi^2 - n_{\chi,{\rm eq}}^2)$$



[http://wmap.gsfc.nasa.gov]

- Numerical calculation needed in evaluating the relic density in many cases

→ Analytic methods should be developed in various scenarios

- Approximate analytical solutions established for particles that are either relativistic (hot) or non-relativistic (cold) at decoupling

- No analytical formula for the relic density of particles that are semi-relativistic at decoupling

Outline

This work

- Analytic treatment that connects
 the hot and cold relic solutions
- Late entropy production by semi-relativistic relics

1. Motivation
2. Thermal abundances of hot and cold relics (review)
3. Thermal abundance of semi-relativistic relics
4. Application of semi-relativistic relics
5. Summary

2. Thermal abundances of hot and cold relics

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- The abundance of thermal relics (e.g. DM) is determined by the Boltzmann eq.:

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma_{\rm eff} v \rangle (n_\chi^2 - n_{\chi,{\rm eq}}^2)$$

$n_{\chi,({\rm eq})}$: (Equilibrium) number density H : Hubble parameter

$\langle \sigma v \rangle$:Thermally averaged annihilation cross section times velocity

$Y_\chi = n_\chi / s$

Co-moving number density

- At high temperatures:

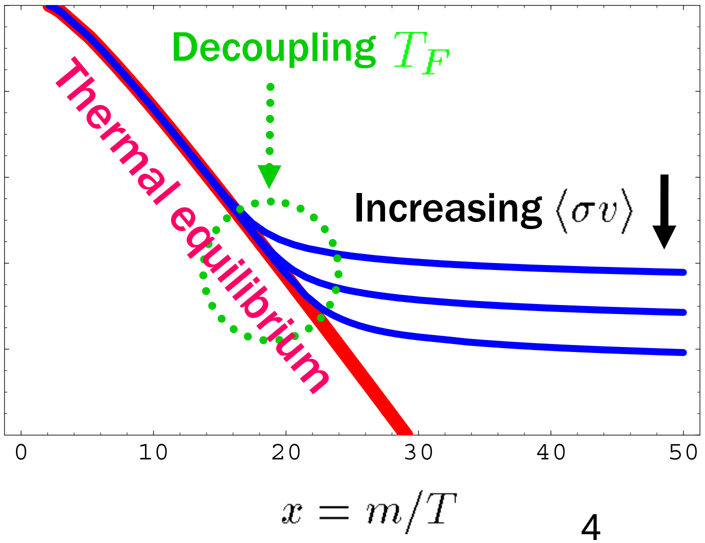
$$\Gamma = n_\chi \langle \sigma v \rangle > H = R/\dot{R}$$

Thermal equilibrium was maintained

- When $\Gamma < H$, the number density is fixed:

Decoupling, freeze-out

- χ decoupled in the RD epoch: $H = \frac{\pi T^2}{M_{\rm Pl}} \sqrt{\frac{90}{g_*}}$



Thermal abundances of hot and cold relics

- Hot relics (decouple for $x_F < 3$):

$$Y_{\chi,\mathrm{eq}} \equiv \frac{n_{\chi,\mathrm{eq}}}{s}$$ almost constant

➡ Final abundance is insensitive to the freeze out temperature:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F) = \frac{45}{2\pi^4}\frac{g_\chi}{g_{*s}(x_F)}$$

- Cold relics (decouple for $x_F > 3$):

$$\langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2),$$

$$n_{\chi,\mathrm{eq}} = g_\chi \, (m_\chi T/2\pi)^{3/2} \mathrm{e}^{-m_\chi/T}$$

$$\Omega_\chi h^2 = 2.7 \times 10^8 \, Y_\chi \left(\frac{m_\chi}{1 \text{ GeV}}\right)$$

➡

$$\Omega_{\chi,\mathrm{standard}} h^2 \simeq 0.1 \times \left(\frac{a + 3b/x_F}{10^{-9} \text{ GeV}^{-2}}\right)^{-1} \left(\frac{x_F}{22}\right) \left(\frac{g_*}{90}\right)^{-1/2} \sim \Omega_{\mathrm{DM}} h^2$$

3. Thermal abundance of semi-relativistic relics

- Precise evaluation of the abundance of particles that freeze out when they are semi-relativistic $(x_F \sim 3)$ is complicated

→ **Goal: simple analytic treatment that describes the transition from non-relativistic to relativistic relics**

- Assume the Maxwell-Boltzmann distribution:

$$Y_{\chi,\mathrm{eq}} \equiv \frac{n_{\chi,\mathrm{eq}}}{s} = 0.115 \frac{g_\chi}{g_{*s}} x^2 K_2(x)$$

($K_n(x)$: modified Bessel function)

→ Thermal average of cross section σ :

$$\langle \sigma v \rangle = \frac{1}{8 m_\chi^4 T K_2^2(m_\chi/T)} \int_{4m_\chi^2}^{\infty} \mathrm{d}s \ \sigma (s - 4m_\chi^2) \sqrt{s} \ K_1(\sqrt{s}/T)$$

Ansatz for approximate cross sections

- Consider neutrinos as stable relic:
- Annihilation cross section:

$$\sigma v^{\mathrm{Dirac}\ \nu} = \frac{G^2 s}{16\pi}$$

$$\sigma v^{\mathrm{Majorana}\ \nu} = \frac{G^2 s v^2}{16\pi}$$

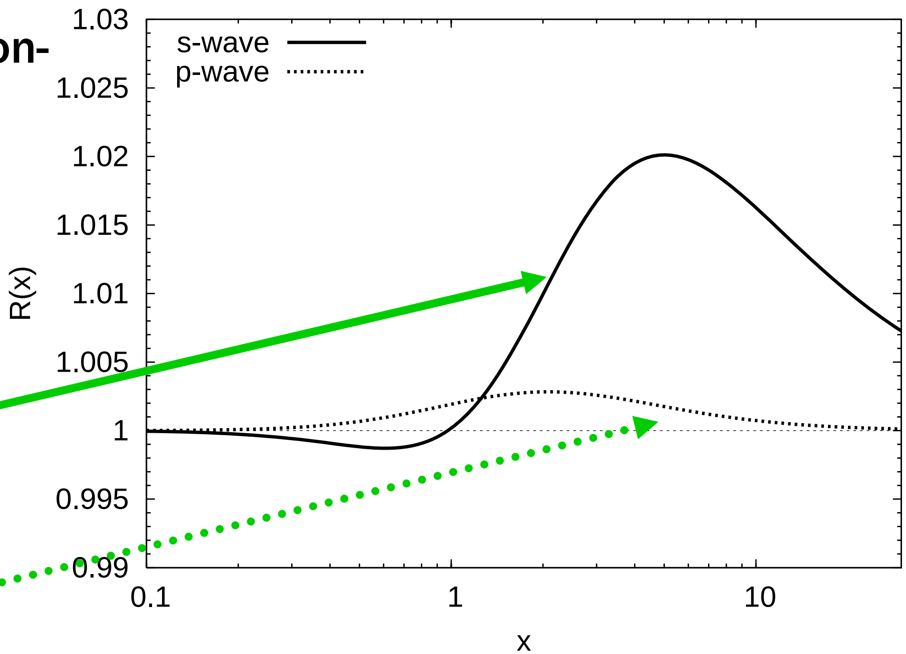
G : Effective dimension-six coupling

- Ansatz for the thermally-averaged annihilation cross section:

$$\langle \sigma v \rangle_{\mathrm{app}}^{\mathrm{Dirac}} = \frac{G^2 m_\chi^2}{16\pi}\left(\frac{12}{x^2} + \frac{5+4x}{1+x}\right)$$

$$\langle \sigma v \rangle_{\mathrm{app}}^{\mathrm{Majorana}} = \frac{G^2 m_\chi^2}{16\pi}\left(\frac{12}{x^2} + \frac{3+6x}{(1+x)^2}\right)$$

- $\langle \sigma v \rangle_{\mathrm{app}} / \langle \sigma v \rangle_{\mathrm{exact\ MB}}$:



The approx. cross sections reproduce the exact results with accuracy of a few %

Approximate abundance of semi-relativistic relics

- Define the freeze–out temperature by

$$\Gamma(x_F) = H(x_F)$$

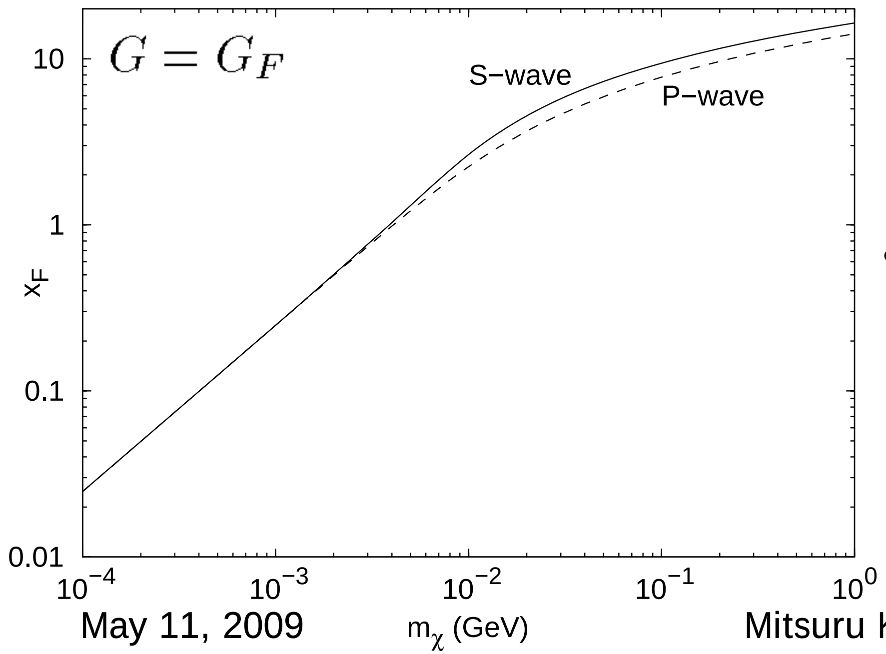
$$\Gamma(x_F) = n_{\chi,\mathrm{eq}}(x_F)\langle\sigma v\rangle(x_F)$$

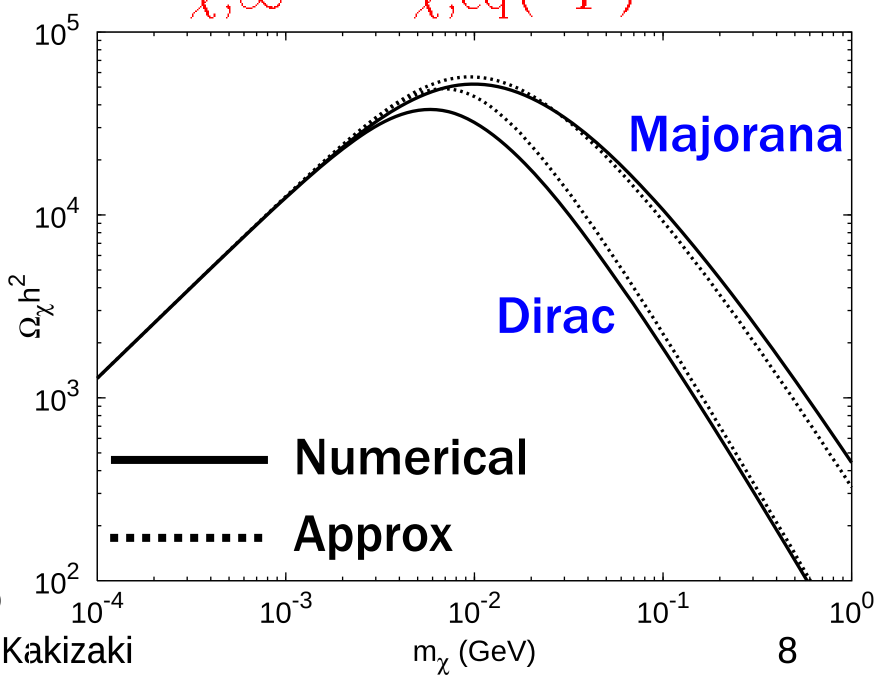
(different from the standard x_F)

- Assume the relic abundance does not change after decoupling

Final abundance:

$$Y_{\chi,\infty} = Y_{\chi,\mathrm{eq}}(x_F)$$





4.1. Semi-relativistic dark matter?

- Observed dark matter abundance: $\Omega_{\rm DM}h^2 \simeq 0.1$

  Final density of a semi-relativistic particle: $Y_{\chi,{\rm eq}}(x \simeq 3) \sim 10^{-2}$

  ⇒ $m_\chi \sim 100\ {\rm eV}, \quad T_F \sim \mathcal{O}(10)\ {\rm eV}$

  Semi-relativistic dark matter affects BBN

- Such light particle decouples when semi-relativistic

  ⇒ Effective dimension-six coupling is too large:

  $$G \simeq 10^3\ {\rm GeV}^{-2}$$

4.2. Entropy production by decaying particles

- Suppose a decaying particle dominates the energy of Universe
  - Out-of-eq. decay ⇒ Large entropy production ⇒ Unwanted relics diluted
  - Ratio of the final to initial entropy: $\frac{S_f}{S_i} = g_*^{1/4} \frac{m_\chi Y_{\chi,i} \tau_\chi^{1/2}}{M_{\rm Pl}^{1/2}} \propto \Omega_\chi h^2$
    [Steinhardt, Turner (1983)]
- In the case of the late decay of non-relativistic particles
  - χ energy density can dominate at low temperate: $T \ll {\rm e}^{-x_F} T_F$
  - Very large mass, very long lifetime needed for large entropy production
- In the case of the late decay of semi-relativistic particles
  - The abundance at decoupling is large

⇒ **Significant entropy can be produced even if the mass is small**

Example: sterile neutrino

- Consider a sterile neutrino mixed with an active neutrino (mixing angle: θ)
- Decay rate of the sterile neutrino:

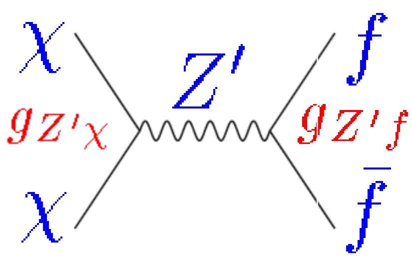
$$\Gamma_\chi = \frac{G_F^2 m_\chi^5}{192\pi^3}\sin^2\theta, \qquad \frac{G_F m_\chi^3}{16\pi}\sin^2\theta$$

(for small m_χ) (for large m_χ)

should be large enough not to spoil BBN

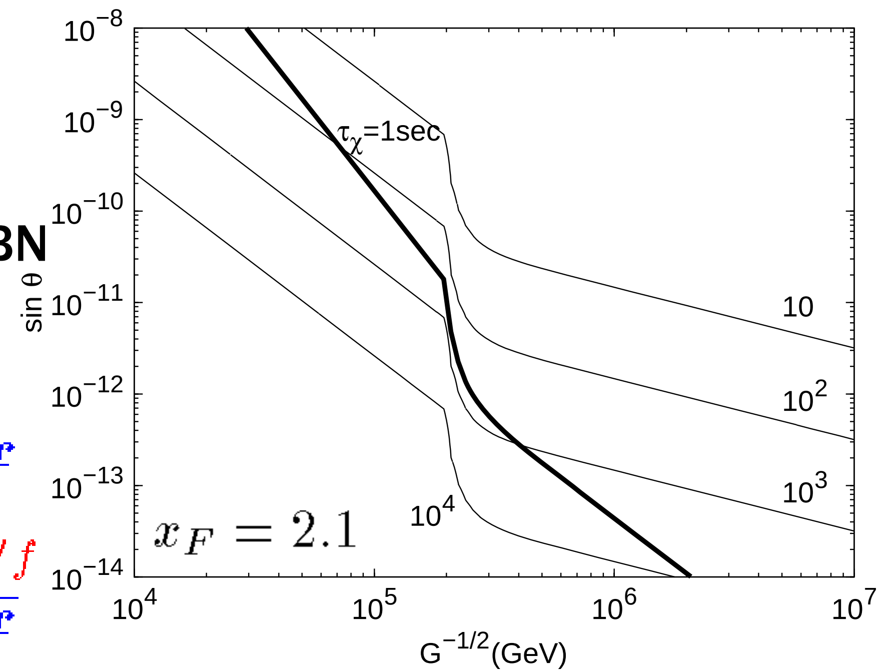
- By introducing a new heavy particle, large pair annihilation can be induced:

$$G = \frac{g_{Z'\chi}^2 g_{Z'f}^2}{M_Z'^4}$$



➡ $x_F \sim 3$ possible

- Entropy production S_f/S_i by the decay of semi-relativistic sterile neutrinos



5. Summary

- We find an useful approximate analytic formula for the abundance of semi-relativistic relics

- Semi-relativistic relics are useful for producing a large amount of entropy

Backup slides

- Observations of
  - cosmic microwave background
  - structure of the universe
  - etc.

→ **Non-baryonic dark matter:** $\Omega_{\rm DM}h^2 = 0.1143 \pm 0.0034$

Physics beyond the standard model (SM) of particle physics necessary

- Weakly interacting massive particles (WIMPs) χ are good candidates for dark matter (DM)

The predicted thermal relic abundance naturally explains the observed dark matter abundance: $\Omega_{\chi,\rm standard}h^2 \sim 0.1$

- Neutralino (LSP); 1^{st} KK mode of the B boson (LKP); etc.

2. Standard calculation of the WIMP relic abundance

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- Conventional assumptions for WIMPs as DM particle:
  - $\chi = \bar{\chi}$, single production of χ is forbidden
- WIMP abundance n_χ is determined by the Boltzmann eq.:

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

$H = \dot{R}/R$: Hubble expansion parameter

$\langle \sigma v \rangle$: thermal average of the annihilation cross section $\sigma(\chi\chi \to \mathrm{SM\ particles})$ times relative velocity v

$n_{\chi,\mathrm{eq}}$: equilibrium number density

- Introduce $Y_{\chi(,\mathrm{eq})} = \frac{n_{\chi(,\mathrm{eq})}}{s}, x = \frac{m_\chi}{T}$

$$\Rightarrow \frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle \sigma v \rangle s}{Hx}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2)$$