A Higher Derivative Lee-Wick Standard Model

Chris Carone
William and Mary

• C. D. Carone and R. F. Lebed,
"A Higher-Derivative Lee-Wick Standard Model,"
JHEP **0901**, 043 (2009) [arXiv:0811.4150 [hep-ph]].

And a brief advertisement for:

• C. D. Carone,
"Higher-Derivative Lee-Wick Unification,"
arXiv:0904.2359 [hep-ph].

The Lee-Wick Idea

Include higher-derivative (HD) kinetic terms for each field:

$$-\frac{1}{2}\hat{\phi}\,\Box\,\hat{\phi} \to -\frac{1}{2}\hat{\phi}\,\Box\,\hat{\phi} - \frac{1}{2M^2}\hat{\phi}\,\Box^2\hat{\phi}\,.$$

Propagators fall off more quickly with p,

$$\tilde{D}(p) = \frac{i}{p^2 - m^2 - p^4/M^2}$$

- In the Lee-Wick Standard Model (LWSM), quadratic divergences are eliminated.
- $N > 1$ poles in $\tilde{D}(p)$ indicate new states.
- Equivalent to theory without HD terms, but with LW partners for each particle.
- LWSM of Grinstein, et al., was an $N = 2$ theory. Here we consider $N = 3$ generalizations.

Gauge unification can be improved in $N = 3$ theories.

Real Scalar Field ($N = 2$)

Higher-derivative (HD) Lagrangian:

$$\mathcal{L}_{\text{HD}} = -\frac{1}{2}\hat{\phi}\,\Box\,\hat{\phi} - \frac{1}{2M^2}\hat{\phi}\,\Box^2\hat{\phi} - \frac{1}{2}m_\phi^2\hat{\phi}^2 + \mathcal{L}_{\text{int}}(\hat{\phi})\,.$$

Equivalent Auxiliary Field (AF) Lagrangian:

$$\mathcal{L}_{\text{AF}} = -\frac{1}{2}\hat{\phi}\,\Box\,\hat{\phi} - \frac{1}{2}m_\phi^2\hat{\phi}^2 - \tilde{\phi}\,\Box\,\hat{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 + \mathcal{L}_{\text{int}}(\hat{\phi})\,.$$

EOM gives: $\tilde{\phi} = \frac{1}{M^2}\Box\,\hat{\phi}$.

Kinetic terms diagonalized via $\hat{\phi} = \phi - \tilde{\phi}$:

$$\mathcal{L} = -\frac{1}{2}\phi\,\Box\,\phi + \frac{1}{2}\tilde{\phi}\,\Box\,\tilde{\phi} - \frac{1}{2}m_\phi^2(\phi - \tilde{\phi})^2 + \frac{1}{2}M^2\tilde{\phi}^2 + \mathcal{L}_{\text{int}}(\phi - \tilde{\phi})\,.$$

Mass matrix diagonalized via a symplectic transformation. The final Lagrangian:

$$\mathcal{L}_{\text{LW}} = -\frac{1}{2}\phi_0\,\Box\,\phi_0 + \frac{1}{2}\tilde{\phi}_0\,\Box\,\tilde{\phi}_0 - \frac{1}{2}m_0^2\phi_0^2 + \frac{1}{2}M_0^2\tilde{\phi}_0^2 + \mathcal{L}_{\text{int}}[e^{-\theta}(\phi_0 - \tilde{\phi}_0)]\,.$$

The LW partner $\tilde{\phi}_0$ corresponds to a physical particle; consistent with unitarity and macroscopic causality if LW particles decay.

Real Scalar Field ($N=3$)

$$\mathcal{L}_{\rm HD}^{N=3} = -\frac{1}{2}\widehat{\phi}\,\Box\,\widehat{\phi} - \frac{1}{2M_1^2}\widehat{\phi}\,\Box^2\widehat{\phi} - \frac{1}{2M_2^4}\widehat{\phi}\,\Box^3\widehat{\phi} - \frac{1}{2}m_\phi^2\widehat{\phi}^2 + \mathcal{L}_{\rm int}(\widehat{\phi})\,.$$

We desire

$$\mathcal{L}_{\rm LW}^{N=3} = \sum_{i=1}^{3} c_i\left[-\frac{1}{2}\phi^{(i)}(\Box + m_i^2)\phi^{(i)}\right] + \mathcal{L}_{\rm int}(\{\phi^{(i)}\})\,,$$

where the $c_i = \pm 1$ and $m_i^2 > 0$. Example: consider $m_\phi = 0$ ($m_1 = 0$). The missing link:

$$\mathcal{L}_{\rm AF} = -\frac{1}{2}\widehat{\phi}\,\Box\,\widehat{\phi} - \chi\,\Box\,\widehat{\phi} + m_2 m_3\,\chi\psi - \frac{1}{2}\psi\,\Box\,\psi - \frac{1}{2}(m_2^2 + m_3^2)\,\psi^2 + \mathcal{L}_{\rm int}(\widehat{\phi})\,.$$

Linear in the AF χ, yielding the constraint

$$\psi = \Box\,\widehat{\phi}/(m_2 m_3)\,.$$

Substituting, one obtains

$$\mathcal{L}_{\rm HD} = -\frac{1}{2m_2^2 m_3^2}\widehat{\phi}\,\Box\,(\Box + m_2^2)(\Box + m_3^2)\,\widehat{\phi} + \mathcal{L}_{\rm int}(\widehat{\phi})\,.$$

i.e., $M_1^2 = m_2^2 m_3^2/(m_2^2 + m_3^2)$ and $M_2^4 = m_2^2 m_3^2$.

The LW form is obtained via field redefinitions

$$\hat{\phi} = \phi^{(1)} - \frac{1}{(m_3^2 - m_2^2)^{1/2}} \left[m_3 \phi^{(2)} - m_2 \phi^{(3)} \right] ,$$

$$\chi = \frac{1}{(m_3^2 - m_2^2)^{1/2}} \left[m_3 \phi^{(2)} - m_2 \phi^{(3)} \right] ,$$

$$\psi = \frac{1}{(m_3^2 - m_2^2)^{1/2}} \left[m_2 \phi^{(2)} - m_3 \phi^{(3)} \right] .$$

Substitute in the AF Lagrangian:

$$\mathcal{L} = -\frac{1}{2} \phi^{(1)} \Box \, \phi^{(1)} + \frac{1}{2} \phi^{(2)} (\Box + m_2^2) \, \phi^{(2)} - \frac{1}{2} \phi^{(3)} (\Box + m_3^2) \, \phi^{(3)} + \mathcal{L}_{\text{int}}(\hat{\phi}) \, .$$

- Easily generalizable to m_ϕ, $m_1 \neq 0$.
- Second LW partner is a normal (positive norm) particle!
- Field redefinitions imply very specific form for the interaction terms, $\mathcal{L}_{\text{int}}(\hat{\phi})$.

$N = 3$ LWSM: One example

Pure SU(N) Yang Mills sector:

$$\mathcal{L}_{\text{HD}} = -\frac{1}{2}\,\text{Tr}\,\widehat{F}_{\mu\nu}\widehat{F}^{\mu\nu} - \frac{1}{M_1^2}\text{Tr}\widehat{F}_{\mu\nu}\widehat{D}^{\mu}\widehat{D}_{\alpha}\widehat{F}^{\alpha\nu}$$
$$-\frac{1}{M_2^4}\,\text{Tr}\widehat{F}_{\mu\nu}\widehat{D}^{\mu}\widehat{D}_{\alpha}\widehat{D}^{[\alpha}\widehat{D}_{\beta}\widehat{F}^{\beta\nu]}\,,$$

$M_{1,2}$ defined as in scalar example, brackets antisymmetrize the first and last indices only,

$$\begin{aligned} \widehat{F}^{\mu\nu} &\equiv \partial^{\mu}\widehat{A}^{\nu} - \partial^{\nu}\widehat{A}^{\mu} - ig\,[\widehat{A}^{\mu}, \widehat{A}^{\nu}]\,, \\ \widehat{D}^{\mu}X &\equiv \partial^{\mu}X - ig\,[\widehat{A}^{\mu}, X]\,. \end{aligned}$$

Equivalent Lagrangian, new fields χ^{μ}, ω^{μ}:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{2}\,\text{Tr}\,\widehat{F}_{\mu\nu}\widehat{F}^{\mu\nu} - \text{Tr}\,\widehat{F}^{\mu\nu}(\widehat{D}_{\mu}\chi_{\nu} - \widehat{D}_{\nu}\chi_{\mu})$$
$$-\frac{1}{2}\,\text{Tr}\,(\widehat{D}_{\mu}\omega_{\nu} - \widehat{D}_{\nu}\omega_{\mu})^2 - 2m_2 m_3\,\text{Tr}\,\chi_{\mu}\omega^{\nu}$$
$$+(m_2^2 + m_3^2)\,\text{Tr}\,\omega_{\mu}\omega^{\mu}\,,$$

Original form recovered using χ^{μ} EOM:

$$\widehat{D}_{\nu}\widehat{F}^{\nu\mu} - m_2 m_3\,\omega^{\mu} = 0\,.$$

Field redefinitions yield the canonical LW form

$$
\begin{aligned}
A_1^\mu &\equiv \widehat{A}^\mu + \chi^\mu\,, \\
A_2^\mu &\equiv \frac{1}{(m_3^2 - m_2^2)^{1/2}}\left[m_3\,\chi^\mu - m_2\,\omega^\mu\right]\,, \\
A_3^\mu &\equiv \frac{1}{(m_3^2 - m_2^2)^{1/2}}\left[m_2\,\chi^\mu - m_3\,\omega^\mu\right]\,.
\end{aligned}
$$

• A_1 transforms as a gauge field, $A_{2,3}$ as adjoint matter.

Define $F_1^{\mu\nu}$ and D^μ with $\widehat{A}^\mu \to A_1^\mu$, then

$$
\mathcal{L}_{\mathrm{YM,\,LW}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2\,,
$$

$$
\begin{aligned}
\mathcal{L}_0 = &-\frac{1}{2}\,\mathrm{Tr}\,F_1^{\mu\nu}F_{1\mu\nu} + \frac{1}{2}\,\mathrm{Tr}(D_\mu A_{2\nu} - D_\nu A_{2\mu})^2 \\
&-\frac{1}{2}\,\mathrm{Tr}(D_\mu A_{3\nu} - D_\nu A_{3\mu})^2 - m_2^2\,\mathrm{Tr}A_2^\mu A_{2\mu} \\
&+m_3^2\,\mathrm{Tr}A_3^\mu A_{3\mu}\,.
\end{aligned}
$$

• A_1 is massless ($m_1 = 0$), only A_2 has wrong-sign quadratic terms.

• There are numerous new three- and four-boson interactions:

$$
\begin{aligned}
\mathcal{L}_1 = & \frac{-ig}{m_3^2 - m_2^2} \text{Tr}\left(F_{1\mu\nu}\left[m_3 A_2^\mu - m_2 A_3^\mu,\, m_3 A_2^\nu - m_2 A_3^\nu\right]\right) \\
& + \frac{ig}{(m_3^2 - m_2^2)^{1/2}} \{\text{Tr}\left(D_\mu A_{2\nu} - D_\nu A_{2\mu}\right) \cdot \\
& \qquad \left(2m_3\left[A_2^\mu, A_2^\nu\right] - m_2\left[A_2^\mu, A_3^\nu\right] - m_2\left[A_3^\mu, A_2^\nu\right]\right) \\
& + \text{Tr}\left(D_\mu A_{3\nu} - D_\nu A_{3\mu}\right) \cdot \\
& \qquad \left(2m_2\left[A_3^\mu, A_3^\nu\right] - m_3\left[A_2^\mu, A_3^\nu\right] - m_3\left[A_3^\mu, A_2^\nu\right]\right)\}
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{L}_2 = & \frac{g^2}{2(m_3^2 - m_2^2)^2} \times \{m_3^2(4m_2^2 - 3m_3^2)\text{Tr}\left[A_2^\mu, A_2^\nu\right]^2 \\
& + 2m_2^2 m_3^2 \,\text{Tr}\left[A_2^\mu, A_2^\nu\right]\left[A_{3\mu}, A_{3\nu}\right] \\
& + m_2^2(4m_3^2 - 3m_2^2)\text{Tr}\left[A_3^\mu, A_3^\nu\right]^2 \\
& + 2m_2 m_3(m_3^2 - 2m_2^2)\text{Tr}\left[A_2^\mu, A_2^\nu\right]\left(\left[A_{2\mu}, A_{3\nu}\right] + \left[A_{3\mu}, A_{2\nu}\right]\right) \\
& + 2m_2 m_3(m_2^2 - 2m_3^2)\text{Tr}\left[A_3^\mu, A_3^\nu\right]\left(\left[A_{2\mu}, A_{3\nu}\right] + \left[A_{3\mu}, A_{2\nu}\right]\right) \\
& + (m_2^4 - m_2^2 m_3^2 + m_3^4)\text{Tr}\left(\left[A_2^\mu, A_3^\nu\right] + \left[A_3^\mu, A_2^\nu\right]\right)\left(\left[A_{2\mu}, A_{3\nu}\right]\right. \\
& \left. + \left[A_{3\mu}, A_{2\nu}\right]\right)\} \,.
\end{aligned}
$$

Distinctive triple gauge boson couplings contribute to gauge coupling running.

One-loop unification: Predictions for $\alpha_3^{-1}(m_Z)$

model	$N=3$ fields	$\alpha_3^{-1}(m_Z)$	error
SM	-	14.04	$+50.8\sigma$
MSSM	-	8.55	$+2.9\sigma$
$N=2$ 1H LWSM	none	14.03	$+50.6\sigma$
$N=3$ 1H LWSM	all	13.76	$+48.3\sigma$
$N=2$ 8H LWSM	none	7.76	-4.01σ
$N=3$ 6H LWSM	all	7.85	-3.16σ
$N=2$ 1H LWSM	gluons	7.81	-3.55σ
$N=2$ 1H LWSM	gluons, 1 gen. quarks	8.40	$+1.55\sigma$
$N=2$ 1H LWSM	1 gen. LH fields	8.03	-1.66σ
$N=2$ 2H LWSM	LH leptons	7.76	-4.01σ
$N=2$ 2H LWSM	gluons, quarks, 1H	8.21	-0.06σ

$$\alpha_3^{-1}(m_Z)_{exp} = 8.2169 \pm 0.1148$$

Abbreviations used: H=Higgs doublets, gen.=generation, LH=left handed.

- Unification scale ranges from 4×10^7 to 4×10^8 GeV. No proton decay from dimension-6 operators.
- Some models are compatible with trinifinication.
- Some models require incomplete GUT multiplets + proton decay suppression: possibly realized via orbifold GUTs?