

$Z \rightarrow b\bar{b}$ in Higgsless Models

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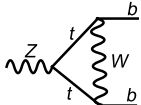
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Why Calculate $Z \rightarrow b_L \bar{b}_L$?

- The top quark is special in the SM model.
- Solutions to the hierarchy problem necessarily involve modifications / additions to the top sector, and the top quark mass sets a scale for flavor hierarchy.
- Through $SU(2)_L$, the coupling between Z and b_L feels these modifications and the well-measured Z -pole observables (ex: $\Gamma(Z \rightarrow b_L \bar{b}_L)$ and A_{LR}^b) serve as strong constraints.
- In this talk, we elucidate the *gaugeless* technique of calculating *flavor-dependent* corrections to $Z \rightarrow b_L \bar{b}_L$, and apply this technique to the Three-Site Higgsless Model.
- The reference contains many other ways of understanding the result of this calculation.


$Z \rightarrow b_L \bar{b}_L$ in the SM

The brute-force calculation is straightforward ($m_b = 0$, $m_W^2 \ll m_t^2$):



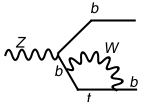
A Feynman diagram showing a Z boson (wavy line) on the left, which splits into a top quark (t) and an anti-top quark (t-bar). These two quarks form a loop with a W boson (wavy line) connecting them. The loop then splits into a bottom quark (b) and an anti-bottom quark (b-bar) on the right.

$$= i \frac{g_Z m_t^2}{192 \pi^2 v^2} \left[6 \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_t^2} \right) (4s_W^2 - 4) + (20s_W^2 - 12) \right]$$



A Feynman diagram showing a Z boson (wavy line) on the left, which splits into a W boson (wavy line) and a top quark (t). The W boson then splits into another W boson and a bottom quark (b). The top quark and the second W boson form a box loop with a bottom quark (b) and an anti-bottom quark (b-bar) on the right.

$$= i \frac{g_Z m_t^2}{192 \pi^2 v^2} \left[6 \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_t^2} \right) (7 - 6s_W^2) + (39 - 30s_W^2) \right]$$

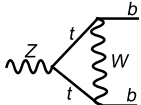



A Feynman diagram showing a Z boson (wavy line) on the left, which splits into a bottom quark (b) and an anti-bottom quark (b-bar). These two quarks form a triangle loop with a top quark (t) and a W boson (wavy line) connecting them. The loop then splits into a bottom quark (b) and an anti-bottom quark (b-bar) on the right.

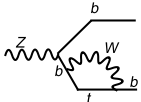
$$= i \frac{g_Z m_t^2}{192 \pi^2 v^2} \left[6 \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m_t^2} \right) (2s_W^2 - 3) + (10s_W^2 - 15) \right]$$

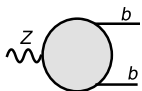
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$$= i \frac{g_Z m_t^2}{16 \pi^2 v^2} \Leftarrow$$

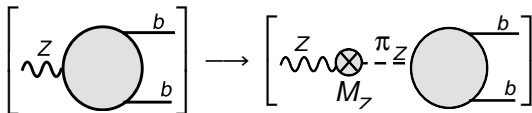
The result is finite, as expected, because there is no flavor-dependent counter-term in the SM.

The Gaugeless Calculation

Barbieri, Beccaria, Ciafaloni, Curi, and Vicere.

Phys. Lett. **B288**, 95 (1992). Nucl. Phys. **B409**, 105 (1993).

The key idea: treat Z -boson as external field that couples to $J_3^\mu - s_\theta^2 J_Q^\mu$.



For the gaugeless calculation, we compute the coefficient to the operator $\partial^\mu \pi_Z \bar{b}_L \gamma_\mu b_L$ and relate to $Z \rightarrow b_L \bar{b}_L$ through a Ward-Takahashi identity

The diagrammatic equation shows a loop diagram with a wavy 'Z' line on the left and two 'b' lines on the right, followed by a tilde symbol and the label 'M_Z'. This is followed by a loop diagram with a dashed 'π_Z' line on the left and two 'b' lines on the right, with 'p_μ' written below the loop. To the right of this is a plus sign and a large right-facing parenthesis containing the text 'Wavefunction renormalization contributions'.

$\pi_Z \rightarrow b_L \bar{b}_L$ in the SM

This is easier than the $Z \rightarrow b_L \bar{b}_L$ calculation:

$$M_Z \left[\begin{array}{c} \text{Diagram 1} \\ \text{---} \pi_Z \text{---} \\ \text{---} p_\mu \text{---} \end{array} \right] = i \frac{g_Z m_t^2}{16\pi^2 v^2}$$

$$M_Z \left[\begin{array}{c} \text{Diagram 2} \\ \text{---} \pi_Z \text{---} \\ \text{---} p_\mu \text{---} \end{array} \right] = 0 \quad (\text{No } \pi\pi\pi\text{-vertex})$$

$$M_Z \left[\begin{array}{c} \text{Diagram 3} \\ \text{---} \pi_Z \text{---} \\ \text{---} p_\mu \text{---} \end{array} \right] = 0 \quad (\text{No } \pi \bar{b}_L b_R\text{-vertex when } m_b = 0.)$$

$\pi_Z \rightarrow b_L \bar{b}_L$ in the SM

This is easier than the $Z \rightarrow b_L \bar{b}_L$ calculation:

$$M_Z \left[\begin{array}{c} \text{Diagram 1: } \pi_Z \text{ splits into } t \text{ and } \bar{t}, \text{ which then meet at a } \pi_W \text{ vertex to produce } b \text{ and } \bar{b}. \\ \text{---} \pi_Z \\ \text{---} t \\ \text{---} \bar{t} \\ \text{---} \pi_W \\ \text{---} b \\ \text{---} \bar{b} \end{array} \right]_{p_\mu} = i \frac{g_Z m_t^2}{16\pi^2 v^2}$$

$$M_Z \left[\begin{array}{c} \text{Diagram 2: } \pi_Z \text{ splits into } \pi_W \text{ and } \bar{\pi}_W, \text{ which then meet at a } t \text{ vertex to produce } b \text{ and } \bar{b}. \\ \text{---} \pi_Z \\ \text{---} \pi_W \\ \text{---} \bar{\pi}_W \\ \text{---} t \\ \text{---} b \\ \text{---} \bar{b} \end{array} \right]_{p_\mu} = 0 \quad (\text{No } \pi\pi\pi\text{-vertex})$$

$$M_Z \left[\begin{array}{c} \text{Diagram 3: } \pi_Z \text{ splits into } b \text{ and } \bar{b}, \text{ which then meet at a } \pi_W \text{ vertex to produce } t \text{ and } \bar{t}. \\ \text{---} \pi_Z \\ \text{---} b \\ \text{---} \bar{b} \\ \text{---} \pi_W \\ \text{---} t \\ \text{---} \bar{t} \end{array} \right]_{p_\mu} = 0 \quad (\text{No } \pi \bar{b}_L b_R\text{-vertex when } m_b = 0.)$$

$$M_Z \left[\begin{array}{c} \text{Diagram 4: } \pi_Z \text{ splits into } b \text{ and } \bar{b}, \text{ which then meet at a shaded circle representing a fermion loop.} \\ \text{---} \pi_Z \\ \text{---} b \\ \text{---} \bar{b} \\ \text{---} \text{Shaded Circle} \end{array} \right]_{p_\mu} = i \frac{g_Z m_t^2}{16\pi^2 v^2} \Leftarrow$$

There is only one, finite diagram in the SM.

The Gaugeless Limit

Barbieri, Beccaria, Ciafaloni, Curi, and Vicere.

Phys. Lett. **B288**, 95 (1992). Nucl. Phys. **B409**, 105 (1993).

In the *gaugeless* limit, we treat the massive Z -boson as an external, classical field that couples to the conserved (before EWSB) current

$$\begin{aligned}J^\mu &= -M_Z \partial^\mu \pi_Z + \hat{J}^\mu, \\ \hat{J}^\mu &= g_{Zbb}^L \bar{b}_L \gamma^\mu P_L b_L + g_{Zbb}^R \bar{b}_R \gamma^\mu P_R b_R + \dots\end{aligned}$$

The Ward-Takahashi identity then reads

$$\begin{aligned}\partial_\mu^x \left\langle T \hat{J}^\mu(x) b(y) \bar{b}(z) \right\rangle &= M_Z \left\langle T(\square_x \pi_Z(x)) b(y) \bar{b}(z) \right\rangle \\ &\quad - \delta(x-y) \left(g_{Zbb}^L P_L + g_{Zbb}^R P_R \right) \left\langle b(x) \bar{b}(z) \right\rangle \\ &\quad + \delta(x-z) \left\langle b(y) \bar{b}(x) \right\rangle \left(g_{Zbb}^L P_R + g_{Zbb}^R P_L \right),\end{aligned}$$

The Gaugeless Limit - cont.

In terms of amputated Green's functions, we have

$$\begin{aligned} & i(p_1 + p_2)_\mu \left\langle \hat{J}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) \right\rangle_{1\text{PI}} \\ &= -iM_Z \left\langle \pi_Z(p_1 + p_2) b(p_2) \bar{b}(p_1) \right\rangle_{1\text{PI}} \\ & \quad - S_{bb}^{-1}(p_1) \left(g_{Zbb}^L P_L + g_{Zbb}^R P_R \right) + \left(g_{Zbb}^L P_R + g_{Zbb}^R P_L \right) S_{bb}^{-1}(-p_2), \end{aligned}$$

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We can organize the Ward-Takahashi identity with the **Lorentz structure** of the Green's functions

The Gaugeless Limit - cont.

In terms of amputated Green's functions, we have

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 & i(p_1 + p_2)_\mu \left\langle \hat{J}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) \right\rangle_{1\text{PI}} \\
 &= -iM_Z \left\langle \pi_Z(p_1 + p_2) b(p_2) \bar{b}(p_1) \right\rangle_{1\text{PI}} \\
 & \quad - S_{bb}^{-1}(p_1) \left(g_{Zbb}^L P_L + g_{Zbb}^R P_R \right) + \left(g_{Zbb}^L P_R + g_{Zbb}^R P_L \right) S_{bb}^{-1}(-p_2),
 \end{aligned}$$

We can organize the Ward-Takahashi identity with the **Lorentz structure** of the Green's functions

$$\begin{aligned}
 & i(\not{p}_1 + \not{p}_2) \left(J_\mu \begin{array}{c} \nearrow p_1 \quad b \\ \circ \\ \searrow p_2 \quad b \end{array} \right)_{\gamma^\mu} = -iM_Z(\not{p}_1 + \not{p}_2) \left(\frac{\pi_Z}{\not{p}_1 + \not{p}_2} \begin{array}{c} \nearrow p_1 \quad b \\ \circ \\ \searrow p_2 \quad b \end{array} \right)_{\not{p}} \\
 & \quad - \left(\begin{array}{c} \overrightarrow{p_1} \\ \circ \\ \overrightarrow{b} \end{array} \right)_{\not{p}}^{-1} \not{p}_1 \left(g_{Zbb}^L P_L + g_{Zbb}^R P_R \right) + \left(g_{Zbb}^L P_R + g_{Zbb}^R P_L \right) \left(\begin{array}{c} \overrightarrow{p_2} \\ \circ \\ \overrightarrow{b} \end{array} \right)_{\not{p}}^{-1} \not{p}_2,
 \end{aligned}$$

At tree level, this gives the gauge couplings of the Z-boson.

The Gaugeless Limit - cont.

In terms of amputated Green's functions, we have

$$\begin{aligned}
 & i(p_1 + p_2)_\mu \left\langle \hat{J}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) \right\rangle_{1\text{PI}} \\
 &= -iM_Z \left\langle \pi_Z(p_1 + p_2) b(p_2) \bar{b}(p_1) \right\rangle_{1\text{PI}} \\
 & \quad - S_{bb}^{-1}(p_1) \left(g_{Zbb}^L P_L + g_{Zbb}^R P_R \right) + \left(g_{Zbb}^L P_R + g_{Zbb}^R P_L \right) S_{bb}^{-1}(-p_2),
 \end{aligned}$$

We can organize the Ward-Takahashi identity with the Lorentz structure of the Green's functions

$$\begin{aligned}
 & i(p_1 + p_2)_\mu \left(J_\mu \begin{array}{c} \nearrow p_1 \quad b \\ \circ \\ \searrow p_2 \quad b \end{array} \right) \quad (= 0) = -iM_Z \left(\frac{\pi_Z}{p_1 + p_2} \begin{array}{c} \nearrow p_1 \quad b \\ \circ \\ \searrow p_2 \quad b \end{array} \right) \\
 & \quad \quad \quad \gamma^\mu \gamma_5 \quad \quad \quad \gamma_5 \\
 & \quad - \left(\begin{array}{c} \overleftarrow{p_1} \\ \circ \\ \overrightarrow{b} \end{array} \right)_\perp^{-1} \left(g_{Zbb}^L P_L + g_{Zbb}^R P_R \right) + \left(g_{Zbb}^L P_R + g_{Zbb}^R P_L \right) \left(\begin{array}{c} \overleftarrow{p_2} \\ \circ \\ \overrightarrow{b} \end{array} \right)_\perp^{-1},
 \end{aligned}$$

At tree level, this relates the $\pi_Z b \bar{b}$ coupling to m_b .

The Ward Identity at Loop-Level

The one-loop $Zb_L\bar{b}_L$ coupling is given by

$$i g_{Zbb}^{L,1\text{-loop}} = i \left(J_\mu \begin{array}{c} \nearrow p_1 \quad b \\ \searrow p_2 \quad b \end{array} \right) \gamma^\mu$$

The Ward Identity at Loop-Level

The one-loop $Zb_L\bar{b}_L$ coupling is given by

$$i g_{Zbb}^{L,1\text{-loop}} = i \sqrt{Z_b^L} \left(J_\mu \begin{array}{c} p_1 \nearrow b \\ \bullet \\ p_2 \searrow b \end{array} \right)_{\gamma^\mu} \sqrt{Z_b^L}$$

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 i g_{Zbb}^{L,1\text{-loop}} &= i \sqrt{Z_b^L} \left(\text{Diagram 1} \right)_{\gamma^\mu} \sqrt{Z_b^L} \\
 &= -i M_Z \sqrt{Z_b^L} \left(\text{Diagram 2} \right)_{\not{p}'} \sqrt{Z_b^L} - (g_{Zbb}^L) \sqrt{Z_b^L} \left(\text{Diagram 3} \right)_{\not{p}'}^{-1} \sqrt{Z_b^L}
 \end{aligned}$$

Diagram 1: A vertex labeled J_μ with two outgoing fermion lines labeled b and momenta p_1 and p_2 .

Diagram 2: A vertex labeled π_Z with two outgoing fermion lines labeled b and momenta p_1 and p_2 , and an incoming boson line labeled Z with momentum p_1+p_2 .

Diagram 3: A vertex with two outgoing fermion lines labeled b and momenta p_1 and p_2 .

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 &= -i M_Z \sqrt{Z_b^L} \left(\frac{\pi_Z}{p_1+p_2} \begin{array}{c} p_1 \nearrow b \\ \bullet \\ p_2 \searrow b \end{array} \right)_{\not{p}'} \sqrt{Z_b^L} - (g_{Zbb}^L) \sqrt{Z_b^L} \left(\begin{array}{c} p_1 \rightarrow b \\ \bullet \\ p_2 \rightarrow b \end{array} \right)_{\not{p}'}^{-1} \sqrt{Z_b^L} \\
 &= -i M_Z \left(\frac{\pi_Z}{p_1+p_2} \begin{array}{c} p_1 \nearrow b \\ \bullet \\ p_2 \searrow b \end{array} \right)_{\not{p}'} + i (g_{Zbb}^{L,\text{tree}}) \left(1 + \frac{1}{2} \delta Z_b^L \right) (1 - \delta Z_b^L) \left(1 + \frac{1}{2} \delta Z_b^L \right)
 \end{aligned}$$

The Ward Identity at Loop-Level

The one-loop $Zb_L\bar{b}_L$ coupling is given by

$$\begin{aligned}
 i g_{Zbb}^{L,1\text{-loop}} &= i \sqrt{Z_b^L} \left(J_\mu \begin{array}{c} p_1 \nearrow b \\ \circlearrowleft \\ p_2 \searrow b \end{array} \right)_{\gamma^\mu} \sqrt{Z_b^L} \\
 &= -i M_Z \sqrt{Z_b^L} \left(\frac{\pi_Z}{p_1+p_2} \begin{array}{c} p_1 \nearrow b \\ \circlearrowleft \\ p_2 \searrow b \end{array} \right)_{\not{p}'} \sqrt{Z_b^L} - (g_{Zbb}^L) \sqrt{Z_b^L} \left(b \begin{array}{c} p_1 \rightarrow \\ \circlearrowleft \\ b \end{array} \right)_{\not{p}'}^{-1} \sqrt{Z_b^L} \\
 &= -i M_Z \left(\frac{\pi_Z}{p_1+p_2} \begin{array}{c} p_1 \nearrow b \\ \circlearrowleft \\ p_2 \searrow b \end{array} \right)_{\not{p}'} + i (g_{Zbb}^{L,\text{tree}}) \left(1 + \frac{1}{2} \delta Z_b^L \right) (1 - \delta Z_b^L) \left(1 + \frac{1}{2} \delta Z_b^L \right) \\
 &= i g_{Zbb}^{L,\text{tree}} - i M_Z \left(\frac{\pi_Z}{p_1+p_2} \begin{array}{c} p_1 \nearrow b \\ \circlearrowleft \\ p_2 \searrow b \end{array} \right)_{\not{p}'}
 \end{aligned}$$

The Ward Identity at Loop-Level

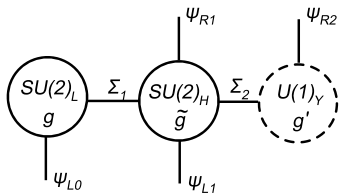
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 &= -i M_Z \sqrt{Z_b^L} \left(\frac{\pi_Z}{p_1+p_2} \right)_{\not{p}'} \sqrt{Z_b^L} - (g_{Zbb}^L) \sqrt{Z_b^L} \left(\frac{p_1}{b} \right)_{\not{p}'}^{-1} \sqrt{Z_b^L} \\
 &= -i M_Z \left(\frac{\pi_Z}{p_1+p_2} \right)_{\not{p}'} + i (g_{Zbb}^{L,\text{tree}}) \left(1 + \frac{1}{2} \delta Z_b^L \right) (1 - \delta Z_b^L) \left(1 + \frac{1}{2} \delta Z_b^L \right) \\
 &= i g_{Zbb}^{L,\text{tree}} - i M_Z \left(\frac{\pi_Z}{p_1+p_2} \right)_{\not{p}'} \quad \Leftarrow \quad \text{This justifies the earlier calculation.}
 \end{aligned}$$

The Three-Site Higgsless Model - Set up

Chivukula, Coleppa, Di Chiara, Simmons, He, Kurachi, and Tanabashi. PRD 74, 075011 (2006).

*Additional references in the paper.



Three-site \rightarrow maximally deconstructed.

Higgsless \rightarrow non-linear Σ -fields.

Implements *ideal delocalization*:
 $\alpha S = 0$ at tree-level.

$$\mathcal{L}_G = \frac{f_1^2}{4} \text{Tr} \left[(D_\mu \Sigma_1)^\dagger (D_\mu \Sigma_1) \right] + \frac{f_2^2}{4} \text{Tr} \left[(D_\mu \Sigma_2)^\dagger (D_\mu \Sigma_2) \right],$$

$$\Sigma = \text{Exp} \left[i \frac{\pi^A}{f_1} \sigma^A \right], \quad D^\mu \Sigma_1 = \partial^\mu \Sigma_1 + i\tilde{g} (W_H^\mu \cdot \Sigma_1) - ig (\Sigma_1 \cdot W_L^\mu),$$

$$-\mathcal{L}_F = M_D \left(\epsilon_L \bar{\psi}_{L0} \Sigma_1 \psi_{R1} + \bar{\psi}_{L1} \psi_{R1} + \epsilon_R \bar{\psi}_{L1} \Sigma_2 \psi_{R2} \right)$$

The Three-Site Higgsless Model - Spectra and Features

Spectra and mass-generation of the Three-Site model:

- Additional gauge bosons $M_{W',Z'} \sim \tilde{g}f > 400$ GeV
Additional fermions $M_F \sim M_D \gtrsim 3$ TeV.
- SM fermion masses generated through a seesaw-type mechanism ($M_f \sim \epsilon_L \epsilon_R M_D$).
- Flavor-dependencies stored in ϵ_R : ϵ_L is flavor-blind at tree-level and crucial for implementing *ideal-delocalization*.

The new ingredients to $Z \rightarrow b_L \bar{b}_L$ in 3SHM

- The physical states are no longer gauge eigenstates, even at tree-level.
- The presence of **off-diagonal couplings to the Z** (e.g. $Z_\mu \bar{B}_L \gamma^\mu b_L$).
- We can not take the *gaugeless* limit for $SU(2)_h$: it remains gauged.

The Ward-Takahashi Identity in 3-Site Higgsless Model

The current now contains extra contributions ...

$$\begin{aligned}J_{3S}^\mu &= \hat{J}_{3S}^\mu - M_Z \partial^\mu \pi_Z, \\ \hat{J}_{3S}^\mu &= \bar{b} \gamma^\mu g_{Zbb} b + (\bar{B} \gamma^\mu g_{ZBb} b + \text{h.c.}) + \dots, \\ g_{Zbb} &\equiv g_{Zbb}^L P_L + g_{Zbb}^R P_R, \quad \tilde{g}_{Zbb} \equiv g_{Zbb}^L P_R + g_{Zbb}^R P_L,\end{aligned}$$

and so does the Ward-Takahashi identity

$$\begin{aligned}\partial_\mu^x \left\langle T \hat{J}_{3S}^\mu(x) b(y) \bar{b}(z) \right\rangle &= M_Z \left\langle T(\square_x \pi_Z(x)) b(y) \bar{b}(z) \right\rangle \\ &\quad - \delta(x-y) \left[g_{Zbb} \left\langle b(x) \bar{b}(z) \right\rangle + g_{ZBb} \left\langle B(x) \bar{b}(z) \right\rangle \right] \\ &\quad + \delta(x-z) \left[\left\langle b(y) \bar{b}(x) \right\rangle \tilde{g}_{Zbb} + \left\langle b(y) \bar{B}(x) \right\rangle \tilde{g}_{ZBb} \right].\end{aligned}$$

Amputation with Kinetic Mixing

The Ward-Takahashi identity is a relationship among **full Green's functions**:

$$\begin{aligned} \partial_\mu^x \left\langle T \hat{J}_{3S}^\mu(x) b(y) \bar{b}(z) \right\rangle &= M_Z \left\langle T(\square_x \pi_z(x)) b(y) \bar{b}(z) \right\rangle \\ &\quad - \delta(x-y) \left[g_{zbb} \left\langle b(x) \bar{b}(z) \right\rangle + g_{Zbb} \left\langle B(x) \bar{b}(z) \right\rangle \right] \\ &\quad + \delta(x-z) \left[\left\langle b(y) \bar{b}(x) \right\rangle \tilde{g}_{Zbb} + \left\langle b(y) \bar{B}(x) \right\rangle \tilde{g}_{ZBb} \right]. \end{aligned}$$

With **kinetic-mixing**, we have to **properly amputate** full Green's functions.

$$\begin{aligned} \left(J_\mu \begin{array}{c} b \\ \circlearrowleft \\ b \end{array} \right) &= \left(\begin{array}{c} \cancel{b} \\ \circlearrowleft \\ \cancel{b} \end{array} \right) \left(J_\mu \begin{array}{c} b \\ \circlearrowleft \\ \cancel{b} \end{array} \right) \left(\begin{array}{c} \cancel{b} \\ \circlearrowleft \\ \cancel{b} \end{array} \right) + \left(\begin{array}{c} \cancel{b} \\ \circlearrowleft \\ \cancel{B} \end{array} \right) \left(J_\mu \begin{array}{c} B \\ \circlearrowleft \\ \cancel{b} \end{array} \right) \left(\begin{array}{c} \cancel{b} \\ \circlearrowleft \\ \cancel{b} \end{array} \right) \\ &+ \left(\begin{array}{c} \cancel{b} \\ \circlearrowleft \\ \cancel{b} \end{array} \right) \left(J_\mu \begin{array}{c} b \\ \circlearrowleft \\ \cancel{B} \end{array} \right) \left(\begin{array}{c} \cancel{B} \\ \circlearrowleft \\ \cancel{b} \end{array} \right) + \left(\begin{array}{c} \cancel{b} \\ \circlearrowleft \\ \cancel{B} \end{array} \right) \left(J_\mu \begin{array}{c} B \\ \circlearrowleft \\ \cancel{B} \end{array} \right) \left(\begin{array}{c} \cancel{B} \\ \circlearrowleft \\ \cancel{b} \end{array} \right) \end{aligned}$$

Some Intermediate Steps

Compared to the SM, we have many more terms ...

$$\begin{aligned}
 & i(p_1 + p_2)_\mu \left\{ \left(J_\mu \text{diagram} \right) + \left(\text{diagram} \right)^{-1} \left(\text{diagram} \right) \left(J_\mu \text{diagram} \right) \right. \\
 & \quad \left. + \left(J_\mu \text{diagram} \right) \left(\text{diagram} \right) \left(\text{diagram} \right)^{-1} \right\} \\
 & = -iM_Z \left\{ \left(\pi_Z \text{diagram} \right) + \left(\text{diagram} \right)^{-1} \left(\text{diagram} \right) \left(\pi_Z \text{diagram} \right) \right. \\
 & \quad \left. + \left(\pi_Z \text{diagram} \right) \left(\text{diagram} \right) \left(\text{diagram} \right)^{-1} \right\} \\
 & \quad - \left(\text{diagram} \right)^{-1} g_{Zbb} - \left(\text{diagram} \right)^{-1} g_{ZBb} \left(\text{diagram} \right) \left(\text{diagram} \right)^{-1} \\
 & \quad + \tilde{g}_{Zbb} \left(\text{diagram} \right)^{-1} + \left(\text{diagram} \right)^{-1} \left(\text{diagram} \right) \tilde{g}_{ZBb} \left(\text{diagram} \right)^{-1},
 \end{aligned}$$

Some Intermediate Steps

Compared to the SM, we have many more terms ... but they are all **one-loop at leading order**.

$$\begin{aligned}
 & i(p_1 + p_2)_\mu \left\{ \left(J_\mu \text{diagram} \right) + \left(\text{diagram} \right)^{-1} \left(\text{diagram} \right) \left(J_\mu \text{diagram} \right) \right. \\
 & \quad \left. + \left(J_\mu \text{diagram} \right) \left(\text{diagram} \right) \left(\text{diagram} \right)^{-1} \right\} \\
 & = -iM_Z \left\{ \left(\pi_Z \text{diagram} \right) + \left(\text{diagram} \right)^{-1} \left(\text{diagram} \right) \left(\pi_Z \text{diagram} \right) \right. \\
 & \quad \left. + \left(\pi_Z \text{diagram} \right) \left(\text{diagram} \right) \left(\text{diagram} \right)^{-1} \right\} \\
 & \quad - \left(\text{diagram} \right)^{-1} g_{Zbb} - \left(\text{diagram} \right)^{-1} g_{ZBb} \left(\text{diagram} \right) \left(\text{diagram} \right)^{-1} \\
 & \quad + \tilde{g}_{Zbb} \left(\text{diagram} \right)^{-1} + \left(\text{diagram} \right)^{-1} \left(\text{diagram} \right) \tilde{g}_{ZBb} \left(\text{diagram} \right)^{-1},
 \end{aligned}$$

The final result

$$\begin{aligned}
 i g_{Zbb,3S}^{L,1\text{-loop}} &= i g_{Zbb}^{L,\text{tree}} - i M_Z \left[\left(\text{Diagram 1} \right)_{\gamma^\mu} - \frac{g \pi_Z \bar{b}_L b_L}{M_B} \left(\text{Diagram 2} \right) \right], \\
 &= g_{Zbb}^{L,\text{tree}} + g_Z \frac{m_t^2}{16\pi^2 v^2} \left(1 + \frac{1}{8} \ln \frac{\Lambda^2}{M_D^2} \right).
 \end{aligned}$$

Some features:

- Only **kinetic-mixing** (not mass-mixing) enters the calculation: we are after the coefficient of the operator $(\partial_\mu \pi_Z) \bar{b}_L \gamma^\mu b_L$.
- Only $\ln(\Lambda^2/M^2)$ and **not** $\ln(M^2/m_t^2)$:
We can understand $\ln(\Lambda^2/M^2)$ from RGE effects of ϵ_L , and there is no flavor-non-universal operator that gives further scaling below M .

Other Checks & Conclusions

In the paper, we obtain the same result and further insights via several other calculations:

- direct calculation in unitary gauge,
- renormalization group analysis,
- chiral current.

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In the paper, we obtain the same result and further insights via several other calculations:

- direct calculation in unitary gauge,
- renormalization group analysis,
- chiral current.

We can also apply this technique to other models and derive constraints on the relevant parameters. One particular model is the general Universal Extra Dimension model (in progress: N. Christensen, T. Flacke, K. Hsieh, and A. Menon).