$Z \rightarrow b\bar{b}$ in Higgsless Models

PRD **79**, 075016 (2009) [arXiv:0902.3910]

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Pheno 09

Madison, Wisconsin

May 11, 2009

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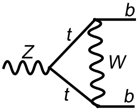
Why Calculate $Z \to b_L \overline{b}_L$?

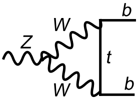
- The top quark is special in the SM model.
- Solutions to the hierarchy problem necessarily involve modifications / additions to the top sector, and the top quark mass sets a scale for flavor hierarchy.
- Through $SU(2)_L$, the coupling between Z and b_L feels these modifications and the well-measured Z-pole observables (ex: $\Gamma(Z \to b_L \overline{b}_L)$ and A^b_{LR}) serve as strong constraints.
- In this talk, we elucidate the *gaugeless* technique of calculating *flavor-dependent* corrections to $Z \to b_L \overline{b}_L$, and apply this technique to the Three-Site Higgsless Model.
- The reference contains many other ways of understanding the result of this calculation.

$Z \to b_L \bar{b}_L$ in the SM

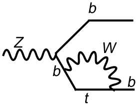
The brute-force calculation is straightforward ($m_b = 0$, $m_W^2 \ll m_t^2$):



$$= \mathrm{i}\,\frac{g_Z m_t^2}{192\pi^2 v^2}\left[6\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m_t^2}\right)(4s_W^2 - 4) + (20s_W^2 - 12)\right]$$



$$= \mathrm{i}\,\frac{g_Z m_t^2}{192\pi^2 v^2}\left[6\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m_t^2}\right)(7 - 6s_W^2) + (39 - 30s_W^2)\right]$$



$$= \mathrm{i}\,\frac{g_Z m_t^2}{192\pi^2 v^2}\left[6\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m_t^2}\right)(2s_W^2 - 3) + (10s_W^2 - 15)\right]$$

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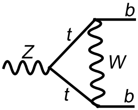

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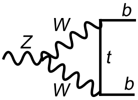

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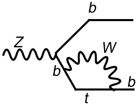

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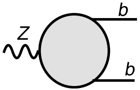


$$= \mathrm{i}\,\frac{g_Z m_t^2}{16\pi^2 v^2} \Leftarrow$$

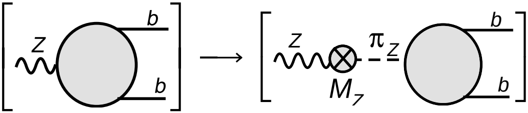
The result is finite, as expected, because there is no flavor-dependent counter-term in the SM.

The *Gaugeless* Calculation

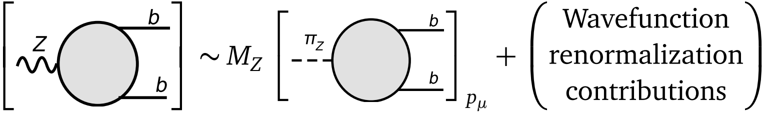
Barbieri, Beccaria, Ciafaloni, Curi, and Vicere.

Phys. Lett. **B288**, 95 (1992). Nucl. Phys. **B409**, 105 (1993).

The key idea: treat Z-boson as external field that couples to $J_3^\mu - s_\theta^2 J_Q^\mu$.

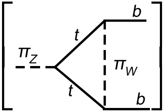


For the gaugeless calculation, we compute the coefficient to the operator $\partial^\mu \pi_Z \overline{b}_L \gamma_\mu b_L$ and relate to $Z \to b_L \overline{b}_L$ through a Ward-Takahashi identity

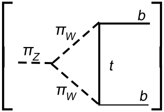


$\pi_Z \to b_L \overline{b}_L$ in the SM

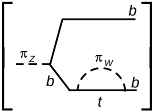
This is easier than the $Z \to b_L \overline{b}_L$ calculation:

M_Z 

p_μ $= i \dfrac{g_Z m_t^2}{16\pi^2 v^2}$

M_Z 

p_μ $= 0$ (No $\pi\pi\pi$-vertex)

M_Z 

p_μ $= 0$ (No $\pi \overline{b}_L b_R$-vertex when $m_b = 0$.)

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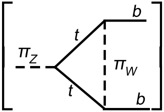
M_Z 

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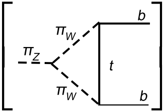
M_Z 

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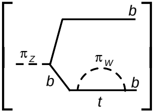
M_Z 

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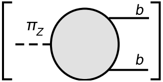
M_Z 

p_μ $= i \dfrac{g_Z m_t^2}{16\pi^2 v^2} \Leftarrow$ There is only one, finite diagram in the SM.

The *Gaugeless* Limit

Barbieri, Beccaria, Ciafaloni, Curi, and Vicere.

Phys. Lett. **B288**, 95 (1992). Nucl. Phys. **B409**, 105 (1993).

In the *gaugeless* limit, we treat the massive Z-boson as an external, classical field that couples to the conserved (before EWSB) current

$$J^\mu = -M_Z \partial^\mu \pi_z + \hat{J}^\mu,$$
$$\hat{J}^\mu = g^L_{Zbb} \overline{b_L} \gamma^\mu P_L b_L + g^R_{Zbb} \overline{b_R} \gamma^\mu P_R b_R + \cdots .$$

The Ward-Takahashi identity then reads

$$\partial^x_\mu \left\langle T\hat{J}^\mu(x) b(y) \bar{b}(z) \right\rangle = M_Z \left\langle T(\Box_x \pi_z(x)) b(y) \bar{b}(z) \right\rangle$$
$$- \delta(x-y) \left(g^L_{Zbb} P_L + g^R_{Zbb} P_R \right) \left\langle b(x) \bar{b}(z) \right\rangle$$
$$+ \delta(x-z) \left\langle b(y) \bar{b}(x) \right\rangle \left(g^L_{Zbb} P_R + g^R_{Zbb} P_L \right),$$

The *Gaugeless* Limit - cont.

In terms of amputated Green's functions, we have

$$\mathbb{i}(p_1 + p_2)_\mu \left\langle \hat{J}^\mu(p_1 + p_2) b(p_2) \bar{b}(p_1) \right\rangle_{1\text{PI}}$$

$$= -\mathbb{i} M_Z \left\langle \pi_Z(p_1 + p_2) b(p_2) \bar{b}(p_1) \right\rangle_{1\text{PI}}$$

$$- S_{bb}^{-1}(p_1) \left(g_{Zbb}^L P_L + g_{Zbb}^R P_R \right) + \left(g_{Zbb}^L P_R + g_{Zbb}^R P_L \right) S_{bb}^{-1}(-p_2),$$

The *Gaugeless* Limit - cont.

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$$
\begin{aligned}
&\mathbb{i}(p_1+p_2)_\mu \left\langle \hat{J}^\mu(p_1+p_2) b(p_2)\bar{b}(p_1) \right\rangle_{1\mathrm{PI}} \\
&= -\mathbb{i} M_Z \left\langle \pi_Z(p_1+p_2) b(p_2)\bar{b}(p_1) \right\rangle_{1\mathrm{PI}} \\
&\quad - S^{-1}_{bb}(p_1)\left(g^L_{Zbb}P_L + g^R_{Zbb}P_R\right) + \left(g^L_{Zbb}P_R + g^R_{Zbb}P_L\right) S^{-1}_{bb}(-p_2),
\end{aligned}
$$

We can organize the Ward-Takahashi identity with the Lorentz structure of the Green's functions

The *Gaugeless* Limit - cont.

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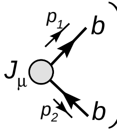
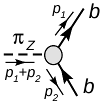
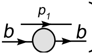
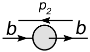
$$- S_{bb}^{-1}(p_1) \left(g^L_{Zbb} P_L + g^R_{Zbb} P_R \right) + \left(g^L_{Zbb} P_R + g^R_{Zbb} P_L \right) S_{bb}^{-1}(-p_2),$$

We can organize the Ward-Takahashi identity with the Lorentz structure of the Green's functions

$\mathbb{i}(\not{p}_1+\not{p}_2)$ ()$_{\gamma^\mu}$ $= -\mathbb{i} M_Z (\not{p}_1+\not{p}_2)$ ()$_{\not{p}}$

$-$ ()$^{-1}_{\not{p}}$ $\not{p}_1 \left(g^L_{Zbb} P_L + g^R_{Zbb} P_R \right) + \left(g^L_{Zbb} P_R + g^R_{Zbb} P_L \right)$ ()$^{-1}_{\not{p}}$ $\not{p}_2,$

At tree level, this gives the gauge couplings of the Z-boson.

The *Gaugeless* Limit - cont.

In terms of amputated Green's functions, we have

$$\mathbb{i}(p_1+p_2)_\mu \left\langle \hat{J}^\mu(p_1+p_2)b(p_2)\bar{b}(p_1)\right\rangle_{\rm 1PI}$$
$$= -\mathbb{i}M_Z \left\langle \pi_Z(p_1+p_2)b(p_2)\bar{b}(p_1)\right\rangle_{\rm 1PI}$$
$$- S^{-1}_{bb}(p_1)\left(g^L_{Zbb}P_L + g^R_{Zbb}P_R\right) + \left(g^L_{Zbb}P_R + g^R_{Zbb}P_L\right)S^{-1}_{bb}(-p_2),$$

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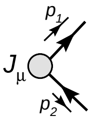
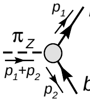
$\mathbb{i}(p_1+p_2)_\mu \Big($

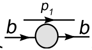


$\Big)_{\gamma^\mu\gamma_5}$ $(=0) = -\mathbb{i}M_Z \Big($

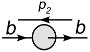


$\Big)_{\gamma_5}$

$-\Big($



$\Big)^{-1}_{\mathbb{1}} \left(g^L_{Zbb}P_L + g^R_{Zbb}P_R\right) + \left(g^L_{Zbb}P_R + g^R_{Zbb}P_L\right) \Big($



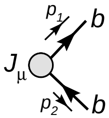
$\Big)^{-1}_{\mathbb{1}}$,

At tree level, this relates the $\pi_Z b\bar{b}$ coupling to m_b.

The Ward Identity at Loop-Level

The one-loop $Zb_L\overline{b_L}$ coupling is given by

$$i\, g_{Zbb}^{L,\text{1-loop}} = i \left(\quad \right)_{\gamma^\mu}$$



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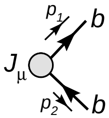
The one-loop $Zb_L\overline{b_L}$ coupling is given by

$$\mathbb{i}\, g^{L,\text{1-loop}}_{Zbb} = \mathbb{i}\sqrt{Z_b^L}\left(\quad \right)_{\gamma^\mu}\sqrt{Z_b^L}$$



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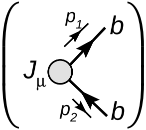
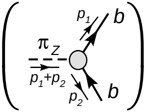
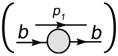


$$\sqrt{Z^L_b} \quad (\ldots)_{\gamma^\mu}$$

$$= -\,\mathbb{i}\, M_Z \sqrt{Z^L_b}$$



$$\Big|_{\not p} \sqrt{Z^L_b} - \left(g^L_{Zbb}\right) \sqrt{Z^L_b}$$



$$\left(\ldots\right)^{-1}_{\not p} \sqrt{Z^L_b}$$

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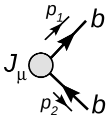
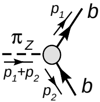
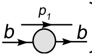
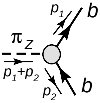


$$= -\mathrm{i}\, M_Z \sqrt{Z_b^L}\left(\quad \right)_{\not{p}} \sqrt{Z_b^L} - \left(g_{Zbb}^L\right)\sqrt{Z_b^L}\left(\quad \right)_{\not{p}}^{-1}\sqrt{Z_b^L}$$





$$= -\mathrm{i}\, M_Z \left(\quad \right)_{\not{p}} + \mathrm{i}\left(g_{Zbb}^{L,\text{tree}}\right)\left(1+\frac{1}{2}\delta Z_b^L\right)\left(1-\delta Z_b^L\right)\left(1+\frac{1}{2}\delta Z_b^L\right)$$



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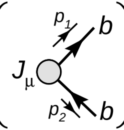
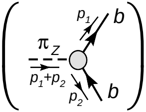
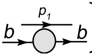
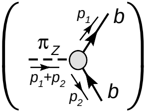
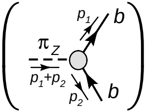
$\mathbb{i}\, g_{Zbb}^{L,\text{1-loop}} = \mathbb{i} \sqrt{Z_b^L}$  $\Big|_{\gamma^\mu} \sqrt{Z_b^L}$

$= -\,\mathbb{i}\, M_Z \sqrt{Z_b^L}$  $\Big|_{\not p} \sqrt{Z_b^L} - \left(g_{Zbb}^L\right) \sqrt{Z_b^L}$  $\Big|_{\not p}^{-1} \sqrt{Z_b^L}$

$= -\,\mathbb{i}\, M_Z$  $\Big|_{\not p} + \mathbb{i}\left(g_{Zbb}^{L,\text{tree}}\right)\left(1 + \frac{1}{2}\delta Z_b^L\right)\left(1 - \delta Z_b^L\right)\left(1 + \frac{1}{2}\delta Z_b^L\right)$

$= \mathbb{i}\, g_{Zbb}^{L,\text{tree}} - \mathbb{i} M_Z$  $\Big|_{\not p}$

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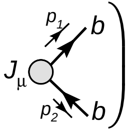
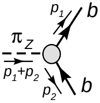
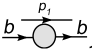
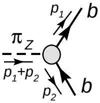


$$\left.\right)_{\gamma^\mu}\sqrt{Z^L_b}$$

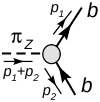
$$= -\,\mathbb{i}\, M_Z \sqrt{Z^L_b}\left(\right.$$


$$\left.\right)_{\not p}\sqrt{Z^L_b} - \left(g^L_{Zbb}\right)\sqrt{Z^L_b}\left(\right.$$


$$\left.\right)^{-1}_{\not p}\sqrt{Z^L_b}$$

$$= -\,\mathbb{i}\, M_Z \left(\right.$$


$$\left.\right)_{\not p} + \mathbb{i}\left(g^{L,\text{tree}}_{Zbb}\right)\left(1+\frac{1}{2}\delta Z^L_b\right)\left(1-\delta Z^L_b\right)\left(1+\frac{1}{2}\delta Z^L_b\right)$$

$$= \mathbb{i}\, g^{L,\text{tree}}_{Zbb} - \mathbb{i} M_Z \left(\right.$$


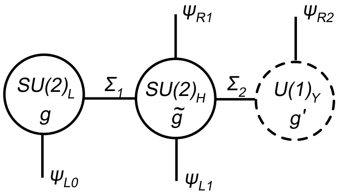
$$\left.\right)_{\not p} \Leftarrow$$

This justifies the earlier calculation.

The Three-Site Higgsless Model - Set up

Chivukula, Coleppa, Di Chiara, Simmons, He, Kurachi, and Tanabashi. PRD **74**, 075011 (2006).

*Additional references in the paper.



Three-site → maximally deconstructed.

Higgsless → non-linear Σ-fields.

Implements *ideal delocalization*:
$\alpha S = 0$ at tree-level.

$$\mathscr{L}_G = \frac{f_1^2}{4}\mathrm{Tr}\left[\left(D_\mu\Sigma_1\right)^\dagger\left(D_\mu\Sigma_1\right)\right] + \frac{f_2^2}{4}\mathrm{Tr}\left[\left(D_\mu\Sigma_2\right)^\dagger\left(D_\mu\Sigma_2\right)\right],$$

$$\Sigma = \mathrm{Exp}\left[\mathrm{i}\frac{\pi_1^A}{f_1}\sigma^A\right], \quad D^\mu\Sigma_1 = \partial^\mu\Sigma_1 + \mathrm{i}\tilde{g}\left(W_H^\mu\cdot\Sigma_1\right) - \mathrm{i}g\left(\Sigma_1\cdot W_L^\mu\right),$$

$$-\mathscr{L}_F = M_D\left(\epsilon_L\overline{\psi}_{L0}\Sigma_1\psi_{R1} + \overline{\psi}_{L1}\psi_{R1} + \epsilon_R\overline{\psi}_{L1}\Sigma_2\psi_{R2}\right)$$

The Three-Site Higgsless Model - Spectra and Features

Spectra and mass-generation of the Three-Site model:

- Additional gauge bosons $M_{W',Z'} \sim \tilde{g}f > 400$ GeV
  Additional fermions $M_F \sim M_D \gtrsim 3$ TeV.
- SM fermion masses generated through a seesaw-type mechanism ($M_f \sim \epsilon_L \epsilon_R M_D$).
- Flavor-dependencies stored in ϵ_R: ϵ_L is flavor-blind at tree-level and crucial for implementing *ideal-delocalization*.

The new ingredients to $Z \to b_L \overline{b_L}$ in 3SHM

- The physical states are no longer gauge eigenstates, even at tree-level.
- The presence of off-diagonal couplings to the Z (e.g. $Z_\mu \overline{B}_L \gamma^\mu b_L$).
- We can not take the *gaugeless* limit for $SU(2)_h$: it remains gauged.

The Ward-Takahashi Identity in 3-Site Higgsless Model

The current now contains extra contributions ...

$$J^\mu_{3S} = \hat{J}^\mu_{3S} - M_Z \partial^\mu \pi_Z,$$
$$\hat{J}^\mu_{3S} = \bar{b}\gamma^\mu g_{Zbb} b + \left(\bar{B}\gamma^\mu g_{ZBb} b + \text{h.c.}\right) + \cdots .$$
$$g_{Zbb} \equiv g^L_{Zbb} P_L + g^R_{Zbb} P_R, \quad \tilde{g}_{Zbb} \equiv g^L_{Zbb} P_R + g^R_{Zbb} P_L,$$

and so does the Ward-Takahashi identity

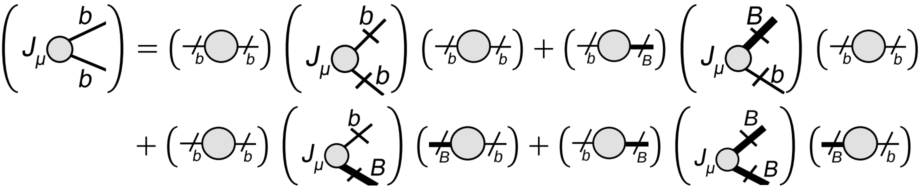
$$\begin{aligned}\partial^x_\mu \left\langle T\hat{J}^\mu_{3S}(x) b(y)\bar{b}(z)\right\rangle &= M_Z \left\langle T(\Box_x \pi_Z(x)) b(y)\bar{b}(z)\right\rangle \\ &\quad - \delta(x-y)\left[g_{Zbb}\left\langle b(x)\bar{b}(z)\right\rangle + g_{ZBb}\left\langle B(x)\bar{b}(z)\right\rangle\right] \\ &\quad + \delta(x-z)\left[\left\langle b(y)\bar{b}(x)\right\rangle \tilde{g}_{Zbb} + \left\langle b(y)\bar{B}(x)\right\rangle \tilde{g}_{ZBb}\right].\end{aligned}$$

Amputation with Kinetic Mixing

The Ward-Takahashi identity is a relationship among full Green's functions:

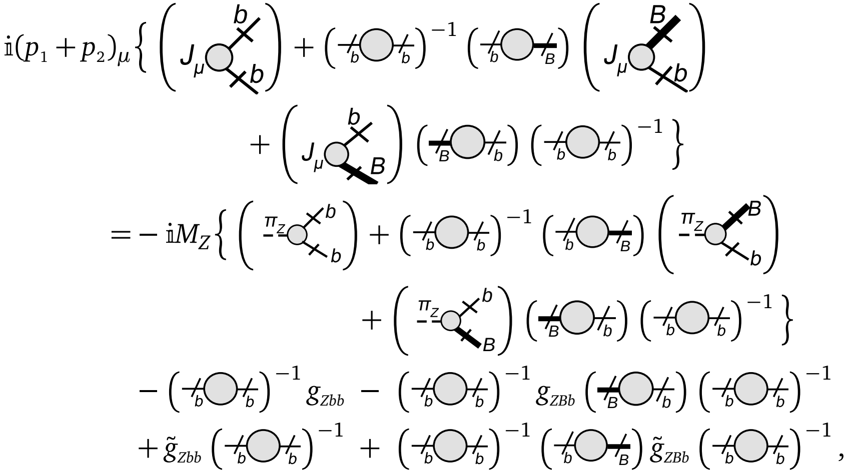
$$\begin{aligned}\partial^x_\mu \left\langle T\hat{J}^\mu_{3S}(x)b(y)\bar{b}(z)\right\rangle &= M_Z \left\langle T(\Box_x \pi_z(x))b(y)\bar{b}(z)\right\rangle \\ &\quad -\delta(x-y)\left[g_{Zbb}\left\langle b(x)\bar{b}(z)\right\rangle + g_{ZBb}\left\langle B(x)\bar{b}(z)\right\rangle\right] \\ &\quad +\delta(x-z)\left[\left\langle b(y)\bar{b}(x)\right\rangle \tilde{g}_{Zbb} + \left\langle b(y)\bar{B}(x)\right\rangle \tilde{g}_{ZBb}\right].\end{aligned}$$

With kinetic-mixing, we have to properly amputate full Green's functions.



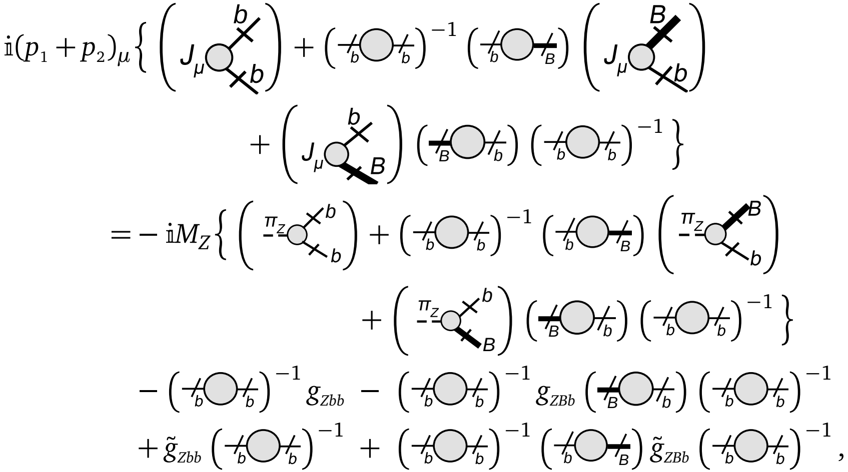
Some Intermediate Steps

Compared to the SM, we have many more terms ...



Some Intermediate Steps

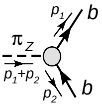
Compared to the SM, we have many more terms ... but they are all one-loop at leading order.



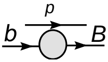
Kinetic Mixing

The final result

$$\mathbb{i}\, g^{L,\text{1-loop}}_{Zbb,3S} = \mathbb{i}\, g^{L,\text{tree}}_{Zbb} - \mathbb{i} M_Z \Bigg[\Bigg($$



$$\Bigg)_{\gamma^\mu} - \frac{g_{\pi_Z \overline{B_L} b_L}}{M_B} \Bigg($$



$$\Bigg)_{\not p} \Bigg] ,$$

$$= g^{L,\text{tree}}_{Zbb} + g_Z \frac{m_t^2}{16\pi^2 v^2} \left(1 + \frac{1}{8} \ln \frac{\Lambda^2}{M_D^2} \right) .$$

Some features:

- Only kinetic-mixing (not mass-mixing) enters the calculation: we are after the coefficient of the operator $(\partial_\mu \pi_Z)\overline{b_L}\gamma^\mu b_L$.
- Only $\ln(\Lambda^2/M^2)$ and not $\ln(M^2/m_t^2)$: We can understand $\ln(\Lambda^2/M^2)$ from RGE effects of ϵ_L, and there is no flavor-non-universal operator that gives further scaling below M.

Other Checks & Conclusions

In the paper, we obtain the same result and further insights via several other calculations:

- direct calculation in unitary gauge,
- renormalization group analysis,
- chiral current.

Other Checks & Conclusions

In the paper, we obtain the same result and further insights via several other calculations:

- direct calculation in unitary gauge,
- renormalization group analysis,
- chiral current.

We can also apply this technique to other models and derive constraints on the relevant parameters. One particular model is the general Universal Extra Dimension model (in progress: N. Christensen, T. Flacke, K. Hsieh, and A. Menon).