Quintessence and Gravitational Waves

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Gravitational waves

- Linearized wave equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = \frac{8\pi}{M_{pl}^{2}} T_{\mu\nu} \longrightarrow \partial_{\alpha}\partial^{\alpha} h_{\mu\nu} = \frac{16\pi}{M_{pl}^{2}} T_{\mu\nu}$$

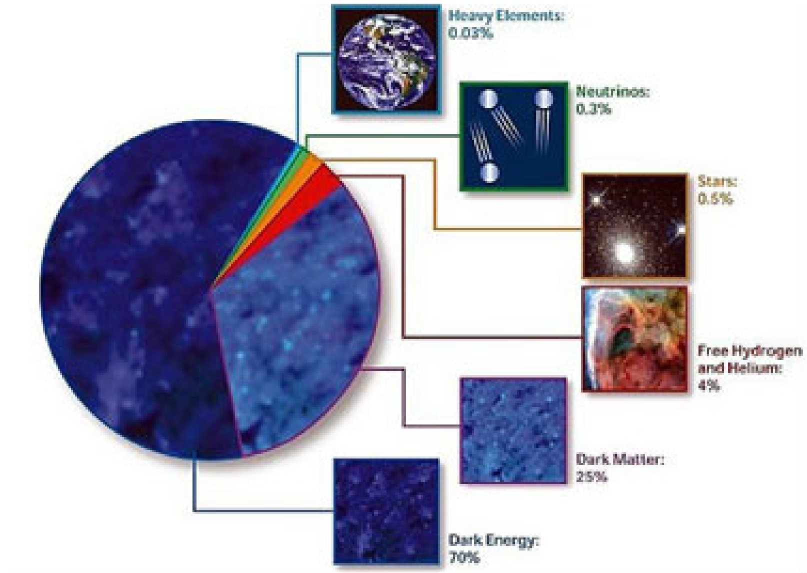
- GW couples to matter weakly
  - Propagate freely
  - Hard to detect

- Sources of GW (coordinated motion of huge mass)
  - Binary rotating stars, supernovae…
  - **First order phase transition (e.g. elelctroweak PT), by bubble collision and turbulence**

- **If some unknown energy exist during EWPT, how would the GW spectrum change?**

Quintessence

- Dark Energy could be…
  - Static? Cosmological constant : Λ
  - Dynamic? Quintessence

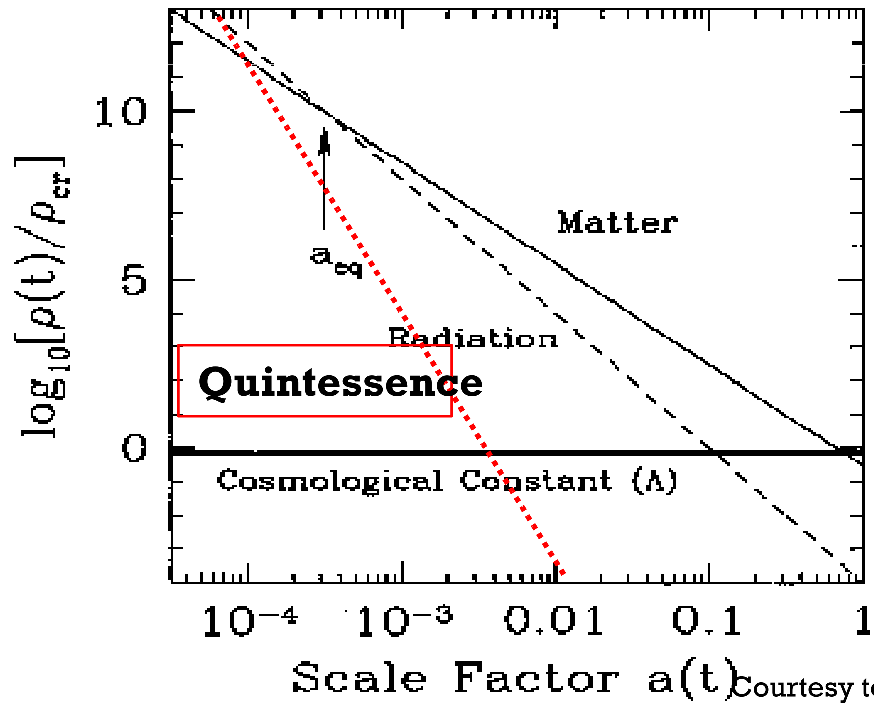


- Quintessence
  - Scalar field
  - Couples to matter by gravity

If kination domination[1]

  - Dilutes faster
  - Dominate in the past

$$\rho \propto \frac{1}{a^6}$$



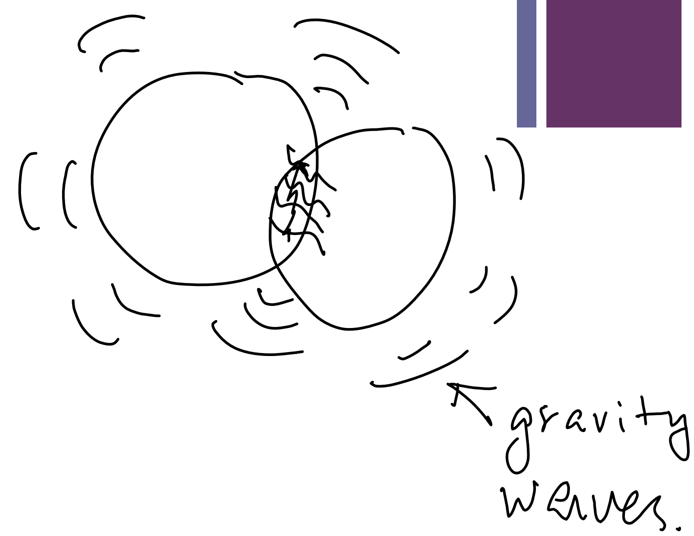
Courtesy to Scott Dodelson's Modern Cosmology

[1] Daniel J.H. Chung et. al. *Connecting LHC, ILC, and Quintessence*. JHEP 0710:016,2007.

+ Literature review: Bubble Collision

- How to describe bubble nucleation?
  - Universe cools down
    -> nucleation start
    -> nucleate faster and faster as T drops

$$\Gamma \propto \exp\left[\left(-S_s^{(3)} - (t - t_*)\frac{dS^{(3)}}{dt}\Big|_{t_*}\right)/T\right]$$



- How to describe one bubble's growth[1]?
  - Bubble wall velocity V_w
  - Interior fluid motion and temperature
- How to describe two bubbles collision?
  - Simulation[2]: Thin-wall approximation, envelope approximation
  - Analytic method[3]:
    velocity correlator <vvvv> → stress tensor correlator <TT> → GW's energy tensor

[1] Paul Steinhardt, *Relativistic Detonation waves and bubble growth in false vacuum decay*. PRD V25 #8

[2] Kamionkowski , Arthur Kosowsky, Michael S. Turner , *Gravitational radiation from first order phase transitions.* Phys.Rev.D49:2837-2851,1994

[3] Chiara Caprini , Ruth Durrer , Geraldine Servant *Gravitational wave generation from bubble collisions in first-order phase transitions: An analytic approach Phys.Rev.D77:124015,2008.*

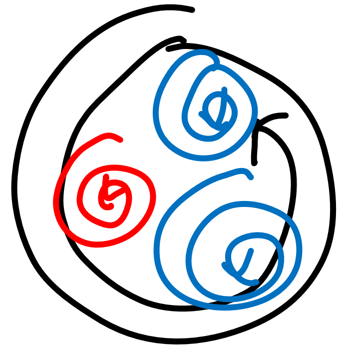
+ Literature review: Turbulence

- With quadrupole approximation for GW generation and dimension analysis for turbulence, Kamionkowski et al[1] derived:

$$\frac{d\rho_{GW}}{\rho d\log(\omega)} \sim \left(\frac{H}{\beta}\right)^2 v_b v_0^6 \left(\frac{\omega}{\omega_0}\right)^{-9/2}$$

- An analytic approach by Durrer et al[2], velocity correlator <vv> → stress tensor correlator <TT> ~ <vv><vv>

$$\frac{d\Omega_G(k,\eta_0)}{d\log(k)} \approx \frac{9}{32\pi}(\mathcal{H}_* L)^2 \left(\frac{\Omega_T^*}{\Omega_{rad}^*}\right)^2 \Omega_{rad} \times \begin{cases} x^3/v_L^2 & \text{for } 0 < x < 1 \\ x^{-2}/v_L^2 & \text{for } 1 < x < (2v_L)^3 \\ 4x^{-8/3} & \text{for } (2v_L)^3 < x < (\frac{L}{\lambda}) \\ 0 & \text{otherwise} \end{cases}$$



Turbulence: superposition of big and small eddies.

1. Energy injection scale
2. Energy dissipation scale
3. Energy dissipation rate

Velocity of eddy of size l:

$$v_l \sim (\epsilon l)^{1/3}$$

[1]Kamionkowski , Arthur Kosowsky, Michael S. Turner , *Gravitational radiation from first order phase transitions.* Phys.Rev.D49:2837-2851,1994
[2]Chiara Caprini, Ruth Durrer *Gravitational waves from stochastic relativistic sources: Primordial turbulence and magnetic fields.* Phys.Rev.D74:063521,2006.

+ Not sure yet..

- Bubble wall velocity uncertainty due to friction (G. Moore[1])
  - Once the plasma inside th e bubble gains mass, it will slow down the bubble wall velocity, breaks down the detonation scenario.
- The predicted GW spectrum is only firm in the large scale region(small k region), where the sources are evenly distributed. Regarding the peak position and the small distance region, it is still under investigation[2].
- Our result is a general statement, not sensitive to these uncertainties.

[1]G.Moore *Electroweak bubble wall friction: Analytic results. JHEP 0003:006,2000.* **hep-ph/0001274**
[2] Chiara Caprini, Ruth Durrer, Thomas Konstandin, Geraldine Servant , **General Properties of the Gravitational Wave Spectrum from Phase Transitions. arXiv:0901.1661** [astro-ph]

Research Method

- Identify all the factors affecting the GW spectrum, i.e. the parametric dependence of the spectrum. And find out the key factor controlling the spectrum's peak, amplitude and shape.

- Find out how the addition of quintessence will change these parameters and the subsequently the spectrum.

Bubble Collision

$$\rho_{GW} \sim M_p^2 \langle \dot{h}_{ij} \dot{h}_{ij} \rangle$$

$\Box h \sim T$

$$\rho_{GW} \sim \frac{1}{M_p^2} \left(\frac{a_*}{a}\right)^4 \left\langle \frac{d}{dt}\left(\frac{1}{\Box} T_{ij}\right) \frac{d}{dt}\left(\frac{1}{\Box} T_{ij}\right) \right\rangle |_{PT}$$

bubble kinetic E.

$$\left\langle \tilde{T}_{ij}(t_1', \vec{k}_1) \tilde{T}_{ij}^*(t_2', \vec{k}_2) \right\rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 - \vec{k}_2) P(k_1, t_1', t_2') \left[\rho_B^{rest} \gamma^2 v_w^2\right]^2$$

All uncertainties resides is here

$$\frac{d\rho_{GW}}{d\ln k} \sim \frac{1}{M_p^2} \left(\frac{a_*}{a}\right)^4 \left[\rho_B^{rest} \gamma^2 v_w^2\right]^2 \int dt_1' dt_2' \cos\left[k(t_1' - t_2')\right] \left[k^3 P(k, t_1', t_2')\right]$$

Assume only one scale exist, the duration of PT

$$F_{k\Delta t}((t_1' - t_*)/\Delta t, (t_2' - t_*)/\Delta t) \equiv k^3 P(k, t_1', t_2').$$

$$\begin{aligned} \frac{d\rho_{GW}}{d\ln k} &\sim \frac{1}{M_p^2} \left(\frac{a_*}{a}\right)^4 \left[\rho_B^{rest} \gamma^2 v_w^2\right]^2 \int dt_1' dt_2' \cos\left[k(t_1' - t_2')\right] F_{k\Delta t}((t_1' - t_*)/\Delta t, (t_2' - t_*)/\Delta t) \\ &= \underbrace{\frac{1}{M_p^2} \left(\frac{a_*}{a}\right)^4 \left[\rho_B^{rest} \gamma^2 v_w^2\right]^2 (\Delta t)^2}_{\text{magnitude}} \underbrace{\int dq_1' dq_2' \cos\left[k\Delta t(q_1' - q_2')\right] F_{k\Delta t}(q_1', q_2')}_{\text{shape}} \end{aligned}$$

+

$$\frac{1}{M_p^2}\left(\frac{a_*}{a}\right)^4\left[\rho_B^{rest}\gamma^2 v_w^2\right]^2(\Delta t)^2\int dq_1' dq_2' \cos\left[k\Delta t(q_1'-q_2')\right]F_{k\Delta t}(q_1',q_2') \quad \rightarrow$$

$$\frac{1}{M_p^2}\left(\frac{a_*}{a}\right)^4\left[\rho_B^{rest}\gamma^2 v_w^2\right]^2(\Delta t/\xi)^2\int dq_1' dq_2' \cos\left[k\Delta t(q_1'-q_2')/\xi\right]F_{k\Delta t/\xi}(q_1',q_2')$$

$$\frac{d\rho_{GW}(k)}{d\ln k} \rightarrow \frac{1}{\xi^2}\frac{d\rho_{GW}(k/\xi)}{d\ln k}.$$

Turbulence

- comparable contribution as bubble collision
- Same parametric dependence on Hubble expansion rate.
- Turbulence has a new scale, the dissipation scale, but it only affect the GW spectrum's cut-off frequency. So previous conclusion still holds.

Detonation (from Durrer's model)

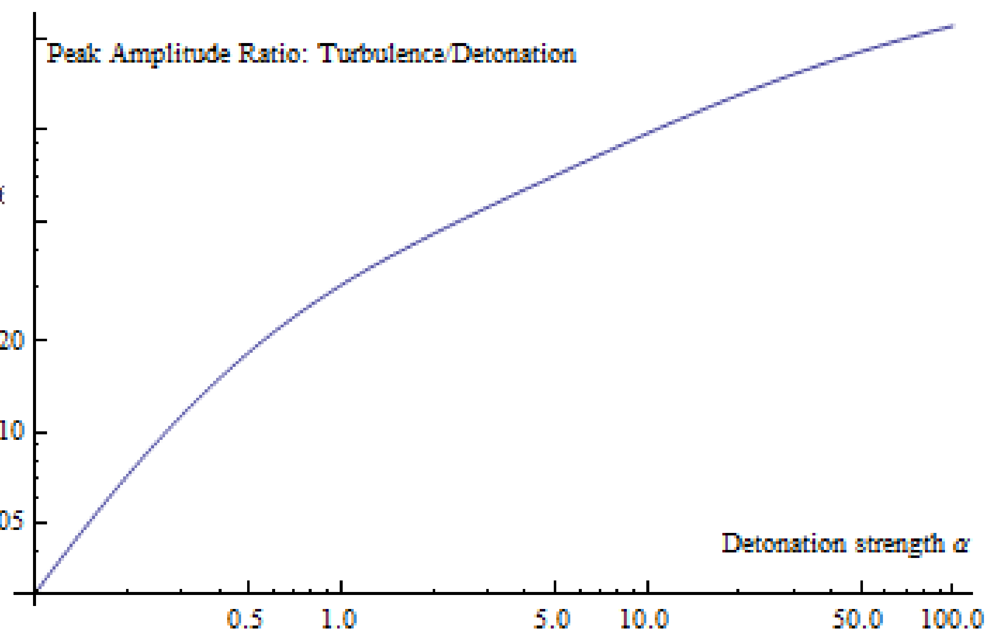
$$\left.\frac{d\Omega(k,\eta_0)h^2}{d\ln k}\right|_{\rm deto} \sim \frac{3}{2\pi^3}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}\Omega_{\rm rad}h^2\left(\frac{\Omega^*_{\rm kin}}{\Omega^*_{\rm rad}}\right)^2\left(\frac{\mathcal{H}_*}{\beta}\right)^2\frac{(1-s^3)^2}{s^4}\times {\rm int}|_{\rm deto}(Z)$$

ere $\quad {\rm int}|_{\rm deto}(Z) = \dfrac{0.21\left(\frac{Z}{Z_m}\right)^3}{1+\frac{1}{2}\left(\frac{Z}{Z_m}\right)^2+\left(\frac{Z}{Z_m}\right)^{4.8}}, \qquad Z_m = 3.8\,. \qquad$ (7

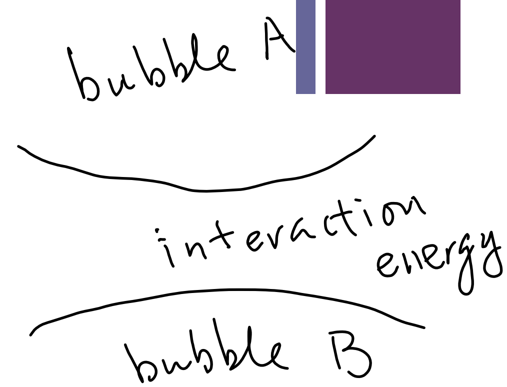
Turbulence(from Durrer's model)

$$\frac{d\Omega_G(k,\eta_0)}{d\log(k)} \approx \frac{9}{32\pi}(\mathcal{H}_*L)^2\left(\frac{\Omega_T^*}{\Omega_{rad}^*}\right)^2\Omega_{rad}\times$$

$$\begin{cases} x^3/v_L^2 & \text{for } 0 < x < 1 \\ x^{-2}/v_L^2 & \text{for } 1 < x < (2v_L)^3 \\ 4x^{-8/3} & \text{for } (2v_L)^3 < x < (\frac{L}{\lambda}) \\ 0 & \text{otherwise} \end{cases}$$



Interaction between bubbles



- So far we have approximating
  - No interaction between bubble before they collide
  - Only true for vacuum case, when bubble wall velocity=c

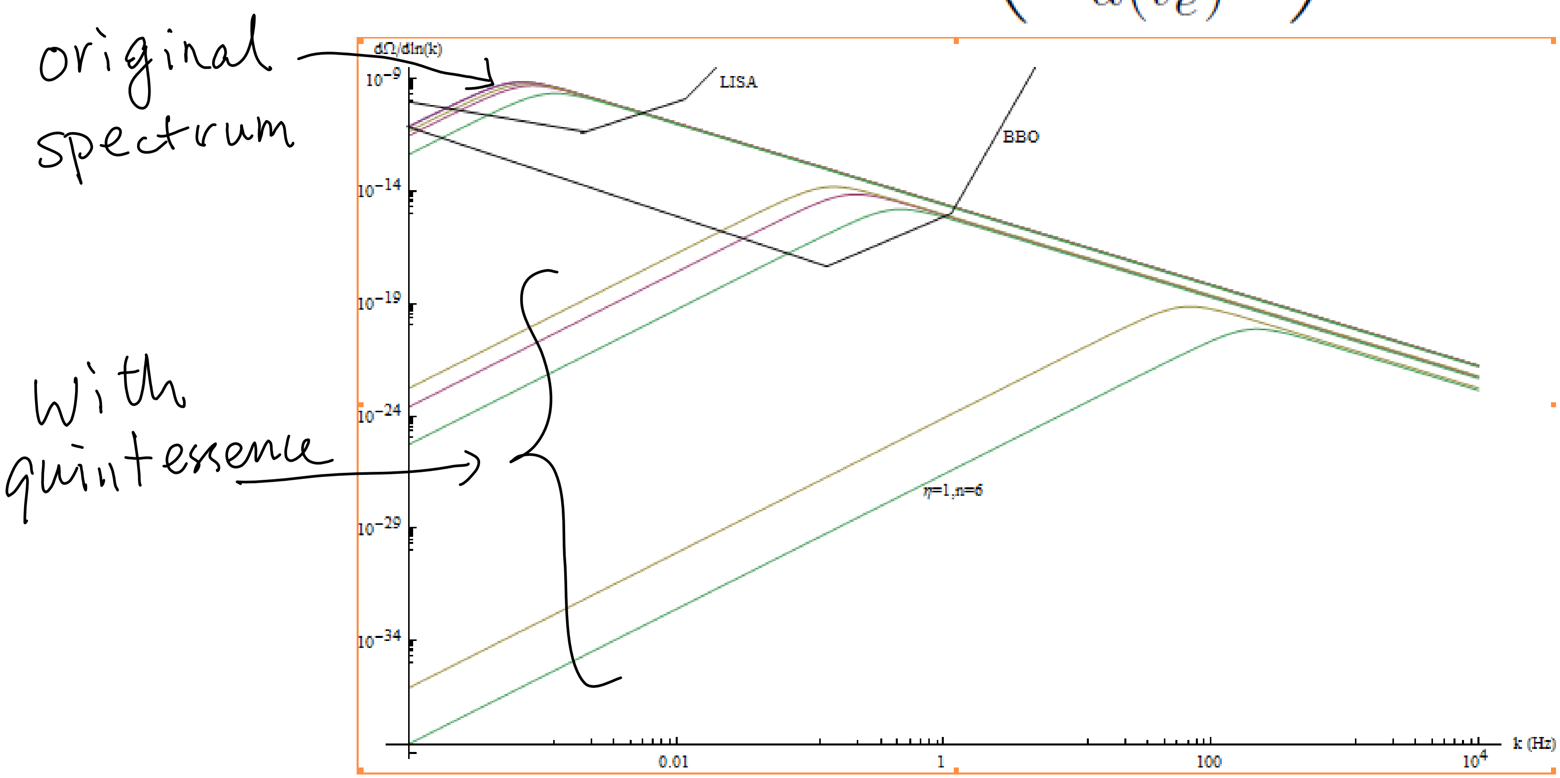
- If we consider interaction between bubble walls
  - exchange Z boson would give the dominant contribution.
  - But that contribution to stress tensor turns out to be negligible compared with bubble wall's stress tensor.

Interaction Energy $\ll$ Fluid energy

Our result

- Parametrize the quintessence's amplitude and evolution behavior as

$$\rho_q(t_e) = \rho_\gamma(t_{BBN})\eta\left(\frac{a(t_{BBN})}{a(t_e)}\right)^{n_q}$$



Summary

- The addition of quintessence will not alter the plasma's energy density, pressure or temperature.
- Quintessence will accelerate the expansion rate of universe then, make the universe cool down faster, thus shortening the phase transition duration
- Quintessence will shift the peak of the spectrum and lower the amplitude according to

$$\frac{d\rho_{GW}(k)}{d\ln k} \rightarrow \frac{1}{\xi^2}\frac{d\rho_{GW}(k/\xi)}{d\ln k}.$$