Pseudo-Dirac See-Saw Neutrinos

Northwestern University

Wei-Chih Huang

05.12.2009

Pheno 2009, Madison, May 11-13

Motivations

- **What are the current experimental constraints on right-handed neutrino masses?**
- **What are the consequences of right-handed neutrinos masses much smaller than Dirac neutrino masses?**
- **What are the consequences of the first Borexino experiment measurement of the ^{7}Be neutrino flux?**

What is the Pseudo-Dirac Neutrino?

- In normal see-saw mechanism, the mass matrix for neutrino is:

$$\begin{pmatrix} 0 & yv \\ (yv)^T & M \end{pmatrix}$$

Where y is the Yukawa coupling, v is the expectation value of the Higgs field and M is Majarona mass which is much larger than y*v to account for tiny neutrino mass.

- In the Pseudo-Dirac case, M is much smaller than yv; therefore, the mass spectrum are nearly degenerate.

Diagonalization

- **General n active and n sterile neutrinos case**

$$M_\nu = \begin{pmatrix} 0 & m \\ m^T & \epsilon_m \end{pmatrix}$$

$$M_\nu \simeq \begin{pmatrix} 1 & -\delta^* \\ \delta^T & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} U^* & -U^* \\ 1 & 1 \end{pmatrix} \begin{pmatrix} m^D(1+\epsilon^D) & 0 \\ 0 & -m^D(1-\epsilon^D) \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} U^\dagger & 1 \\ -U^\dagger & 1 \end{pmatrix} \begin{pmatrix} 1 & \delta \\ -\delta^\dagger & 1 \end{pmatrix}$$

$$m = U^* m^D$$

$$\delta = U\left(\frac{\epsilon^D}{2} + \varepsilon\right),$$

$$\epsilon_m = 2\epsilon^D m^D + \varepsilon^T m^D + m^D \varepsilon, \quad \text{and} \quad m^D \varepsilon^T = -\varepsilon m^D.$$

The 2+1 case

$$M_\nu = \begin{pmatrix} 0 & 0 & m\sin\theta \\ 0 & 0 & m\cos\theta \\ m\sin\theta & m\cos\theta & \epsilon_m \end{pmatrix},$$

$$V^T M_\nu V = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m\left(1+\frac{\epsilon}{2}\right) & 0 \\ 0 & 0 & -m\left(1-\frac{\epsilon}{2}\right) \end{pmatrix},$$

$$\epsilon_m = m * \epsilon$$

$$V = \begin{pmatrix} V_{e1} & V_{e2} & V_{e2'} \\ V_{a1} & V_{a2} & V_{a2'} \\ V_{s1} & V_{s2} & V_{s2'} \end{pmatrix} = \begin{pmatrix} \cos\theta & \frac{\sin\theta}{\sqrt{2}}\left(1-\frac{\epsilon}{4}\right) & \frac{\sin\theta}{\sqrt{2}}\left(1+\frac{\epsilon}{4}\right) \\ -\sin\theta & \frac{\cos\theta}{\sqrt{2}}\left(1-\frac{\epsilon}{4}\right) & \frac{\cos\theta}{\sqrt{2}}\left(1+\frac{\epsilon}{4}\right) \\ 0 & -\frac{1}{\sqrt{2}}\left(1+\frac{\epsilon}{4}\right) & \frac{1}{\sqrt{2}}\left(1-\frac{\epsilon}{4}\right) \end{pmatrix}$$

$$1 - P_{ee} = \sin^2 2\theta \sin^2\left(\frac{m^2 L}{4E}\right) + \mathcal{O}(\epsilon^2).$$

$$P_{es} = \sin^2\theta \sin^2\left(\frac{2m^2\epsilon L}{4E}\right),$$

$$P_{\mu s} = \cos^2\theta \sin^2\left(\frac{2m^2\epsilon L}{4E}\right).$$

Neutrino Propagation

- **In vacuum(for simplicity, just two active neutrinos)**

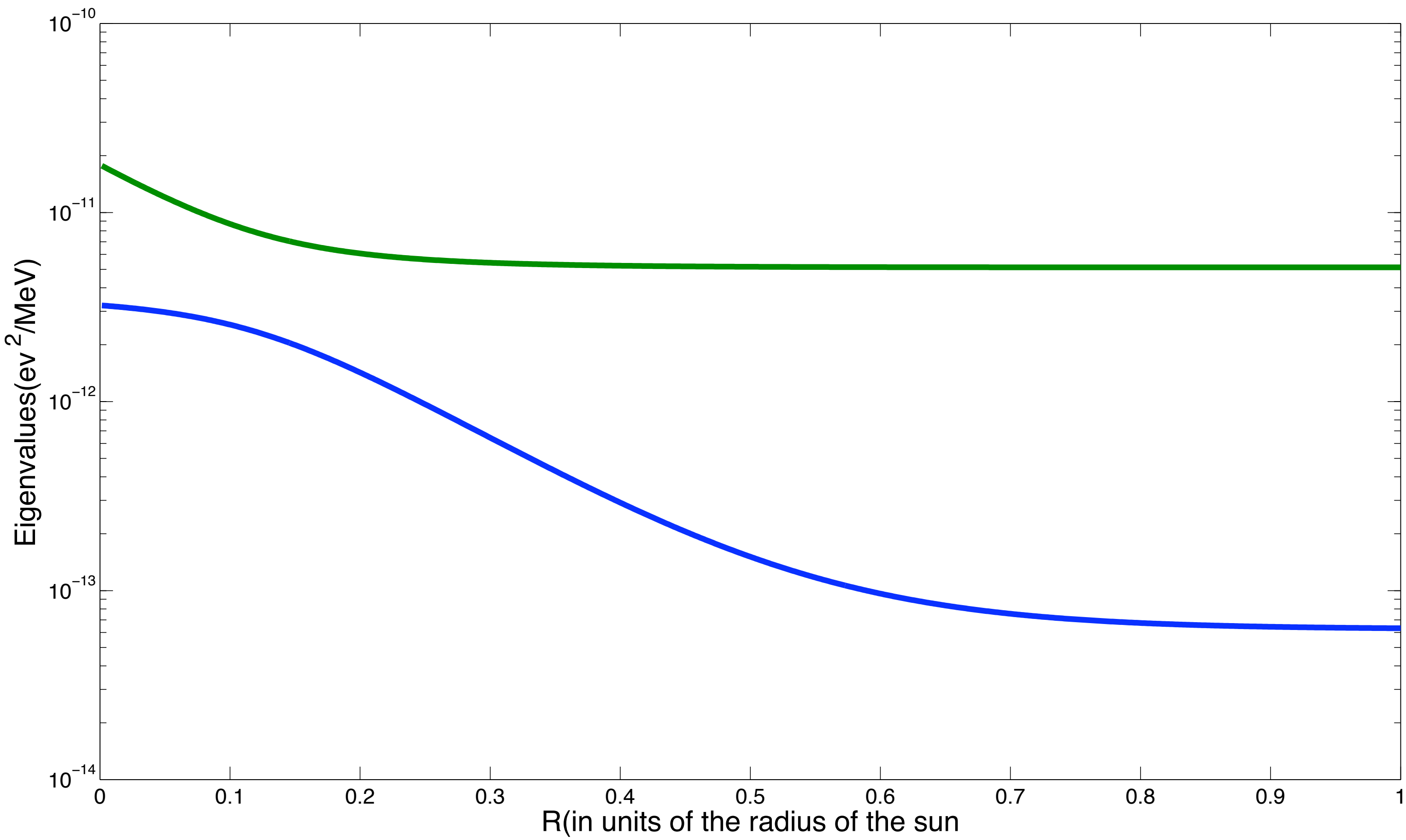
$$\imath \frac{d}{dL} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \frac{\Delta m^2}{2E} \begin{pmatrix} \sin^2\theta & \cos\theta\sin\theta \\ \cos\theta & \cos^2\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

where $\Delta m^2 = m_2^2 - m_1^2$, θ is the mixing angle and E is the neutrino energy

$$\imath \frac{d}{dL} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} A & \frac{\Delta}{2}\sin 2\theta \\ \frac{\Delta}{2}\sin 2\theta & \Delta\cos 2\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

where $\Delta = \frac{\Delta m^2}{2E}$ and A is the matter potential

Evolution of simultaneous Hamiltonian eigenstates in the case of 2 active neutrinos



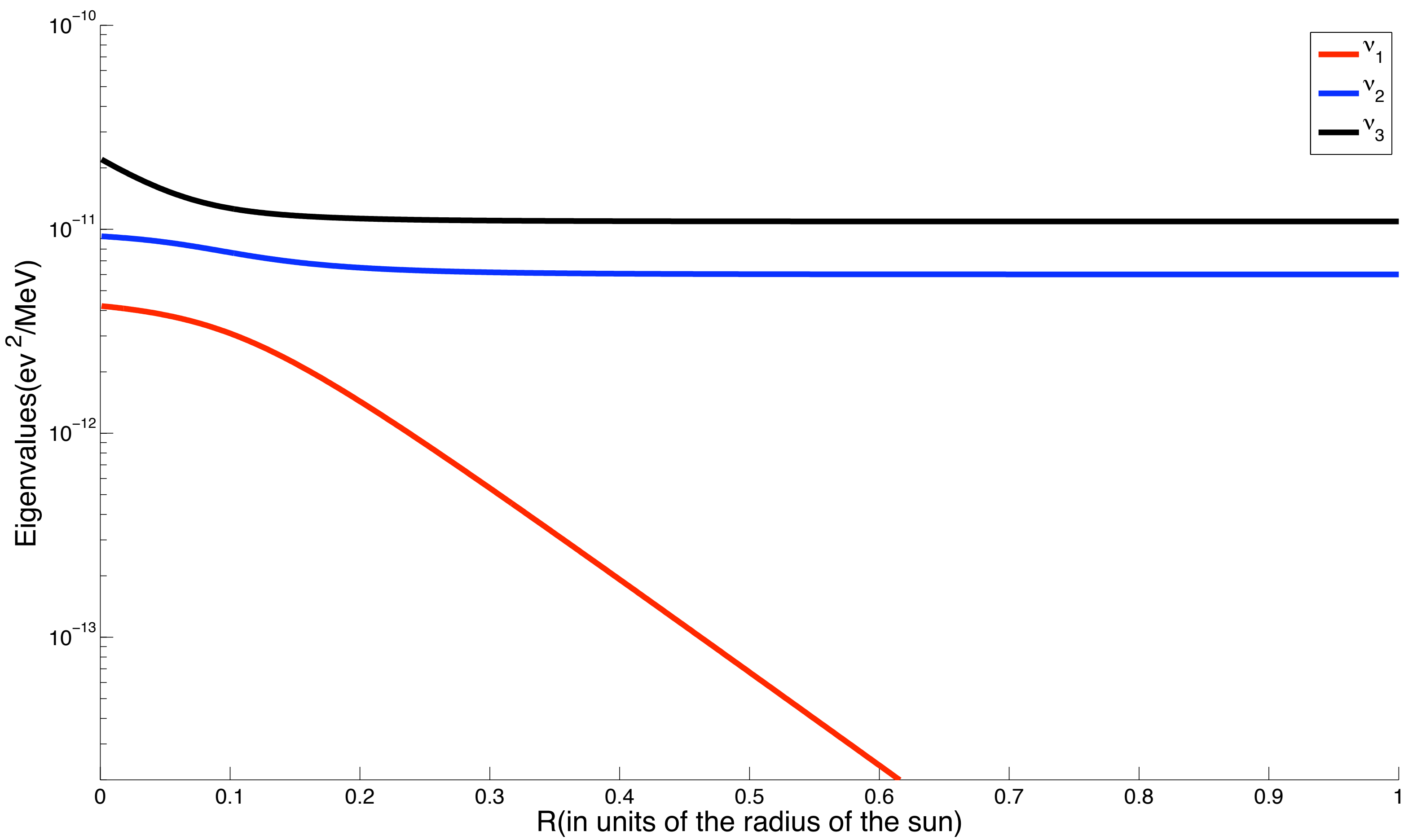
Transition Probability in Active Neutrinos only

For an exponential electron number density $N_e = N_{e0}e^{-L/r_0}$, P_c, also referred to as the crossing probability, is exactly calculable[29]

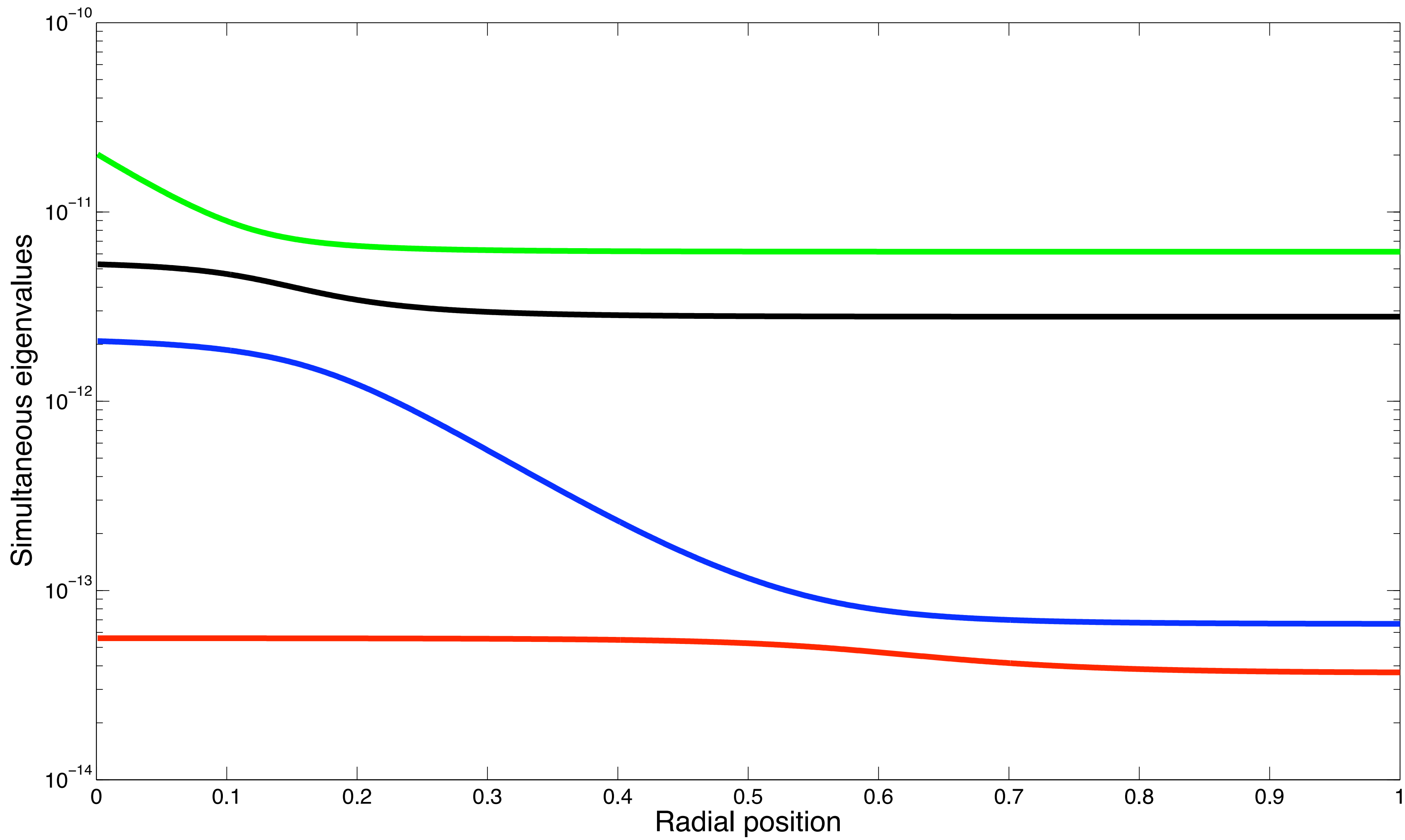
$$P_c = \frac{e^{-\gamma \sin^2\theta} - e^{-\gamma}}{1 - e^{-\gamma}}, \quad \gamma = 2\pi r_0 \Delta. \tag{53}$$

(from A. de Gouvêa, hep-ph/0411274)

Evolution of simultaneous Hamiltonian eigenstates in the case of 2 active and 1 sterile neutrinos



Evolution of simultaneous Hamiltonian eigenstates in the case of 2 active and 2 sterile neutrinos



Transition Probability in 2+1 case

$$H \longrightarrow V^T H V = \begin{pmatrix} A\cos^2\theta & A\cos\theta\sin\theta & 0 \\ A\cos\theta\sin\theta & A\sin^2\theta + \frac{m^2}{2E} & \frac{m\epsilon}{2E} \\ 0 & \frac{m\epsilon}{2E} & \frac{m^2}{2E} \end{pmatrix}.$$

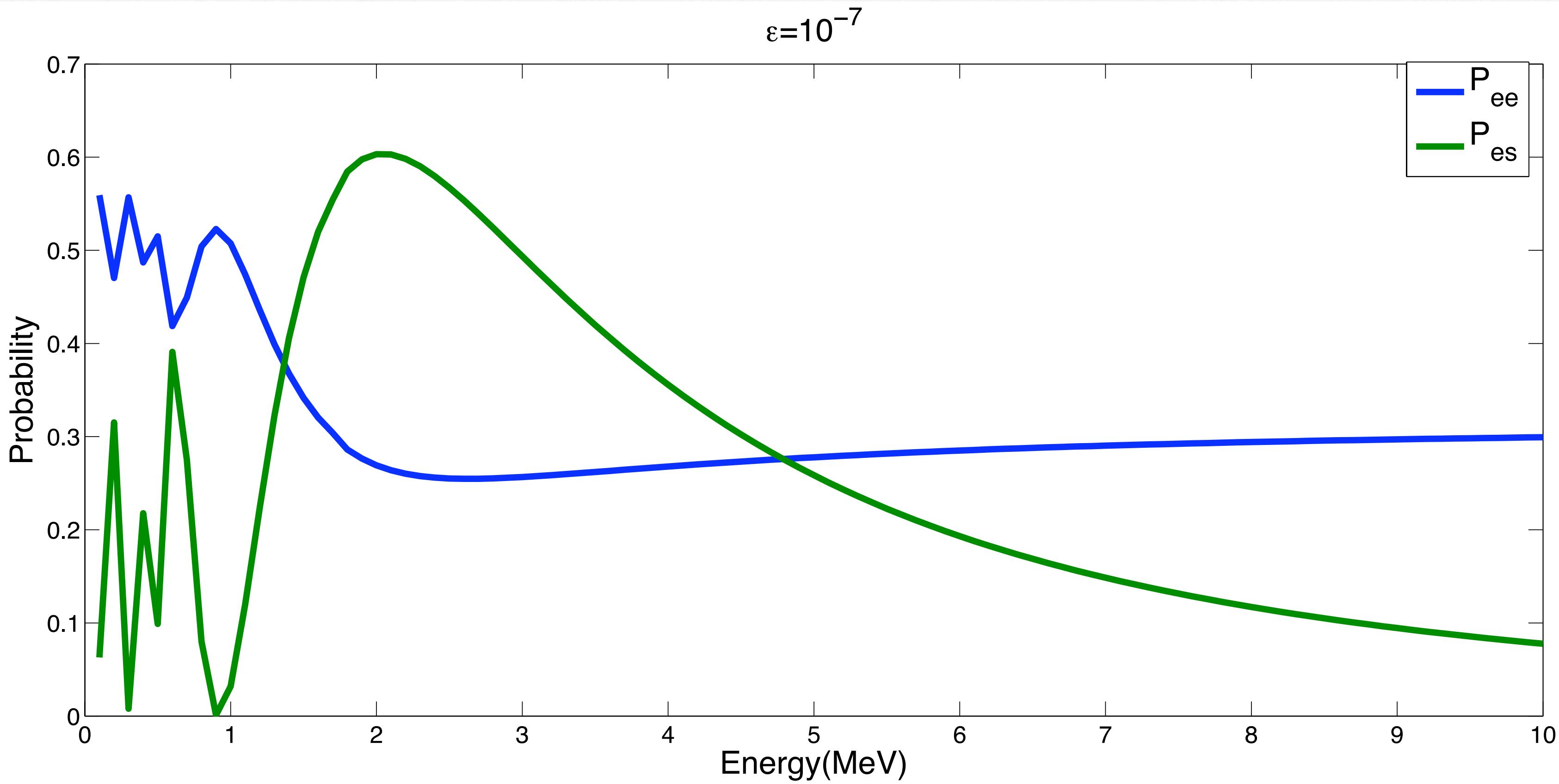
$$P_c = \frac{e^{-\gamma|V_{s2}|^2} - e^{-\gamma}}{1 - e^{-\gamma}}, \qquad \gamma \simeq 9.8\left(\frac{\epsilon}{10^{-4}}\right)\left(\frac{m^2}{8\times 10^{-5}\ \mathrm{eV}^2}\right)\left(\frac{0.862\ \mathrm{MeV}}{E}\right),$$

$$P_{ee} = \left| \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} V \begin{pmatrix} \exp(-i\phi_1') & 0 & 0 \\ 0 & \exp(-i\phi_2') & 0 \\ 0 & 0 & \exp(-i\phi_3') \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-P_c} & \sqrt{P_c} \\ 0 & -\sqrt{P_c} & \sqrt{1-P_c} \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} \exp(-i\phi_1) & 0 & 0 \\ 0 & \exp(-i\phi_2) & 0 \\ 0 & 0 & \exp(-i\phi_3) \end{pmatrix} V_{\mathrm{mat}}^\dagger \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right|^2,$$

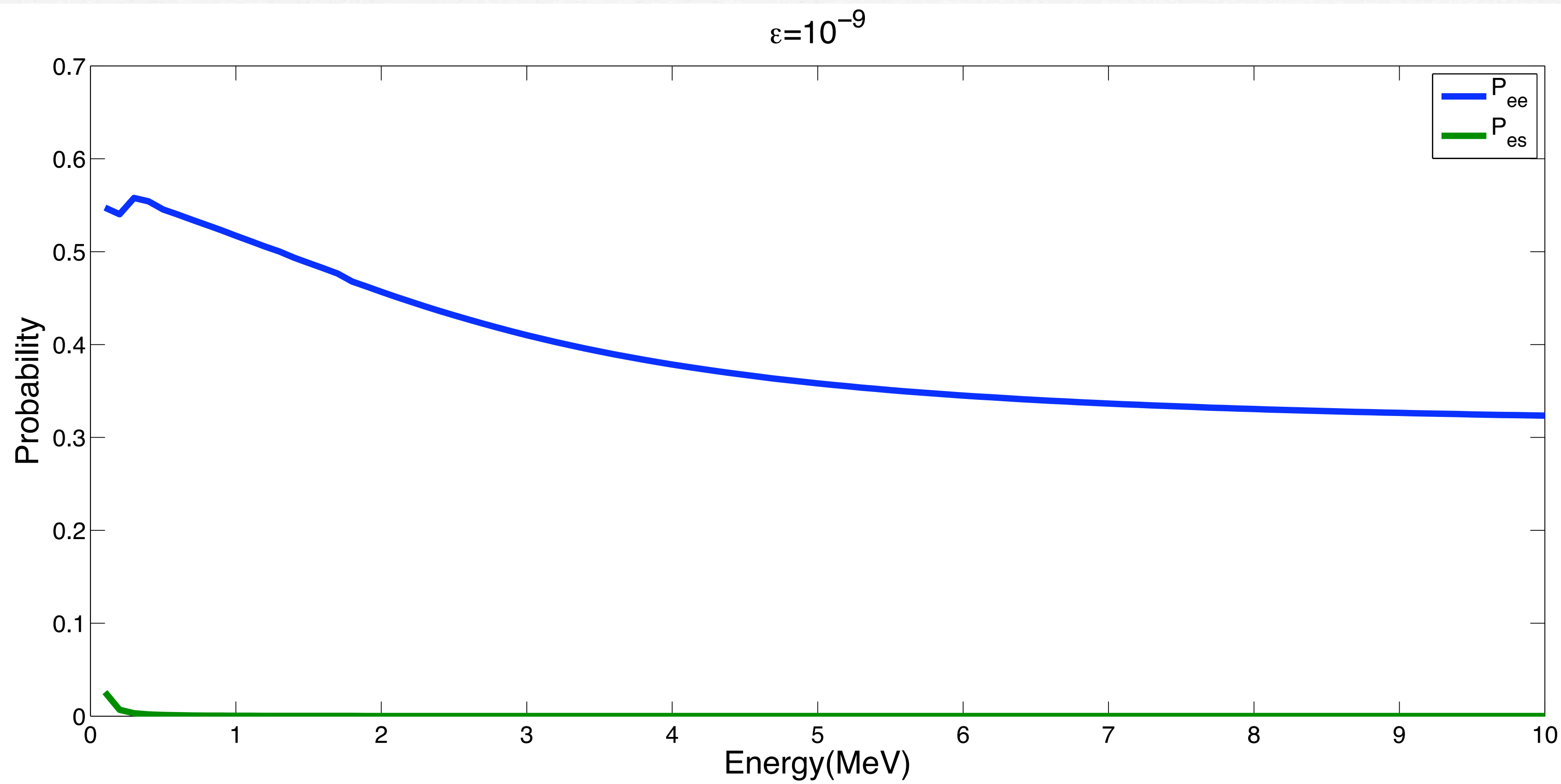
Results for 2+1 case

Here, we use $\theta = 33.9$ and $\Delta m^2 = 7.58 * 10^{-5}$ eV^2



Results for 2+1 case

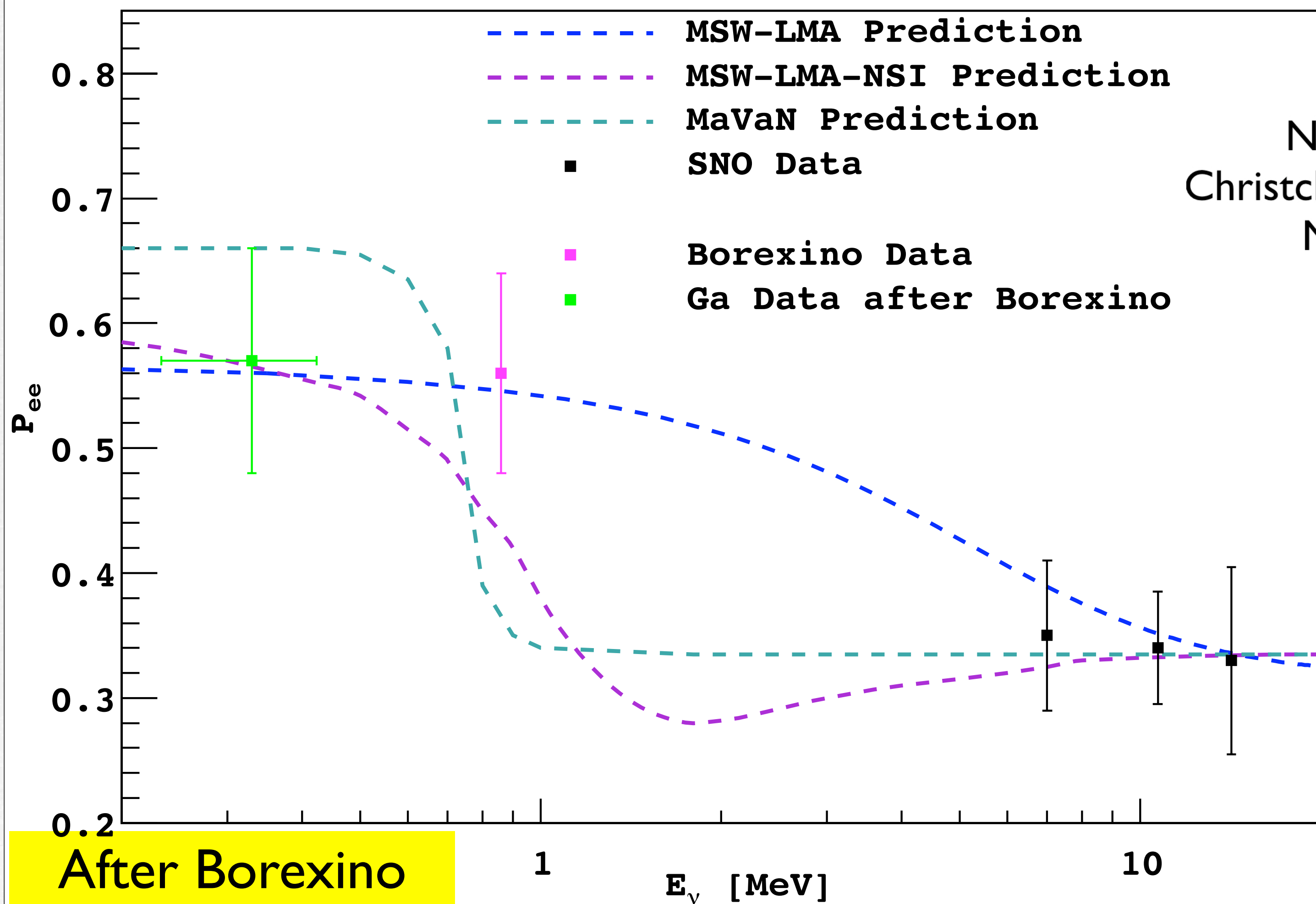
Here, we use $\theta = 33.9$ and $\Delta m^2 = 7.58 * 10^{-5}$ eV^2



Experimental data

- Super-K elastic-scattering
- SNO phase-III charged-current and neutral-current
- Borexino
- GALLEX+GNO
- Homestake

Solar Neutrino Survival Probability

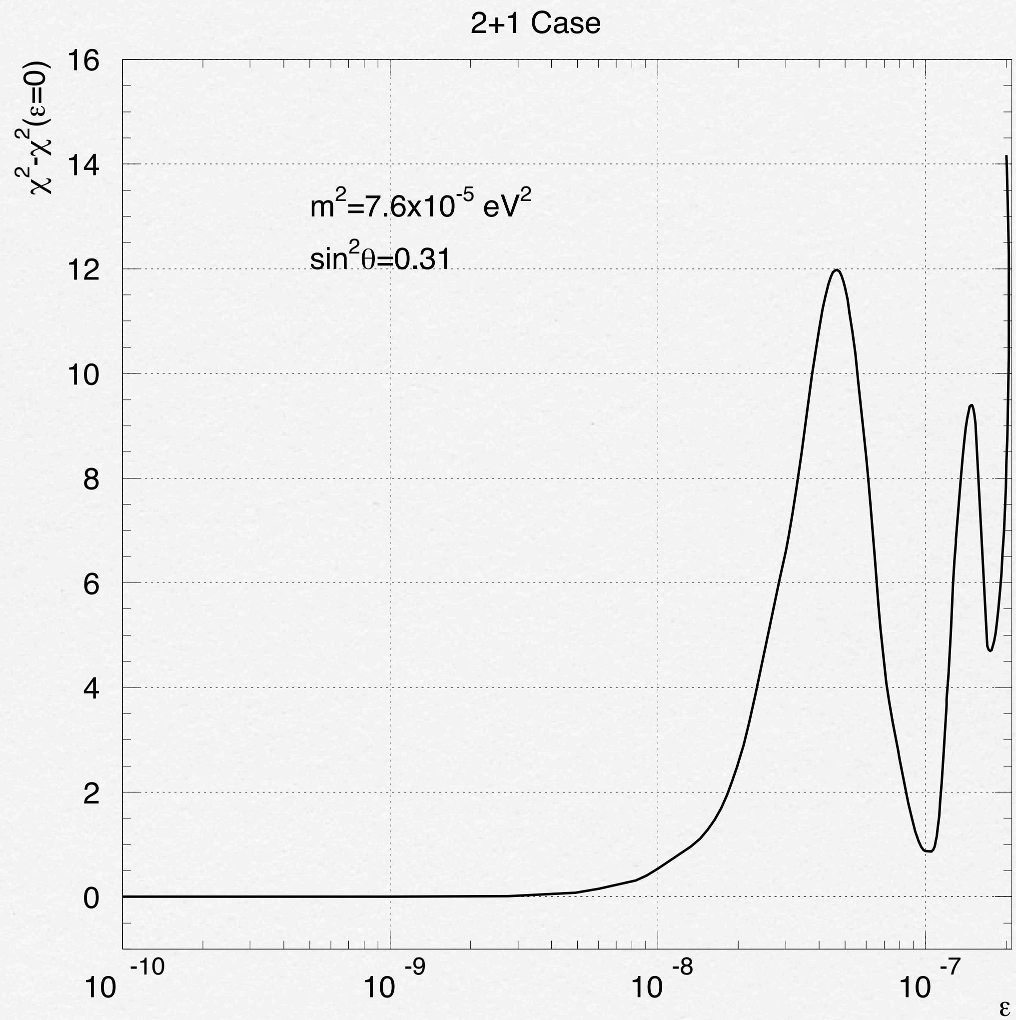


After Borexino

Neutrino 2008
Christchurch, New Zeland
May 26, 2008

Cristiano Galbiati

Chi-Squared Fit



Conclusion

- We analytically diagonalized the neutrino mass matrix in the limit of the small Majorana mass term.
- By using a special basis, the resonance region could be identified and the transition probability could be computed analytically.
- Constraint on right-handed Majorana masses (from solar data): $M<10^{-9}$ eV(for 2+1 case)