A Novel Origin of CP Violation

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Tri-bimaximal Mixing

- Neutrino Oscillation Parameters (2σ) — Schwetz, Tortola, Valle (Aug 2008)

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sin^2\theta_{23} = 0.5^{+0.14}_{-0.12}, \quad \sin^2\theta_{12} = 0.304^{+0.044}_{-0.032}$$

- indication for non-zero θ_{13}: — Bari group, June 2008

$$\sin^2\theta_{13} = 0.01^{+0.016}_{-0.011}\ (1\sigma)$$

consistent with $\theta_{13} = 0$

- Tri-bimaximal neutrino mixing: — Harrison, Perkins, Scott, 1999

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\sin^2\theta_{\rm atm,\,TBM} = 1/2 \qquad \sin\theta_{13,\rm TBM} = 0.$$

$$\sin^2\theta_{\odot,\rm TBM} = 1/3 \qquad \tan^2\theta_{\odot,\rm TBM} = 1/2$$

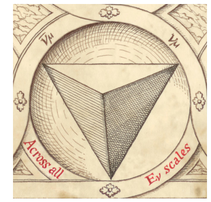
$$\tan^2\theta_{\odot,\rm exp} = 0.429$$

new KamLAND result: $\tan\theta^2_{\odot,exp} = 0.47^{+0.06}_{-0.05}$

Group Theory of T′

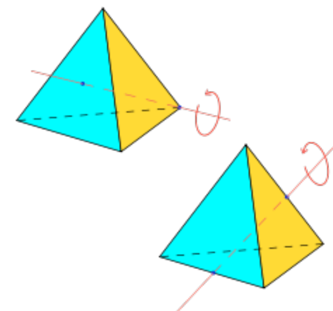
Frampton & Kephart, IJMPA (1995)

A4:



- even permutations of four objects

  S: (1234) → (4321) T: (1234) → (2314)
- geometrically -- invariant group of tetrahedron
- does NOT give rise to CKM mixing: $V_{ckm} = 1$
- all CG coefficients real



- Double covering of tetrahedral group A4:
  - in-equivalent representations of T’:

A4: $1,\ 1',\ 1'',\ 3$ → TBM for neutrinos

other: $2,\ 2',\ 2''$ → 2 +1 assignments for quarks

- generators:

$$S^2 = R,\ T^3 = 1,\ (ST)^3 = 1,\ R^2 = 1$$

R=1: $1,\ 1',\ 1'',\ 3$

R= -1: $2,\ 2',\ 2''$

Group Theory of T'

- product rules:

$$1^0 \equiv 1,\ 1^1 \equiv 1',\ 1^{-1} \equiv 1''$$

$$1^a \otimes r^b = r^b \otimes 1^a = r^{a+b} \qquad \text{for } r = 1, 2 \qquad a, b = 0, \pm 1$$
$$1^a \otimes 3 = 3 \otimes 1^a = 3$$
$$2^a \otimes 2^b = 3 \oplus 1^{a+b}$$
$$2^a \otimes 3 = 3 \otimes 2^a = 2 \oplus 2' \oplus 2''$$
$$3 \otimes 3 = 3 \oplus 3 \oplus 1 \oplus 1' \oplus 1''$$

J. Q. Chen & P. D. Fan, J. Math Phys 39, 5519 (1998)

★ complex CG coefficients in T'

complexity cannot be avoided by different basis choice

- spinorial x spinorial ⊃ vector:

$$2 \otimes 2 = 2' \otimes 2'' = 2'' \otimes 2' = 3 \oplus 1 \qquad 3 = \begin{pmatrix} \left(\frac{1-i}{2}\right)(\alpha_1\beta_2 + \alpha_2\beta_1) \\ i\alpha_1\beta_1 \\ \alpha_2\beta_2 \end{pmatrix}$$

- spinorial x vector ⊃ spinorial:

$$2 \otimes 3 = 2 \oplus 2' \oplus 2'' \qquad 2 = \begin{pmatrix} (1+i)\alpha_2\beta_2 + \alpha_1\beta_1 \\ (1-i)\alpha_1\beta_3 - \alpha_2\beta_1 \end{pmatrix}$$

A Novel Origin of CP Violation

- Conventionally:
  - Explicit CP violation: complex Yukawa couplings
  - Spontaneous CP violation: complex Higgs VEVs

★ complex CG coefficients in T' $\Rightarrow$ explicit CP violation

- real Yukawa couplings, real Higgs VEVs
- CP violation determined by complex CG coefficients
- no additional parameters needed $\Rightarrow$ extremely predictive model!!

The Model

- Symmetry: SU(5) x T′
- Particle Content $10(Q,\, u^c,\, e^c)_L$ $\overline{5}(d^c, \ell)_L$

	T_3	T_a	$\overline{F}$	H_5	$H'_{\overline{5}}$	Δ_{45}	ϕ	ϕ'	ψ	ψ'	ζ	N	ξ	η
SU(5)	10	10	$\overline{5}$	5	$\overline{5}$	45	1	1	1	1	1	1	1	1
T'	1	2	3	1	1	$1'$	3	3	$2'$	2	$1''$	$1'$	3	1
Z_{12}	ω^5	ω^2	ω^5	ω^2	ω^2	ω^5	ω^3	ω^2	ω^6	ω^9	ω^9	ω^3	ω^{10}	ω^{10}
Z'_{12}	ω	ω^4	ω^8	ω^{10}	ω^{10}	ω^3	ω^3	ω^6	ω^7	ω^8	ω^2	ω^{11}	1	1

$\omega = e^{i\pi/6}$.

- additional $Z_{12} \times Z'_{12}$ symmetry:
  - ★ predictive model: only 9 operators allowed up to at least dim-7
  - ★ vacuum misalignment: neutrino sector vs charged fermion sector
  - ★ mass hierarchy: lighter generation masses allowed only at higher dim

The Model

- Lagrangian: only 9 operators allowed!!

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Yuk}} &= \mathcal{L}_{\mathrm{TT}} + \mathcal{L}_{\mathrm{TF}} + \mathcal{L}_{\mathrm{FF}} \\
\mathcal{L}_{\mathrm{TT}} &= y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi \zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3 \\
\mathcal{L}_{\mathrm{TF}} &= \frac{1}{\Lambda^2} y_b H'_{\overline{5}} \overline{F} T_3 \phi \zeta + \frac{1}{\Lambda^3} \left[y_s \Delta_{45} \overline{F} T_a \phi \psi N + y_d H'_{\overline{5}} \overline{F} T_a \phi^2 \psi' \right] \\
\mathcal{L}_{\mathrm{FF}} &= \frac{1}{M_x \Lambda} \left[\lambda_1 H_5 H_5 \overline{F}\,\overline{F} \xi + \lambda_2 H_5 H_5 \overline{F}\,\overline{F} \eta \right] ,
\end{aligned}
$$

Λ: cutoff scale above which the family symmetry T' is exact
M_x: scale at which the lepton number violating operator is generated

Neutrino Sector

- Operators: $$\mathcal{L}_{\rm FF} = \frac{1}{M_x\Lambda}\left[\lambda_1 H_5 H_5 \overline{F}\,\overline{F}\xi + \lambda_2 H_5 H_5 \overline{F}\,\overline{F}\eta\right]$$
- Symmetry breaking:

$$T' \to G_{TST^2}: \qquad \langle\xi\rangle = \xi_0\,\Lambda\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad\qquad T' - \text{invariant:} \qquad \langle\eta\rangle = u_0\Lambda$$

- Resulting mass matrix:

$$M_\nu = \begin{pmatrix} 2\xi_0+u_0 & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & -\xi_0+u_0 \\ -\xi_0 & -\xi_0+u_0 & 2\xi_0 \end{pmatrix}\frac{\lambda v^2}{M_x}$$

only vector representations involved
⇒ all CG are real

⇒ Majorana phases either 0 or π

$$U^T_{\rm TBM} M_\nu U_{\rm TBM} = \text{diag}(u_0+3\xi_0, u_0, -u_0+3\xi_0)\frac{v_u^2}{M_X}$$

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form diagonalizable:
-- no adjustable parameters
-- neutrino mixing from CG coefficients!

General conditions for Form Diagonalizablility
in seesaw: M.-C. Chen, S. F. King, arXiv:0903.0125

Up Quark Sector

- Operators: $\mathcal{L}_{\rm TT} \;=\; y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi\zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3$
- top mass: allowed by T′
- lighter family acquire masses thru operators with higher dimensionality

  ➡ dynamical origin of mass hierarchy

- symmetry breaking:

$$T' \to G_T : \qquad \langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \;,\; \langle\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi'_0 \Lambda$$

dim-6

no contributions to elements involving 1st family; true to all levels

$$T' \to G_{TST^2} : \qquad \langle\phi'\rangle = \phi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

dim-7

- Mass matrix:

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u$$

both vector and spinorial reps involved
⇒ complex CG

Down Quark Sector

- operators: $$\mathcal{L}_{\rm TF} = \frac{1}{\Lambda^2} y_b H'_{\bar{5}} \overline{F} T_3 \phi\zeta + \frac{1}{\Lambda^3}\left[y_s \Delta_{45} \overline{F} T_a \phi\psi N + y_d H'_{\bar{5}} \overline{F} T_a \phi^2 \psi' \right]$$
- generation of b-quark mass: breaking of T′ : dynamical origin for hierarchy between m_b and m_t
- lighter family acquire masses thru operators with higher dimensionality

➡ dynamical origin of mass hierarchy

- symmetry breaking:

$$T' \to G_T : \quad \langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \, , \ \langle\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi'_0 \Lambda \qquad T' \to \text{nothing:} \quad \langle\psi'\rangle = \psi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- mass matrix:

$$M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi'_0 & 0 \\ -(1-i)\phi_0\psi'_0 & \psi_0 N_0 & 0 \\ \phi_0\psi'_0 & \phi_0\psi'_0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0, \qquad M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi'_0 & \phi_0\psi'_0 \\ (1+i)\phi_0\psi'_0 & -3\psi_0 N_0 & \phi_0\psi'_0 \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0$$

complex CG

- consider 2nd, 3rd families only: TBM exact
- Georgi-Jarlskog relations: $m_d \simeq 3m_e \qquad m_\mu \simeq 3m_s$ → corrections to TBM

Quark and Lepton Mixing Matrices

- CKM mixing matrix:

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u \qquad M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi_0' & 0 \\ -(1-i)\phi_0\psi_0' & \psi_0 N_0 & 0 \\ \phi_0\psi_0' & \phi_0\psi_0' & \zeta_0 \end{pmatrix} y_b v_d \phi_0,$$

V_{cb} $\qquad$ V_{ub}

$$\theta_c \simeq \left| \sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c} \right| \sim \sqrt{m_d/m_s},$$

Georgi-Jarlskog relations $\Rightarrow V_{d,L} \neq I$

SU(5) $\Rightarrow M_d = (M_e)^T$

$\Rightarrow$ corrections to TBM related to θ_c

- MNS matrix:

$$M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi_0' & \phi_0\psi_0' \\ (1+i)\phi_0\psi_0' & -3\psi_0 N_0 & \phi_0\psi_0' \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0 \Longrightarrow \theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

$$U_{\rm MNS} = V_{e,L}^\dagger U_{\rm TBM} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,\rm TBM} - \frac{1}{2}\theta_c\cos\delta \qquad \theta_{13} \simeq \theta_c/3\sqrt{2}$$

new QLC relation!

leptonic Dirac CPV

Numerical Results

- Experimentally: $m_d : m_s : m_b \simeq \theta_c^{4.7} : \theta_c^{2.7} : 1, \quad m_u : m_c : m_t \simeq \theta_c^{8} : \theta_c^{3.2} : 1,$
- Model Parameters:

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g+(1-\frac{i}{2})h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u$$

$$k \equiv y' \psi_0 \zeta_0 = -0.029$$
$$h \equiv \phi_0^2 = 0.008$$
$$g \equiv \phi_0'^3 = -9 \times 10^{-6}$$

7 parameters in charged fermion sector

$$\frac{M_d}{y_b v_d \phi_0 \zeta_0} = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & c & 0 \\ b & b & 1 \end{pmatrix}$$

$$y_b \phi_0 \zeta_0 \simeq m_b/m_t \simeq (0.011)$$
$$c \equiv \psi_0 N_0/\zeta_0 = -0.0169$$
$$b \equiv \phi_0 \psi_0'/\zeta_0 = 0.0029$$

$$y_t = 1$$

- CKM Matrix:

$$V_{CKM} = \begin{pmatrix} 0.975e^{-i26.8^o} & 0.225e^{i21.1^o} & 0.00293e^{i164^o} \\ 0.224e^{i124^o} & 0.974e^{-i8.19^o} & 0.032e^{i180^o} \\ 0.00557e^{i103^o} & 0.0317e^{-i7.33^o} & 0.999 \end{pmatrix}$$

$$\beta \equiv \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = 21.3^o, \ \sin 2\beta = 0.676 ,$$
$$\alpha \equiv \arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = 114^o ,$$
$$\gamma \equiv \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \delta_q = 44.9^o ,$$
$$J \equiv \mathrm{Im}(V_{ud}V_{cb}V_{ub}^*V_{cs}^*) = 1.45 \times 10^{-5} ,$$

$$\lambda = 0.225, \ A = 0.637, \ \overline{\rho} = 0.280 \ \text{and} \ \overline{\eta} = 0.280.$$

predicting:
9 masses, 3 mixing angles, 1 CP Phase

Numerical Results

- diagonalization matrix for charged leptons:

$$\begin{pmatrix} 0.997e^{i177^o} & 0.08e^{i132^o} & 1.2\times10^{-5}e^{-i45^o} \\ 0.08e^{i41.9^o} & 0.997e^{i177^o} & 1.40\times10^{-4}e^{-i3.47^o} \\ 10^{-6} & 1.4\times10^{-4} & 1 \end{pmatrix}$$

- MNS Matrix:

$$V_{\rm PMNS} = \begin{pmatrix} 0.837e^{-i179^o} & 0.544e^{-i173^o} & 0.0566e^{i138^o} \\ 0.364e^{-i3.86^o} & 0.609e^{-i173^o} & 0.705e^{i3.45^o} \\ 0.408e^{i180^o} & 0.577 & 0.707 \end{pmatrix}$$

→ $$\sin^2\theta_{\rm atm} = 1, \ \tan^2\theta_\odot = 0.422 \ \text{and} \ |U_{e3}| = 0.0566$$

prediction for Dirac CP phase: δ = -46.9 degrees

$$J_\ell = -0.0094$$

Note that these predictions do NOT depend on *u* and ξ_0

- neutrino masses:

$$u_0 = -0.0593, \quad \xi_0 = 0.0369, \quad M_X = 10^{14} \text{ GeV}$$
$$m_1 = 0.0156 \text{ eV}, \quad m_2 = 0.0179 \text{ eV}, \quad m_3 = 0.0514 \text{ eV}$$

2 parameters in neutrino sector

- Majorana phases $\alpha_{21} = \pi \qquad \alpha_{31} = 0.$

predicting:3 masses,
3 mixing angles, 3 CP Phases

Neutrino Mass Sum Rule

- sum rule among three neutrino masses:

$$m_1 - m_3 = 2m_2$$

- the mass eigenvalues:

$$\begin{aligned} m_1 &= u_0 + 3\xi_0 \\ m_2 &= u_0 \\ m_3 &= -u_0 + 3\xi_0 \end{aligned}$$

$$\begin{aligned} \Delta m^2_{atm} &\equiv |m_3|^2 - |m_2|^2 = -12u_0\xi_0 \\ \Delta m^2_{\odot} &\equiv |m_2|^2 - |m_1|^2 = -9\xi_0^2 - 6u_0\xi_0 \end{aligned}$$

- leads to sum rule

$$\Delta m^2_{\odot} = -9\xi_0^2 + \frac{1}{2}\Delta m^2_{atm} \longrightarrow \Delta m^2_{atm} > 0$$

normal hierarchy predicted!!

Summary

- SU(5) x T' symmetry: tri-bimaximal lepton mixing & realistic CKM matrix
- complex CG coefficients in T': origin of CPV both in quark and lepton sectors
- Z_{12} x Z_{12}': only 9 parameters in Yukawa sector
  - ★ dynamical origin of mass hierarchy (including m_b vs m_t)
  - ★ forbid proton decay
- interesting sum rules:

leptonic Dirac CP phase: δ = -46.9 degrees

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,\mathrm{TBM}} - \frac{1}{2}\theta_c\cos\delta$$

$$\theta_{13} \simeq \theta_c/3\sqrt{2} \sim 0.05$$

right amount to account for discrepancy between exp best fit value and TBM prediction